AN ENHANCED DFT-BASED CHANNEL ESTIMATION USING VIRTUAL INTERPOLATION WITH GUARD BANDS PREDICTION FOR OFDM

Jeong-Wook Seo, Jung-Wook Wee, Won-Gi Jeon, Jong-Ho Paik
Korea Electronics Technology Institute
Gyeonggi-do, Korea

Dong-Ku Kim
Yonsei University
Seoul, Korea

ABSTRACT
In this paper, we investigate the conventional windowed DFT-based channel estimation for OFDM systems with guard bands. The conventional method suffers from the edge effect and the pilot position mismatch. In order to solve the problems, we propose an enhanced DFT-based channel estimation using virtual interpolation with guard bands prediction, where linear minimum mean square error (LMMSE) smoothing/prediction and circular frequency shifting are employed. This technique can efficiently mitigate the edge effect caused by the DFT of the truncated channel impulse response and compensate the pilot position mismatch that pilot symbols are not allocated at the first and the last subcarriers in a useful band. Simulation results show that the proposed method has about 5 dB SNR gain at BER=$10^{-3}$ compared to the conventional method and is as good as the frequency-domain LMMSE method.

I. INTRODUCTION

Coherent orthogonal frequency division multiplexing (OFDM), which is a promising access technology for next-generation communication systems such as IEEE 802.16e and 3G long-term evolution, requires the accurate channel frequency response (CFR) estimation [1], [2]. Many of the works in the literatures [3]–[7] are devoted to the investigation of pilot-symbol-aided (PSA) channel estimation, where known pilot symbols are periodically inserted in useful subcarriers. Therefore, we focus on the PSA channel estimation techniques in this paper.

The frequency-domain linear minimum mean square error (LMMSE) estimation was proposed as the optimal one [3], whose complexity is usually very high. As low-complexity choice, channel estimation using discrete Fourier transform (DFT) can be used and is implemented efficiently by the fast Fourier transform (FFT). However, the periodic replicas of the channel impulse response (CIR) after inverse DFT (IDFT) would be overlapped and cause aliasing error for non sample-spaced multipath channels. A windowed DFT-based channel estimation was proposed in [5] to reduce the aliasing error and suppress the channel noise. But, it still suffers from the edge effect caused by the DFT of the CIR with limited length. Furthermore, if pilot symbols are not allocated at the first and the last subcarriers in a useful band, the edge effect would be increased, and severe interpolation errors would be introduced. To suppress the edge effect, frequency-domain approaches with slightly high computational complexity was proposed in [6], [7]. So, this paper addresses the edge effect on the windowed DFT-based channel estimation, which is referred to as a time-domain approach, and proposes a new edge-effect suppression method with low complexity. In [8], we already proposed DFT-based PSA channel estimation with LMMSE prediction of the virtual CFRs in guard bands for the purpose of reducing the dispersive distortion of an estimated CIR under sample-spaced multipath channels. In order to mitigate the edge effect, our LMMSE prediction will be adjusted to non-sample spaced multipath channels. Moreover, LMMSE smoothing will be exploited to improve the estimation performance.

This paper is organized as follows. Section II introduces an OFDM baseband model in multipath fading channels. Section III reviews the conventional windowed DFT-based channel estimation. Section IV derives the proposed channel estimation method and its computational complexity. Section V gives some simulation results to evaluate the performance of the proposed method. This paper is concluded in Section VI.

II. OFDM BASEBAND MODEL

Let us consider an OFDM system with $N = N_u + N_v + 1$ total subcarriers which consist of $N_u$ useful subcarriers, $N_v$ virtual carriers, and DC component. Here, $N_u = N_{v,l} + N_{v,r}$ where $N_{v,l}$ and $N_{v,r}$ correspond to left-side and right-side virtual carriers, respectively. It is assumed that $N_u$ is odd number, and $N_v = N_u + 1$. To estimate the CFRs, $N_p$ pilot tones are inserted into the useful subcarriers, resulting in $X_iD_{j+\alpha} = P_i$ for $i = 0, 1, \ldots, N_p - 1$ where $X_i$ is the $i$th useful symbol, $P_i$ is the $i$th pilot symbol, $D_j = N/M$ is minimum pilot spacing, $M$ is a power of 2 ($M \geq N_p$), and $\alpha$ is the initial position of a pilot subcarrier ($\alpha \geq N_{v,l}$). After allocating data symbols $X_k$ for $k \notin \{iD_{j+\alpha} + \alpha\}$, the baseband signal is obtained by an N-point inverse FFT (IFFT) and the addition of cyclic prefix with $N_c$ samples. Subsequently the OFDM signal is transmitted over a multipath fading channel.

At the receiver side, with the assumptions of longer cyclic prefix than the maximum delay spread, quasi-stationary channel, and perfect synchronization, the $k$th subcarrier output after OFDM demodulation is obtained

$$Y_k = X_kH_k + W_k, \quad 0 \leq k \leq N - 1 $$

(1)

where $X_k$, $H_k$, and $W_k$ denote the transmitted symbol, the CFR, and the additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2_w$, respectively. If we consider a frequency selective Rayleigh fading channel modeled by a tapped delay line (TDL) with $L$-non zero taps, the CFR of (1) can be described by

$$H_k = \sum_{l=0}^{L-1} h_l e^{-j2\pi k r_l / N} $$

(2)
estimates of the CFRs at pilot subcarrier frequencies of the windowed DFT-based channel estimation. The equally-

MMSE weighting is also used to suppress the channel noise was proposed for OFDM systems, where frequency-domain

A low-complexity windowed DFT-based channel estimator (WSS) complex Gaussian random variable, and the different normalized by the sampling interval, is a wide sense stationary

SNR=40 dB.

where the lth channel tap gain, $h_l$, impinging with time delay $\tau_l$ normalized by the sampling interval, is a wide sense stationary (WSS) complex Gaussian random variable, and the different tap gains are uncorrelated each other.

III. CONVENTIONAL WINDOWED DFT-BASED CHANNEL ESTIMATION

A low-complexity windowed DFT-based channel estimator was proposed for OFDM systems, where frequency-domain windowing is used to reduce the aliasing errors due to the interpolation and time-domain minimum mean square error (MMSE) weighting is also used to suppress the channel noise [5]. In this section, we review the channel estimation process of the windowed DFT-based channel estimation. The equally-spaced pilot symbols are used to obtain the least squares (LS) estimates of the CFRs at pilot subcarrier frequencies

$$\hat{H}_{iD,j+\alpha} = \frac{Y_{iD,j+\alpha}}{P_i} = \hat{H}_{iD,j+\alpha} + \frac{W_{iD,j+\alpha}}{P_i}. \quad (3)$$

For convenience of explanation, a sequence of LS estimated CFRs containing only pilots is defined as $\{\hat{H}_{p,\cdot}\} = \{\hat{H}_{iD,j+\alpha}\}$. Let $\hat{H}_p = [\hat{H}_{p,0}, \cdots, \hat{H}_{p,N_p-1}]^T$ denote the CFR observation vector where $(\cdot)^T$ represents the transpose operation. $(M-N_p)$ zeros are inserted into both sides of $\{\hat{H}_{p,\cdot}\}$ to obtain an M-point sequence $\{\hat{H}_{v,m}\}, 0 \leq m \leq M - 1$. A Hanning window function is applied to the vector $\hat{H}_v = [\hat{H}_{v,0}, \cdots, \hat{H}_{v,M-1}]^T$ to reduce the aliasing errors. The windowed sequence is

$$\hat{H}_{d,m} = U_{m-M/2} \hat{H}_{v,m} \quad (4)$$

where

$$U_m = \left(0.5 + 0.5 \cos \left(\frac{2\pi m}{M}\right) \right) e^{j\pi(mN_p/M)} \quad (5)$$

is the generalized Hanning window function with the window shape parameter $\Gamma$. By doing an M-point IFFT, we have

$$\hat{h}_n = \frac{1}{M} \sum_{m=0}^{M-1} \hat{H}_{d,m} e^{j2\pi nm/M}. \quad (6)$$

A weighting sequence $\{\theta_n\}$ is applied to the CIR $\{\hat{h}_n\}$ to reduce the channel estimation mean square error (MSE) and zeros are padded to the weighted CIR to create a new sequence denoted by

$$\hat{h}_n = \begin{cases} 
\theta_n \hat{h}_n, & 0 \leq n \leq \frac{M}{2} \\
0, & \text{otherwise} \\
\theta_{n-N+M+1} \hat{h}_{n-N+M}, & N - \frac{M}{2} \leq n \leq N - 1
\end{cases} \quad (7)$$

where $\theta_n$ is obtained by the weighting vector $\Omega = [\theta_0, \cdots, \theta_M]^T$ expressed as

$$\Omega = \left( [GUR\hat{H}_n U^H G] \ast [FHV^{-1}V^{-1}] \right)^{-1}$$

$$\cdot D \left(FHV^{-1}R_{\hat{H}H}V^H G^HG\right) \quad (8)$$

where $G$ is a $(M+1) \times N_p$ IDFT matrix, $F$ is a $(N_a+1) \times (M+1)$ DFT matrix, $U$ is a diagonal matrix with the entries $\{U_m\}$, $V$ is a diagonal matrix with the entries $\{V_m\}$ in (11), $R_{\hat{H}H} = E\{\hat{H}_n\hat{H}_n^H\}$, and $R_{\hat{H}H} = E\{\hat{H}\hat{H}^H\}$. Also, $H$, $(\cdot)^H$, $(\cdot)^*$, and $\ast$ denote the $N \times 1$ CFR vector, Hermitian transpose, complex conjugation, and a pointwise or Hadamard product, respectively. $D(\cdot)$ denotes a vector whose entries are the respective entries of the diagonal of the enclosed matrix. To transform the result into frequency domain, an N-point FFT is performed. That is,

$$\hat{H}_m = \sum_{n=0}^{N-1} \hat{h}_n e^{-j2\pi mn/M}. \quad (9)$$

Finally, the windowing effect are removed to obtain the channel estimation output

$$\tilde{H}_m = \frac{\hat{H}_m}{V_{m-N/2}} \quad (10)$$

where

$$V_m = \left(0.5 + 0.5 \cos \left(\frac{2\pi m}{M}\right) \right) e^{j\pi(mN_p/M)}. \quad (11)$$

IV. PROPOSED CHANNEL ESTIMATION

Channel estimation using IDFT/DFT interpolation introduces the edge effect originating from the DFT of the truncated CIR, which causes the MSE degradation near the band edges [6], [7]. Also, if pilot symbols are not allocated at the first and the last subcarriers in a useful band, which is referred to as the pilot position mismatch, the edge effect will be increased.

Fig. 1 illustrates the MSE performances due to the edge effect. “DFT” represents the DFT-based channel estimation without MMSE weighting, which has the worst MSE performance since it suffers from the aliasing of periodic replicas.
is assumed that the first CFR at a useful band is located at the performance compared to the others. Domain LMMSE channel estimation that shows the best MSE near band edges. “LMMSE" represents the frequency-tensively extends the useful band to improve the MSE performance near band edges. The proposed channel estimation smoothes the LS estimated CFRs to get more reliable channel state information. Moreover, it predicts the equally-spaced virtual CFRs in guard bands, which is referred to as a guard bands prediction, and intensively extends the useful band to improve the MSE performance near band edges. “LMMSE” represents the frequency-domain LMMSE channel estimation that shows the best MSE performance compared to the others.

The block diagram of the proposed channel estimation is shown in Fig. 2(a). It consists of the LS channel estimation, the LMMSE virtual interpolation, and the circular shifting. Fig. 2(b) shows the concept of the LMMSE virtual interpolation. It is assumed that the first CFR at a useful band is located at the \( \alpha \) pilot subcarrier, and the first predicted CFR at a left-side guard band is located at the \( \beta \) virtual pilot subcarrier. The LMMSE smoothing is used to get the MSE performance better than the LS estimation, and the LMMSE prediction, a guard bands prediction method, broadens the useful band to suppress the edge effect. For the interpolation between the smoothed CFRs and the predicted CFRs, the DFT/IDFT interpolation technique is employed.

A. Virtual Interpolation and Circular Shifting

Virtual interpolation consists of LMMSE estimation and IDFT/DFT interpolation. LMMSE estimation smoothes LS estimated CFRs in (3) and predicts equally-spaced virtual CFRs in guard bands, given by

\[
\hat{H}_{v,m} = \sum_{i=0}^{N_p-1} \Phi_i(\Delta m)\hat{H}_{p,i} = \Phi(\Delta m)\tilde{H}_p
\]  

(12)

where the Wiener filter \( \Phi(\Delta m) \) depends on the location of the desired CFR relative to the pilot positions, \( \Delta m = (m-i)D_f + \beta - \alpha \). Here, \( \beta \) denotes the modified initial position of a pilot subcarrier by guard bands prediction \( 0 \leq \beta \leq \alpha \). The Wiener filter is represented by [9]

\[
\Phi(\Delta m) = R_{H_H}(\Delta m)R_{H_H}^{-1}
\]  

(13) where the cross-correlation vector and the auto-correlation matrix are represented by (14) and (15), respectively.

\[
R_{H_H}(\Delta m) = E\left\{H_mH_p^H\right\} = R_{HH}(\Delta m)
\]  

(14)

\[
R_{H_H}(\Delta m) = E\left\{\tilde{H}_pH_p^H\right\} = \left(R_{H_H} + \frac{\lambda}{SNR}I\right)
\]  

(15) where \( \lambda = E\{|X_m|^2\}/|P|^2 \) is the ratio of the average signal power to the pilot power. In order to obtain the \((m,n)\) element of \( R_{H_H}(\Delta m) \) and \( R_{H_H} \), the correlation function between the CFRs at two subcarriers is given by

\[
r_{m,n} = E\left\{H_mH_n^*\right\} = \begin{cases} 1, & m = n \\ 1 - e^{-j2\pi n(m-n)/N}, & m \neq n \end{cases}
\]  

(16) which corresponds to a uniform channel power delay profile matched to a typical worst case scenario [5]. After that, (4)-(11) are used to interpolate the CFRs for data subcarriers with the smoothed and the predicted CFRs.

An M-point IFFT can be used only if \( \beta \) is equal to zero in the following equation.

\[
\hat{h}_n = \frac{N}{M} \cdot \frac{1}{N} \sum_{m=0}^{M-1} H_{mD_f + \beta}e^{j2\pi n(m m D_f + \beta)/N}
\]  

(17)

Otherwise, the CFRs should be multiplied by \( e^{j2\pi \beta n/N} \) as the initial pilot position is changed. Moreover, the position changing induces the pilot position mismatch. In order to maintain an M-point IFFT structure and compensate the pilot position mismatch, the frequency-domain circular shifting is utilized. We use the same channel estimation procedure irrespective of the initial position of a pilot subcarrier. And then, we circularly shift the final channel estimate \( \hat{H}_i \) given in (10) \( \beta \) samples to the right.

The whole channel estimation can be represented in the vector-matrix notation as

\[
\hat{H} = V^{-1}\hat{\Omega}G\hat{H}_H
\]

\[
\hat{H}_H = SR_{H_H}\left(R_{H_H} + \frac{\lambda}{SNR}I\right)^{-1}\hat{H}_p
\]  

(18) where \( S = V^{-1}\hat{\Omega}G \). The MSE of the proposed channel estimation can be expressed in the general form

\[
\rho_o = \frac{1}{N}tr\left[E\left\{H - \hat{H}\right\}\left(H - \hat{H}\right)^H\right]\]

\[
= \frac{1}{N}tr\left[E\left\{HH^H\right\} - E\left\{HH^H\right\}S^H - SE\left\{\hat{H}_H^H\right\}S^H\right]
\]  

(19)
where \( tr(\cdot) \) represents the matrix trace, the autocorrelation matrix is given by \( E\{\hat{H}_v\hat{H}_v^H\} = R_{\hat{H}_v\hat{H}_v}^{-1} R^{H}_{\hat{H}_v\hat{H}_v} R^{H}_{\hat{H}_v\hat{H}_v} \), and the crosscorrelation matrix is given by \( E\{\hat{H}\hat{H}^H\} = R_{\hat{H}\hat{H}} R^{H}_{\hat{H}\hat{H}} R^{H}_{\hat{H}\hat{H}} \). Much of equation (19) is similar to the conventional windowed DFT-based method, except that the smoothed and the predicted CFRs are used to calculate autoand cross-correlation matrices.

### B. Complexity Consideration

In order to measure the computational complexity of different channel estimation methods, we use the number of complex multiplications (CMs). It is assumed that the DFT/IDFT is implemented with the split-radix FFT algorithm which needs about \( N/3 \log_2(N) \) complex multiplications for the \( N \)-point FFT. The frequency-domain LMMSE method requires \( N_p \times (N_u+1) \), the DFT-based method requires \( N_p + M/3 \log_2 M + N/3 \log_2 N \), and the windowed DFT-based method requires about \( N_p + M/3 \log_2 M + N_u + N/3 \log_2 N \) CMs per OFDM symbol [5]. The required CMs of the proposed method are approximately represented by

\[
N_p + N_p \times M + M + \frac{M}{3} \log_2 M + N + \frac{N}{3} \log_2 N. \tag{20}
\]

It should be noted that the computation load of the LMMSE virtual interpolation can be further reduced when the \( D_f \) is increased. Also, if we consider only LMMSE prediction, the \( N_p \times M \) CMs will be reduced to \( N_p \times (M - N_p) \). For the considered system, the required CMs of the proposed method and the others are shown in Table 1. “Proposed-1” represents the proposed method using the LMMSE smoothing and prediction. “Proposed-2” is a low-complexity method, which does not use the LMMSE smoothing but the LS estimation in (3). Namely, it employs only the LMMSE prediction. The computation load of the proposed method is slightly higher than that of the windowed DFT-based method, but much lower than that of the frequency-domain LMMSE method.

Table 2: The ETSI “Vehicular A” channel environment.

<table>
<thead>
<tr>
<th>tap</th>
<th>power [dB]</th>
<th>delay [nsec]</th>
<th>delay [T]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>310</td>
<td>1.55</td>
</tr>
<tr>
<td>3</td>
<td>-9</td>
<td>710</td>
<td>3.55</td>
</tr>
<tr>
<td>4</td>
<td>-10</td>
<td>1090</td>
<td>5.45</td>
</tr>
<tr>
<td>5</td>
<td>-15</td>
<td>1730</td>
<td>8.65</td>
</tr>
<tr>
<td>6</td>
<td>-20</td>
<td>2510</td>
<td>12.55</td>
</tr>
</tbody>
</table>

In this section, the performance of the proposed channel estimation is investigated in a multipath Rayleigh fading channel. A 16QAM-OFDM system with \( N=1024 \) subcarriers occupies a bandwidth of 5 MHz operating in the 2.4 GHz. The sampling interval is given as \( T=0.2 \mu s \). A guard interval with \( N_g=16 \) samples is used. The number of useful subcarriers is \( N_u+1=895 \), and the number of samples at guard bands is \( N_v=129 \). We set the minimum pilot spacing, \( D_f=16 \) and the ratio of the average signal power to the pilot power, \( \lambda=1/1.8 \). Pilot symbols are the outermost four points from the 16QAM constellation. The “Vehicular A” channel model is considered and shown in Table 2, which is defined by ETSI for the evaluation of UMTS radio interface proposals. The maximum Doppler frequency is assumed to be \( f_D=100 \) Hz. The LMMSE method, the windowed DFT-based method, and the proposed method are designed for \( \text{SNR}=40 \) dB.

Fig. 3 compares the MSE performances between proposed method and conventional ones according to SNRs. The DFT-based method presents the worst performance due to the aliasing of periodic replicas of the CIR and the edge effect of the truncated CIR. Since the windowed DFT-based method can effectively reduce the aliasing, the MSE performance is improved compared to the DFT-based method. However, it has an error floor at high SNRs that cannot suppress the edge effect. The proposed method, which can mitigate the aliasing and the edge effect caused by the IDFT/DFT interpolation with the LMMSE smoothing and prediction, outperforms the previous two methods. Even though we use only the LMMSE prediction to reduce the computation complexity, the MSE performance is slightly degraded about 2 dB SNR.

Fig. 4 shows the BER performance of the OFDM system using various channel estimation methods. “Perfect” means
the performance under perfect channel knowledge. When BER=$10^{-3}$, the proposed method has about 5 dB SNR gain compared to the windowed DFT-based method. Especially, “Proposed-2” with low complexity has the performance degradation less than 1 dB SNR compared to the LMMSE method.

**VI. CONCLUSIONS**

In this paper, an enhanced DFT-based channel estimation employing virtual interpolation with guard bands prediction was presented for OFDM systems. Although the conventional windowed DFT-based channel estimation is useful in reducing the aliasing error and suppressing the channel noise, it still suffers from the edge effect and the pilot position mismatch. We added the LMMSE smoothing/prediction and circular frequency shifting techniques to the conventional method to improve the estimation performance. Simulation results show that the proposed method has about 5 dB SNR gain at BER=$10^{-3}$ compared to the conventional method and is as good as the frequency-domain LMMSE method. Also, our method can be applied to clustered orthogonal frequency division multiple access (OFDMA) systems for the users near the band edges.

**REFERENCES**


