Wideband Spectral Matrix Filtering for Multicomponent Sensors Array

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Abstract

As multicomponent (3C or 4C) seismic acquisitions are now commonly used, specific processings for multicomponent datasets are required. The aim of this paper is to present a new wavefield filter for multicomponent seismic data. The proposed method is derived from a recent technique used for single component data, which takes into account the wideband characteristics of the signal. The multicomponent extended method incorporates the polarization information and processes the various components in a global way instead of treating them independently. The proposed method is based on the computation of a global spectral matrix which gathers all the frequent and component interactions. We prove that the eigenvalue decomposition of a reduced matrix issued from this global matrix enables an efficient separation in two complementary spaces: one contains the signal and the other, the noise. The results obtained on synthetic and real data are presented and compared with others methods in order to assess the performances of our algorithm.

Key words: Multicomponent sensors array, Subspace method, Wavefield filter, Polarized wave separation, Wideband spectral matrix, Eigenvalue decomposition (EVD)
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Key words Multicomponent sensors array, Subspace method, Wavefield filter, Polarized wave separation, Wideband spectral matrix, Eigenvalue decomposition (EVD)
1 Introduction

Separation of convolutive mixtures of wideband signals impinging on a linear array is a common problem in signal processing. In geophysical exploration where uniformly spaced linear arrays are widely used, the goals of data processing are the enhancement of signal to noise ratio, and the identification and separation of different wavefields to obtain a better imaging of the earth (for instance, reservoir imaging on seismic exploration) [19].

In case of single component sensors arrays, many techniques have already been developed. For instance, in the time-distance domain, median filters can be used to eliminate spikes but are not efficient to remove coherent noise [3]. Semblance filters [9] also provide efficient algorithm to remove noise and improve velocity estimation. Radon transform is also useful to remove noise or to separate events. In case of plane waves, linear $\tau - p$ [20,27,28,38] is efficient, whereas pseudo-hyperbolic Radon [7,38] provides better results to enhance reflection events in seismic reflection. F-K filters [4] transforms the data into the frequency-wavenumber domain where plane waves can be easily identified. Nonetheless, all these filters are deficient in presence of non-plane waves or in case of short arrays. Other separation techniques based on blind algorithm [31] or on Singular Value Decomposition [8] are also used to remove noise. However, in order to be efficient for wavefield separation, these methods require pre-processing such as wave alignment [1].

Filtering could also be performed in the frequency domain. Trickett has recently developed a method called F-x eigenfiltering (for the 2D case) or F-xy eigenfiltering for 3D seismic volumes [32]. For each frequency bin, each 2D slice ($x,y$) is replaced with the sum of the first weighted eigenimages in order to enhance signal to noise ratio. This method takes into account both spatial dimensions simultaneously. Another method, called spectral matrix filtering (SMF) or multidimensional filtering [18], decomposes an initial dataset into a new space free of noise, with the smallest possible dimension. Signal is separated from noise by finding a subspace defined by the eigenvectors relative to the dominant eigenvalues of the spectral matrix $\Gamma(f)$ issued from the recorded signal. Coherent energy maps onto the first eigenvectors whereas incoherent energy is more equally distributed. The eigenvalue decomposition allows a separation between signal and noise subspace at each frequency bin $f$. The signal is viewed as a juxtaposition of short band signals since each frequency bin is treated independently. Nevertheless, as seismic signals are wideband, the previous two methods (SMF and F-xy eigenfiltering) are not optimal to tackle this problem. Moreover, working independently on each frequency could cause problems on the subbands where signal is weak compared to noise. To take into account the broadband characteristics of the signal, wideband spectral matrix filtering has been developed in [12].
As more and more surveys use multicomponent sensors (geophones and hydrophone), specific filtering methods adapted to multicomponent dataset are required. 3CSVD [14], SVD of quaternion matrices [15], 3D-SVD-ICA compound with partial Independent Component Analysis [34] techniques are efficient to separate wavefield but require wave alignment as they are based on SVD. To avoid this pre-processing, we extend spectral matrix filtering to a multicomponent wideband spectral matrix filtering. This method is more suitable for multicomponent wideband signals and takes jointly into account polarization and interactions between all the frequencies and between all the components. Polarization diversity has been proven useful in several topics such as wireless communication, radar systems, identification of acoustics and elastic signals or direction finding for arrays [22,23,39]. A detailed analysis of polarized seismic wave is given in [2,21].

After this brief introduction, we will present in part 2 the classical spectral matrix and wideband spectral matrix filtering methods for single component sensors array. Then, an extension to the multicomponent case will be submitted in part 3 of this work. In this part, after describing the multicomponent model formulation and estimating the multicomponent wideband spectral matrix, the projection onto signal and noise subspaces is described. Finally, results are presented on synthetic and real data sets in order to evaluate performances of the algorithm.

2 Spectral matrix filtering methods for single component sensors array

2.1 Classical spectral matrix filtering (SMF)

Let us consider an uniform linear array composed of \( N_x \) omni-directional sensors receiving \( P \) waves with \( P < N_x \). A convolutive model of seismic signal was first suggested by Robinson [26]. Using the superposition principle, the signal \( x_k(t) \) recorded on sensor \( k \) is a linear combination of the \( P \) waves received on the array added with noise \( b_k(t) \). Into matrix notation, we write:

\[
\mathbf{X}(t) = \mathbf{S}(t) \ast \mathbf{A}(t) + \mathbf{B}(t)
\]

where:

- \( \mathbf{X}(t) = [x_1(t), x_2(t), \cdots, x_{N_x}(t)]^T \) is the recorded signal (\(^T\) stand for the transpose operation);
\( S(t) = [S_1(t), S_2(t), \cdots, S_P(t)] \) is a \( N_x \) by \( P \) matrix whose columns are
the steering vectors describing the propagation of each wave along the
receiver group with \( S_k(t) = [s_{1,k}(t), s_{2,k}(t), \cdots, s_{N_x,k}(t)]^T \). All these terms are
deterministic;
- \( A = [a_1, a_2, \cdots, a_P]^T \) is the amplitude of the \( P \) waves which follows a ran-
don process;
- \( B(t) = [b_1(t), b_2(t), \cdots, b_{N_x}(t)]^T \) with \( b_i(t) \) an additive random noise. These
noises are supposed to be temporally and spatially white, uncorrelated with
the sources, and to have identical power spectral density \( \sigma_b^2 \);

Applying the Fourier transform to (1), the problem is now divided into a set
of instantaneous mixtures of signals and becomes:
\[
\mathbf{X}(f) = \mathbf{S}(f) \cdot \mathbf{A} + \mathbf{B}(f)
\]
(2)
To reach information on steering vectors of \( \mathbf{S}(f) \), a classical way is to use
spectral matrix \( \Gamma(f) \) defined by:
\[
\Gamma(f) = E[\mathbf{X}(f) \cdot \mathbf{X}(f)^H]
\]
(3)
where \( ^H \) is the transpose conjugate operation and \( E \) the expectation operator.
Thus, we obtain a \( N_x \) by \( N_x \) spectral matrix for each frequency bin \( f \). It follows
from our assumptions that:
\[
\Gamma(f) = \mathbf{S}(f) \cdot \Gamma_A \cdot \mathbf{S}(f)^H + \sigma_b^2 \cdot \mathbf{I}
\]
(4)
with \( \sigma_b^2 \) the noise power spectral density and \( \Gamma_A = E[\mathbf{A} \cdot \mathbf{A}^H] \)

Notice that \( \Gamma_A \) is diagonal when sources are uncorrelated, non-diagonal and
non-singular when the signals are partially correlated, and non-diagonal but
singular when some sources are fully correlated. Assuming that the spacing
between the sensors is less than half a wavelength of the impinging wavefronts,
it follows that the columns of the matrix \( \mathbf{S}(f) \) are all different, and hence,
because of their Vandermonde structure, linearly independent. Thus, if \( \Gamma_A \) is
non-singular, then the rank of \( \mathbf{S}(f) \cdot \Gamma_A \cdot \mathbf{S}(f)^H \) is \( P \).
The eigenvalue decomposition of \( \Gamma(f) \) gives:
\[
\Gamma(f) = \sum_{i=1}^{N_x} \lambda_i(f) \mathbf{u}_i(f) \cdot \mathbf{u}_i^H(f)
\]
(5)
with \( \lambda_1(f) \geq \lambda_2(f) \geq \cdots \geq \lambda_{N_x}(f) \geq 0 \) are the eigenvalues and \( \mathbf{u}_1(f), \mathbf{u}_2(f), \cdots, \mathbf{u}_{N_x}(f) \)
their corresponding eigenvectors. Then, the above rank property implies that:

- the \( N_x - P \) minimal eigenvalues of \( \Gamma(f) \) are equal to \( \sigma_b^2 \):
\[
\lambda_{P+1}(f) = \lambda_{P+2}(f) = \cdots = \lambda_{N_x}(f) = \sigma_b^2
\]
(6)
- the eigenvectors corresponding to these minimal eigenvalues are orthonormal to the columns of the matrix $\mathbf{S}(f)$

The subspace generated by the smallest eigenvalues could be referred as the noise subspace and its orthogonal complement as the signal subspace since it is spanned by the steering vectors of the signal. This justifies the benefit of projecting data on the first eigenvectors to separate signal from noise [30].

Once $\mathbf{\Gamma}(f)$ is estimated by smoothing (see section 3.2), filtering is achieved using the following steps:

- For each frequency:
  * Eigenvalue decomposition of $\mathbf{\Gamma}(f)$ gives eigenvalues and their associated eigenvectors;
  * Selection of the $P$ first eigenvectors associated to the highest $P$ eigenvalues which corresponds to the signal subspace $= \text{Vect}(\mathbf{u}_1(f), \cdots, \mathbf{u}_P(f))$;
  * Projection of the initial data $\mathbf{X}(f)$ onto the signal subspace;

$$\sum_{i=1}^{P} \langle \mathbf{X}(f), \mathbf{u}_i(f) \rangle \cdot \mathbf{u}_i(f) \quad (7)$$

- Computation of the inverse Fourier transform of the projection.

The disadvantages of the spectral matrix filtering is that the method is only effective if the target wave is dominant, otherwise the projection onto the dominant eigenvalues and their corresponding eigenvectors will not always correspond to the selected wave for each frequency bin.

Moreover, since the seismic signal is broadband, the classical spectral matrix filtering has limitations. This implies that $\mathbf{\Gamma}(f)$ does not summarize all information related to the signal. The terms corresponding to the interaction between two different frequencies are not null. It leads to the introduction of the wideband spectral matrix.

### 2.2 Wideband spectral matrix filtering (WBSMF)

To take into account the broadband nature of seismic signal, one should not consider each frequency of the signal spectrum independently of the others. This can be achieved by using a spectral matrix called wideband spectral matrix and defined by:

$$\mathbf{\Gamma} = E[\mathbf{X} \cdot \mathbf{X}^H] \quad (8)$$

with

$$\mathbf{X} = [\mathbf{X}(f_1)^T, \mathbf{X}(f_2)^T, \cdots, \mathbf{X}(f_{N_f})]^T \quad (9)$$

$\mathbf{X}$ is a long-vector (dimension $N_x N_f$) with $N_f$, the number of frequency bins $f$ and $\mathbf{X}(f_i)$ is the vector defined in (2). Consequently, this long-vector corre-
sponds to the concatenation of all frequencies on all the sensors. By developing
the equation with the expression of $X(9)$, $\Gamma$ can be written as:

$$\Gamma = E \begin{pmatrix}
  X(f_1)X(f_1)^H & \cdots & X(f_1)X(f_{N_f})^H \\
  \vdots & \ddots & \vdots \\
  X(f_{N_f})X(f_1)^H & \cdots & X(f_{N_f})X(f_{N_f})^H
\end{pmatrix}$$ (10)

The wideband spectral matrix defined by equations (8) and (10) has the di-
mension $(N_x N_f \times N_x N_f)$. We can notice that all the previous terms $\Gamma(f_i)$, defined in (3), corresponding to the classical spectral matrix (narrow band) are present in the last expression of $\Gamma$ (10). They are located in the blocks of the main diagonal. The other blocks contain information relating to the interaction between frequencies $f_i$ and $f_j$. Consequently, by using them, it is possible to have more information on the signal and consequently to improve filtering results. A detailed description, of the structure of $\Gamma$ and its characteristics can be found in [11].

As in classical case, it is necessary to apply smoothing operators in order to estimate $\Gamma$. Thus, the space generated by the wideband spectral matrix can be split into two subspaces: the signal subspace generated by the first eigenvectors associated to the highest eigenvalues and its complementary, the noise subspace. The filtering scheme in the wideband case is the same as the scheme used in the classical case explained in the above paragraph.

In the following, we will now demonstrate how the wideband spectral matrix filtering is improved and extended to multicomponent sensors arrays.

3 Wideband spectral matrix filtering for multicomponent sensors array (MC-WBSMF)

3.1 3-Component model formulation and hypothesis

Multicomponent seismic data provide the potential to access wavefield po-
larization and consequently to be able to discriminate waves with the same
velocity and the same time delay. 3-Component (3C) or 4-Component (4C)
data recordings depend on 3 parameters: time ($N_t$ samples), distance ($N_x$ sen-
sors) and component ($N_c$ components). Under this assumption, the tensor of the data collected during $N_t$ samples on a set of $N_x$ sensors (each composed of $N_c = 3$ components: X, Y and Z) are written as:

$$Tt \in \mathbb{R}^{N_x \times N_t \times N_c}$$ (11)
Applying the Fourier transform, the problem is divided into a set of instantaneous mixtures of signals and we have:

$$T = TF\{T\} \in \mathbb{C}^{N_x \times N_f \times N_c}$$

(12)

To define the multicomponent wideband spectral matrix, \(T\) is concatenated into a long-vector noted \(T_l\). There are different possible permutations to arrange the data in a long-vector. One arrangement is considered since this choice is independent of the final result (see part 3.3.3). \(T_l\) is defined as a long-vector of dimension \((N_x \times N_f \times N_c)\) which contains all the frequency bins on all sensors for all components:

$$T_l = [X(f_1)^T, \cdots, X(f_{N_f})^T, Y(f_1)^T, \cdots, Y(f_{N_f})^T, Z(f_1)^T, \cdots, Z(f_{N_f})^T]$$

(13)

where \(X(f_i), Y(f_i)\) and \(Z(f_i)\) are the \(i^{th}\) frequential component of the signals received on all the components of each sensors.

The model is then defined as:

$$T_l = S \cdot A + B$$

(14)

where:

- \(S = [S_1(t), S_2(t), \cdots, S_p(t)]\) is a \(N_x \cdot N_f \cdot N_c\) by \(P\) matrix whose columns are the steering vectors describing the propagation of each wave along the receiver group for all frequencies and all components. This matrix contains information about the polarization of the \(P\) waves and information about the waveform emitted by the sources. All these terms are deterministic;
- \(A = [a_1, a_2, \cdots, a_p]^T\) is a vector of size \(P\) which contains the random amplitudes of each waves;
- \(B\) is a vector of dimension \(N_x N_f N_c\) which corresponds to the concatenation of noise vectors obtained at each frequency and on each component. These random noises are supposed to be additive, temporally and spatially white, uncorrelated with the sources, non polarized (i.e. there is no relation between noises received on the various components of one sensor) and to have identical power spectral density \(\sigma_b^2\).

The multicomponent wideband spectral matrix is defined as:

$$\Gamma_{N_c} = E[T_l \cdot T_l^H]$$

(15)

Following our assumptions:

$$\Gamma_{N_c} = S \cdot \Gamma_A \cdot S^H + \sigma_b^2 \cdot I$$

(16)

If \(\Gamma_A\) is non-singular (waves partially correlated or uncorrelated) and if the columns of \(S\) are linearly independent, the rank of \(S \cdot \Gamma_A \cdot S^H\) is \(P\).
\( \Gamma_{Nc} \) has dimensions \((N_x N_f N_c \times N_x N_f N_c)\) and can also be written:

\[
\Gamma_{Nc} = \begin{pmatrix}
\Gamma_{X,X} & \Gamma_{Y,X} & \Gamma_{Z,X} \\
\Gamma_{X,Y} & \Gamma_{Y,Y} & \Gamma_{Z,Y} \\
\Gamma_{X,Z} & \Gamma_{Y,Z} & \Gamma_{Z,Z}
\end{pmatrix}
\]

(17)

where \( \Gamma_{I,I} \) is the monocomponent wideband spectral matrix for the component \( I \) defined in (8) and \( \Gamma_{I,J} \) is the cross-component wideband spectral matrix between component \( I \) and \( J \).

Therefore, the multicomponent wideband spectral matrix method takes into account all information relating to the signal. We can notice that the previous terms \( \Gamma \) (equation (8)) corresponding to the monocomponent wideband spectral matrix are located in the blocks of the main diagonal of \( \Gamma_{Nc} \). The other blocks contain information relating to the interaction between the component \( I \) and \( J \). Since the signals on each component are not uncorrelated, these extra diagonal terms are not null and contain information relating to the polarization of the signal. Consequently, the use of the multicomponent wideband matrix filtering provides more information on the signal and finally, better results rather than applying classical wideband matrix filtering independently on each component.

### 3.2 Estimation of the multicomponent wideband spectral matrix

Since in geophysics only one realization with few samples \((N_t)\) is available, smoothing operators are applied in order to estimate \( \Gamma_{Nc} \). If several realizations \( T_t \) were available, a simple average of these realizations would have given a good estimation of the matrix. In the literature, two types of smoothing can be found: spatial smoothing, firstly introduced by Evans [6] and then used in [24,25,30,33], and frequency smoothing [35,36].

#### 3.2.1 Spatial smoothing

In this part, we will present the spatial smoothing; a pre-processing which enables the spectral matrix to be non-singular even if sources are coherent. Let the uniform linear array with \( N_x \) sensors be subdivided into overlapping subarrays. The idea exploited by spatial smoothing is to have several identical arrays, which will be used to estimate spectral matrices in order to build a smoothed matrix. These subarrays are supposed to be linear and uniform (constant intertrace). If the smoothing order is \( K_s \), the size of each subarray is \( N_x - 2K_s \) and the number of subarrays is \( 2K_s + 1 \). Fig.1 shows an example of subarrays decomposition for \( N_x = 7 \) and \( K_s = 1 \).
The spatially smoothed spectral matrix $\hat{\Gamma}^s_{Nc}$ is defined as the average of the spectral matrices obtained for each subarray:

$$\hat{\Gamma}^s_{Nc} = \frac{1}{2K_s + 1} \sum_{k_s=1}^{2K_s+1} T_{l,k_s}T_{l,k_s}^H \quad (18)$$

where $T_{l,k_s}$ is a long vector of size $((N_x - 2K_s)N_fN_c)$ corresponding to the data received on the $k_s^{th}$ subarray. Shan and al. [30] have proven that if the number of subarrays is greater than or equal to the number of sources $P$, then the spectral matrix of the sources is non-singular. However, to be effective, the spatial smoothing assume that the wave does not vary over the $2K_s + 1$ sensors used in the average. Since each subarray covers $N_x - 2K_s$ sensors, there is a loss of two sensors with each additive order. Consequently, the smoothing order is limited to $K_s \leq \frac{N_x - 1}{2}$.

Another method for estimating the spectral matrix consists in using a sliding window over diagonals of the matrix $\Gamma_{Nc}$. This method is totally comparable with the previous one.

### 3.2.2 Frequential smoothing

In the same way as spatial smoothing, it is possible to design frequential smoothing on the signal spectrum. The concept of spatial subarrays is replaced by the frequential subbands. Considering a frequential smoothing of $K_f$ order applied to the only available realization, we obtain a number of $2K_f + 1$ frequential subbands of size $N_f$. From one long-vector $T_{l}$ such as the one defined in (13), we obtained $2K_f + 1$ frequently shifted long-vectors. Fig. 2 shows an example of subbands decomposition for $N_f = 10$ and $K_s = 2$. Consequently, it is possible to deduce an estimator for the frequentially smoothed matrix $\hat{\Gamma}^f_{Nc}$:

$$\hat{\Gamma}^f_{Nc} = \frac{1}{2K_f + 1} \sum_{k_f=1}^{2K_f+1} T_{l,k_f}T_{l,k_f}^H \quad (19)$$

where $T_{l,k_f}$ is a long vector of size $(N_xN_fN_c)$ corresponding to the data received on the $k_f^{th}$ subband.

To be effective, the frequential smoothing requires the wave to be roughly flattened. As in the spatial case, two implementations are legitimate: either with or without the loss of frequential channels.

### 3.2.3 Joint use of spatial and frequential smoothing

To have a better estimation of the multicomponent wideband spectral matrix, it is possible to design jointly spatial and frequential smoothing. The vectors $T_{l,k_s}$ and $T_{l,k_f}$ are regarded as recurrences as well as the vector $T_{l}$. 
Consequently, an observation $T_l$ generates a set of $(2K_s + 1)$ recurrences $T_{l,k}$ spatially shifted. Subsequently, these recurrences can be frequency-smoothed $(2K_f + 1)$ times. Eventually, a set of $K = (2K_s + 1)(2K_f + 1)$ spatio-frequency shifted recurrences $T_{l,k,k_f}$ is obtained and an expression for the estimated spectral matrix is:

$$\hat{\Gamma}_{N_c} = \frac{1}{(2K_s + 1)(2K_f + 1)} \sum_{k_s=1}^{2K_s+1} \sum_{k_f=1}^{2K_f+1} T_{l,k,k_f} T_{l,k,k_f}^H$$

The estimated multicomponent spectral matrix is of dimension $(M \times M)$ with $M = (N_x - 2K_s) \cdot N_f \cdot N_c$.

### 3.3 Computation of signal subspace

Once the multicomponent wideband spectral matrix is estimated, the space generated by $\hat{\Gamma}_{N_c}$ is separated into two subspaces: the signal subspace generated by the first eigenvectors associated to the highest eigenvalues and its complementary, the noise subspace. Therefore, filtering is achieved by projection of the initial data onto the signal subspace. The estimation procedure is detailed in the following part.

#### 3.3.1 Decomposition of the multicomponent wideband spectral matrix

The estimated multicomponent wideband spectral matrix could be decomposed using the eigenvalue decomposition by:

$$\hat{\Gamma}_{N_c} = U \cdot \Lambda \cdot U^H = \sum_{i=1}^{M} \lambda_i u_i u_i^H$$

where $\Lambda = \text{diag}(\lambda_1, \cdots, \lambda_M)$ are the eigenvalues ordered in decreasing way and $U$ is a unitary $M$ by $M$ matrix whose columns are the orthonormal eigenvectors $u_1, \cdots, u_M$ of $\hat{\Gamma}_{N_c}$. If we consider the equation (21), $\hat{\Gamma}_{N_c}$ has $M$ eigenvalues. Nevertheless, resulting from equation (20), the rank of the matrix is equal to $K$ and $K$ needs to be greater than $P$. In practice, the value of the smoothed order $K$ is much lower than $M$. Consequently, we have:

$$\lambda_i = 0 \quad \text{with} \quad K + 1 \leq i \leq M$$

The eigenvalue 0 has an algebraic multiplicity of $M - K$. As $M \gg K$, the matrix $\hat{\Gamma}_{N_c}$ has a significant deficiency of rank. Hence, it is excessive to diagonalize the whole matrix of size $M \times M$ since the computing time is proportional to $M^3$. Few methods have already been developed in order to compute only the few dominant eigenpairs (the highest eigenvalues and their corresponding
eigenvectors). For example, the Lanczos algorithm is particularly useful when only a few of the largest or smallest eigenvalues are desired (see [10]). Another method is used in order to calculate only the $K$ first eigenpairs. This method is developed in appendix.

3.3.2 Filtering by projection onto the signal subspace

The filtering step corresponds to an orthogonal projection of $\mathbf{T}$ onto the highest eigenvectors corresponding to the signal part. As considered previously, the eigenvectors associated with the $P$ largest eigenvalues belong to the same subspace called signal subspace, by opposition to the noise subspace. Various statistics methods can be used to estimate $P$ [37]. As soon as $P$ is found, the initial data $\mathbf{T}$ (equation (13)) is projected onto the first $P$ eigenvectors, following the expression:

$$\mathbf{T}_s = \sum_{i=1}^{P} \langle \mathbf{T}, \mathbf{u}_i \rangle \cdot \mathbf{u}_i \quad (23)$$

The projection onto the noise subspace is obtained by subtraction of $\mathbf{T}_s$ to the initial data:

$$\mathbf{T}_n = \mathbf{T} - \mathbf{T}_s = \sum_{i=P+1}^{M} \langle \mathbf{T}, \mathbf{u}_i \rangle \cdot \mathbf{u}_i \quad (24)$$

Once the projection onto signal subspace, $\mathbf{T}_s$, and noise subspace, $\mathbf{T}_n$, are computed, the last steps consist in rearranging the long-vectors $\mathbf{T}_s$ and $\mathbf{T}_n$ in the tensor form (reverse of equ.13). An inverse Fourier transform is calculated in order to come back to the time domain.

3.3.3 Influence of the permutation

As seen in section 3.1, the concatenation of the initial seismic data $\mathbf{T}$ into a long-vector $\mathbf{T}$ of dimension $(N_x N_f N_c)$ could be done with various ways of unfolding. We will show that filtering results are independent of the tensor’s unfolding chosen.

Assuming that $\mathbf{T}^i$ and $\mathbf{T}^j$ correspond to two different unfoldings of the initial data $\mathbf{T}$ into a long-vector. Each couple of two long-vectors is linked by the relation:

$$\mathbf{T}^j = \Omega_{ij} \cdot \mathbf{T}^i \quad (25)$$

with $\Omega_{ij}$ a permutation matrix.

Let $\mathbf{\Gamma}$ be the multicomponent wideband spectral matrix obtained for the
initial data $T_i$ and $\Gamma_j$ the one for $T_j^i$, we define:

$$
\Gamma_i = E[ T_i \cdot T_i^H ] = U_i \cdot \Lambda_i \cdot U_i^H
$$

$$
\Gamma_j = E[ T_j \cdot T_j^H ] = U_j \cdot \Lambda_j \cdot U_j^H
$$

(26)

The expression of $\Gamma_j$ can be developed as:

$$
\Gamma_j = E[ T_j \cdot T_j^H ] = E[ \Omega_{i,j} \cdot T_i \cdot T_i^H ] = \Omega_{i,j} \cdot \Gamma_i \cdot \Omega_{i,j}^H = \Omega_{i,j} \cdot U_i \cdot \Lambda_i \cdot U_i^H \cdot \Omega_{i,j}^H
$$

(27)

Finally, identification of equations (26) and (27) gives:

$$
U_j = \Omega_{i,j} \cdot U_i
$$

$$
\Lambda_j = \Lambda_i
$$

(28)

We finally verify that the projection onto signal and noise subspaces is independent of the unfolding chosen by demonstrating that $T_i^{s} = \Omega_{i,j} \cdot T_j^{s}$.

$$
T_j^{s} = \sum_{k=1}^{p} \langle T_i \cdot u_{j_k}, u_{j_k} \rangle \cdot u_{j_k} = \sum_{k=1}^{p} \langle \Omega_{i,j} \cdot T_i, \Omega_{i,j} \cdot u_{k} \rangle \Omega_{i,j} \cdot u_{k} = \Omega_{i,j} \sum_{k=1}^{p} \langle T_i, u_{k} \rangle \cdot u_{k} = \Omega_{i,j} \cdot T_i^{s}
$$

(29)

Finally, the same data are obtained when the long-vectors $T_i^{s}$ and $T_j^{s}$ are rearranged in the seismic tensor form. The multicomponent wideband spectral matrix filtering is independent of the data unfolding chosen.

4 Application on synthetic and real data and comparison with others filtering methods

In this section, multicomponent wideband spectral matrix filtering (MC-WBSMF) is applied on synthetic and real data and results are compared with the results obtained with others well-known filtering methods.

4.1 Synthetic datasets

4.1.1 Simulation 1

The wave used for the first simulation (Fig.3) has an infinite velocity and a linear polarization. Fig.3a represents the model used for each component. In Fig.3b, we present the initial dataset corresponding to the modelled waves with
additive random noise. The signal to noise ratio is relatively poor, especially for Z-component where the wave is totally hidden in the noise. After applying MC-WBSMF with spatial and frequency smoothing ($K_s = 4$ and $K_f = 4$), we obtain a signal subspace issued from the first eigenvector. Projection onto the noise subspace (Fig.4b) is given by the difference between initial dataset and projection onto signal subspace (Fig.4a). The difference between the model and the projection onto signal subspace is given in Fig.6. We can see that this difference is relatively weak and could be quantify by the computation of the Mean Square Error. For the whole profile (3 components), the MSE with MC-WBSMF method is $11.10^{-4}$ whereas, the error is $19.10^{-4}$ with the 3DSVD and $35.10^{-4}$ with the FK filter applied independently on each component. Fig.6 and Fig.7 show respectively the modulus of the multicomponent wideband spectral matrix $\Gamma_{Nc}$ and the amplitude of the ten first eigenvalues. By looking at the spectral matrix modulus and at the structure of $\Gamma_{Nc}$ described in equation (17), we see that the modulus of the cross-component spectral matrices are not null and moreover the modulus of $\Gamma_{X,Z}$ is higher than the one of $\Gamma_{Z,Z}$. Therefore, it is worthwhile to take into account the cross-component interaction. Furthermore, we can see that all the noise has been removed from the projection onto signal subspace (Fig.4a). Consequently, signal to noise ratio has really been improved using the multicomponent wideband matrix filter.

Others classical filtering methods have been applied on this simulation. All results are quantified by calculating the Mean Square Error between the model and the filtered results. Classical spectral matrix filtering, SMF, (section 2.1) and wideband spectral matrix filtering, WBSMF, (section 2.2) have been applied independently on each component. Moreover, FK filter, SVD and 3DSVD have also been applied. All the results are summarized on the Tab.1.

4.1.2 Simulation 2

The wave (Fig.8) used for the second simulation has a non-infinite velocity and a linear polarization. Fig.8a shows the modelled wave and Fig.8b the initial dataset corresponding to the modelled wave with additive random noise. The signal to noise ratio is nearly equal for all the components and is relatively low. We apply multicomponent wideband spectral matrix filtering with a spatial and frequency smoothing ($K_s = 4$ and $K_f = 4$) and we obtain the signal part (Fig.9a) by projecting the initial data on the first eigenvector. The projection onto noise subspace is given in Fig.9b. Although the wave has no infinite velocity, most of the noise has been removed in the projection onto signal subspace. Compared to other wavefield separation filters such as 3DSVD, our method is efficient since it succeeds in removing noise even with a non-infinite velocity wave and without pre-processing. The EQM is $30.10^{-4}$ whereas it is $134.10^{-4}$ with the 3DSVD.

The results obtained with others methods are summarized in the Tab.1. All methods based on SVD give relatively poor results since the wave has not been previously aligned.
4.1.3 Simulation 3

Thanks to this simulation, we would like to show the ability of the algorithm to separate wavefields. The seismic profile of Fig.11b is composed of two waves (Fig.11a) plus noise received on a simulated 3C-sensors array. One wave has an infinite velocity and a linear polarization and the second one is a non-plane wave with a non-constant velocity. The aim is to separate each wave and also to remove noise. The results obtained with MC-WBSMF and 3DSVD are presented. The aligned wave has been obtained with MC-WBSMF by projection on the first eigenvector (Fig.12a) and the second wave by projection onto the second to the sixth eigenvectors (Fig.12b). The filtered waves obtained with 3DSVD are shown on Fig.13.

The results obtained with others methods are summarized in the Tab.1.

4.2 Real datasets

MC-WBSMF has also been tested on real multicomponent seismic datasets such as Verticale Seismic Profile (VSP), OBS (Ocean Bottom Seismometer), OBC (Ocean Bottom Cable). Mostly, a surface seismic record contains direct arrivals, refracted waves, surface waves and reflected waves. Only reflected waves are used for imaging the subsurface in seismic reflection surveying. Since these waves have low energy, the MC-WBSMF is used to remove the waves with the highest energy (direct waves and ground roll) in order to enhance the reflected waves.

In this part, processings of two different types of real dataset are presented. The first comes from an OBS survey and the second from near surface acquisition.

4.2.1 CGG real dataset

This real data set Fig.14 has been purchased by the "Compagnie Générale de Géophysique" (CGG) during a seismic marine campaign using 4C sensor type (Ocean Bottom Seismometer OBS). 4 components data (one hydrophone and 3 geophones) have been recorded. The sampling in time ($\Delta t$) is of 2 ms. The distance between two consecutive sensors ($\Delta x$) is 50 m. Fig.14 shows the initial seismic section (time-distance) for each component (hydrophone Hy and geophone X, Y, Z), where the dispersive guided wave is dominant. We also observe a reflected deep arrival that goes through the section.

One aim of the filtering could be to separate the main wave from the rest of the signal. The multicomponent wideband spectral matrix filtering is applied with a frequency smoothing of order $K_f = 6$. The repartition of the first eigenvalues is given in Fig.15. The projection onto signal subspace which corresponds to the first eigenvector is shown in Fig.16 and the projection onto noise subspace is given in Fig.17. We can see that the totality of the dominant guided wave has been isolated from the other waves present in the seismic section. In this case, the algorithm has been really successful in realizing wave separation.
4.2.2 IFP real dataset

MC-WBSMF has been tested on a real 3 components noise profile issued from a near surface acquisition. The number of 3C-geophones is 32. The minimum offset is 1 m. The distance between 2 adjacent geophones is 2.5 m. The time sampling interval is 0.25 ms. The recording length is 128 ms. The noise profile has been used to define the acquisition parameters for a reflection seismic survey. Fig.18 shows the initial dataset for the in-line horizontal component X and vertical component Z. This noise profile shows different waves such as: the direct wave, a refracted wave train, a low frequency surface wave and a high frequency air wave. In the 30 - 60 m range of offsets, a very high apparent velocity event can be clearly seen at a time of 80 ms on the vertical component. This event is a reflected wave. The velocity of the direct wave is 900 m/s. The velocity of the refracted wave observed in the 16 - 71 m range of offsets is 1620 m/s. The intercept time is 8 ms corresponding at refractor depth of 4.5 m. For the processing, the 16 - 71 m range of offsets has been selected for the processing.

Fig.19 shows the extraction of the refracted wave-field with the MC-WBSMF method applied jointly on the X and Z components. The refracted wave-field has been subtracted on the initial data. The residual section is shown on Fig.20. On Fig.21, reflected wave can be identified. This reflected event at 80 ms, extracted with MC-WBSMF from Fig.20, is associated with the Triassic sandstone bedrock situated at a depth of about 80m.

4.3 Discussion

This part is a brief overlook of the wave separation methods used in seismic and a discussion of the advantages and disadvantages of each of them in comparison with MC-WBSMF.

These methods can be classified in various categories: the methods using masks, the matrix methods and finally the inversion methods.

The first methods require to work in an associated domain. The FK filter belongs to this category. The disadvantages of FK filter are that it requires a large number of traces with short distance sampling and that it is very cost effective.

The second category includes the Singular Value Decomposition (SVD) [8,10], the 3DSVD [14], the Karhunen-Loeve method [13], and the various spectral matrix filtering methods: SMF [29], WBSMF [12] and MC-WBSMF. These methods build a matrix (data, covariance or spectral matrix) which is then decomposed according to its eigenvectors. These methods are used to separate waves but also to improve signal to noise ratio by breaking down the data into a signal subspace and a noise subspace. SVD-based and Karhunen-Loeve methods require the extracted wave to be flattened (infinite velocity) before separation. In contrast, the spectral matrix can operate without flattening and without a-priori information.

The parametric separation [5,16,17] belongs to the last category. This method, which operates in the space-frequency domain, separates multicomponent VSP data into its P and S wavefields by formulating it as a parametric inversion. Each wave is
modelled as its Fourier components and by two frequency independent parameters: the apparent slowness and the polarization angle. The advantages of this method are that it is applicable to a small number of traces and that it does not require prior alignment of the extracted waves. Nonetheless, the disadvantages are as follows: it requires a lengthy computation time and it is advisable to enter right initial values for slowness and polarisation in order to be effective.

In comparison with all these previous methods, MC-WBSMF is a low cost method which works well without pre-treatment and without a-priori information.

5 Conclusions

As multicomponent seismic acquisitions are now widely used, specific processings are required. This paper presents a new method for wavefield filtering of multicomponent seismic data called multicomponent wideband spectral matrix filtering. This proposed method is derived from the classical matrix filtering and more specifically from the wideband spectral matrix filtering used in the single component case where wideband characteristics of the seismic signal are taken into account. We have first presented the classical frequential filtering methods for single component sensors array. Then, the extension to the multicomponent has been developed. The filtering process starts with the estimation of the multicomponent wideband spectral matrix with spatial and frequential smoothings. Once the matrix is estimated, the space generated is separated in two subspaces: the signal subspace generated by the first eigenvectors associated to the highest eigenvalues and its complementary, the noise subspace. We also show that the eigenvalue decomposition of a reduced matrix issued from the global matrix enables an efficient wavefield separation. Therefore, filtering is achieved by the projection of the initial dataset onto the signal subspace. In terms of enhancement of signal to noise ratio and waves separation, the results on synthetic and real data are very good. Compared to other filters applied on multicomponent as 3DSVD, our method is efficient and does not require any pre-processing. A better modelling of the signal, the taking into account of the wideband characteristics and the polarization of the signal are the main reasons for this achievement.

6 Acknowledgements

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References


Appendix: Computation of the dominant eigenpairs of the multicomponent wideband spectral matrix

Since the matrix $\hat{\Gamma}^{N_c}$ has a significant deficiency of rank, it is excessive to diagonalize the whole matrix of size $M$ by $M$ since the computing time is proportional to $M^3$. We have seen that the eigenvalue 0 has an algebraic multiplicity of $M - K$ with $K = (2K_s + 1)(2K_f + 1)$. As $M \gg K$, the following method is used to calculate the $K$ first eigenvalues and eigenvectors.

The eigenvalue decomposition of $\hat{\Gamma}^{N_c}$ gives:

$$\hat{\Gamma}^{N_c} = U \cdot \Lambda \cdot U^H = \sum_{i=1}^{M} \lambda_i u_i u_i^H$$

(30)

where $\Lambda = diag(\lambda_1, \ldots, \lambda_M)$ are the eigenvalues and $U$ contains the orthonormal eigenvectors $u_1, \ldots, u_M$ of $\hat{\Gamma}^{N_c}$. The global spectral matrix has $M$ eigenvalues. Nonetheless, resulting from equation (20), the rank of the matrix is equal to $K$. In practice, the value of the smoothed order $K$ is much lower than $M$. Consequently, we have:

$$\lambda_i = 0 \quad \text{with } K + 1 \leq i \leq M$$

(31)

Accordingly, a new matrix of size $(M \times K)$, called the observation matrix and noted $C$, is introduced:

$$C = \begin{bmatrix}
\vdots & \vdots & \vdots & \vdots \\
T_{l,1,1} & T_{l,2K_s+1,1} & \cdots & T_{l,2K_s+1,2K_f+1} \\
\vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}_{K=(2K_s+1)(2K_f+1)}$$

(32)

All the long-vectors resulting from the spatial and frequency smoothing are concatenated in $C$. This matrix has dimension $M$ (long-vector dimension) by $K$ (number of smoothed vectors). The link between the matrices $C$ and $\hat{\Gamma}^{N_c}$ is:

$$\hat{\Gamma}^{N_c} = \frac{1}{K} C \cdot C^H$$

(33)

The spectral decomposition of $C^H \cdot C$ of size $K$ by $K$ is a good alternative to the costly diagonalisation of the global spectral matrix of size $M$ by $M$. The first eigenvectors and eigenvalues of $\hat{\Gamma}^{N_c}$ are linked with the ones of $C^H \cdot C$.

The Singular Value Decomposition of $C$ is given by:

$$C = U \cdot \Delta \cdot V^H$$

(34)

where $U$ and $V$ are orthonormal matrices with left and right singular vectors in their columns. $\Delta$ is pseudo diagonal and contains the singular values of $C$ noted $\sqrt{\delta_i}$. The
computation of $C \cdot C^H$ which is equal to $\tilde{\Gamma}_{Nc}$ except for a constant multiplier is given by:

$$C \cdot C^H = U \cdot \Delta \cdot V^H \cdot V \cdot \Delta^{-1} \cdot U^H = U \cdot \Delta \cdot I \cdot \Delta \cdot U^H = U \cdot \Delta^2 \cdot U^H$$  \hspace{1cm} (35)

By identification between (30) and (35), we obtain $\Delta^2 = \Lambda$. We consider the diagonalisation of $C^H \cdot C$:

$$C^H \cdot C = V \cdot \Delta \cdot U^H \cdot U \cdot \Delta \cdot V^H = V \cdot \Delta \cdot I \cdot \Delta \cdot V^H = V \cdot \Delta^2 \cdot V^H$$  \hspace{1cm} (36)

Thanks to this decomposition, we obtain the matrices $V$ and $\Delta^2$ and consequently $\Lambda$ from which we can compute the matrix $U$ containing the first eigenvectors of $\tilde{\Gamma}_{Nc}$:

$$C \cdot V \cdot \Delta^{-1} = U \cdot \Delta \cdot V^H \cdot V \cdot \Delta^{-1} = U$$  \hspace{1cm} (37)

Since the long-vectors contained in the observation matrix are linearly independent, $C$ is a full rank matrix. Hence, we have:

$$\text{rank}(C) = K = \text{rank}(C \cdot C^H) = \text{rank}(C^H \cdot C) = \text{rank}(\tilde{\Gamma}_{Nc})$$  \hspace{1cm} (38)

Finally, if we consider the equation (30), we obtain all the terms coming from the decomposition of the global spectral matrix. The matrix $\Lambda$ which contain the eigenvalues is given by:

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ & \lambda_2 & \cdots & 0 \\ & & \ddots & \vdots \\ & & & \lambda_K \\ 0 & \cdots & \cdots & 0 \\ \end{bmatrix}$$  \hspace{1cm} with $\lambda_i = \delta_i$ for $1 \leq i \leq K$  \hspace{1cm} (39)

and the $K$ first eigenvectors of $\tilde{\Gamma}_{Nc}$ are:

$$U_i = \frac{1}{\sqrt{\delta_i}} C \cdot V_i \text{ with } 1 \leq i \leq K$$  \hspace{1cm} (40)

As a result, we obtain all eigenvectors and eigenvalues of $\tilde{\Gamma}_{Nc}$ by computing the decomposition of $C^H \cdot C$ of size $(K \times K)$ instead of the one of $C \cdot C^H$ of size $(M \times M)$ with $(M \gg K)$. The computing time is significantly improved by a factor of $\left(\frac{K}{M}\right)^3$. 

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Table 1
Mean Square Error for different filters on simulation 1, 2 and 3
Fig. 1. Spatial smoothing: Subarrays’ design

Fig. 2. Fictitious frequential bands

Fig. 2. Frequential smoothing: Subbands’ design
Fig. 3. Simulation 1: Model (a) and initial data set (b)

Fig. 4. Simulation 1: Projection onto signal subspace (a) and noise subspace (b)

Fig. 5. Simulation 1: Difference between model and projection onto signal subspace
Fig. 6. Simulation 1: Modulus of the multicomponent wideband spectral matrix

Fig. 7. Simulation 1: Amplitude of the eigenvalues
Fig. 8. Simulation 2: Model (a) and initial data set (b)

Fig. 9. Simulation 2: Projection onto signal subspace (a) and noise subspace (b)

Fig. 10. Simulation 2: Difference between model and projection onto signal subspace
Fig. 11. Simulation 3: Model (a) and initial data set (b)

Fig. 12. Simulation 3: First (a) and second (b) wave after filtering with MC-WBSMF

Fig. 13. Simulation 3: First (a) and second (b) wave after filtering with 3DSVD
Fig. 14. Real dataset 1: Initial real data set

Fig. 15. Real dataset 1: Amplitude of the eigenvalues
Fig. 16. Real dataset 1: Projection onto signal subspace

Fig. 17. Real dataset 1: Projection onto noise subspace
Fig. 18. Real dataset 2: Initial real data set

Fig. 19. Real dataset 2: Refracted wave
Fig. 20. Real dataset 2: Difference between initial data and refracted wave

Fig. 21. Real dataset 2: Reflected wave