Modelling the early steps of transduction in insect olfactory receptor neurons

Jean-Pierre Rospars a,*, Philippe Lucas a, Mathieu Coppey a,b

a UMR1272 UPMC–INRA–INA-PG “Physiologie de l’insecte”, INRA, 78026 Versailles Cedex, France
b Laboratoire de Physique Théorique des Liquides, Université Pierre et Marie Curie (UPMC), 75252 Paris Cedex 05, France

Received 24 November 2005; accepted 22 May 2006

Abstract

Olfactory transduction is a multistep process whose basic function is to convert a low energy reaction, the odorant-receptor interaction that may involve a single odorant molecule, into a whole cell electrical response, the receptor potential, which triggers the firing of one or more action potentials. Although much effort has been devoted to the experimental analysis of transduction in olfactory receptor neurons (ORNs), especially in the favorable moth sexpheromone receptor neuron, its modelling is less advanced. The model we investigated, which takes into account the translocation of pheromone molecules from air to the extracellular space, their deactivation and their interaction with receptors, focuses on the membrane cascade. It involves the interaction of receptors, G-proteins and effector enzymes, whose reaction rates are limited by lateral diffusion in the membrane. The evolutions in time of these species in response to single pulse stimulation of various intensities were compared to one another and to the experimentally measured electrical response. The results obtained suggest that the receptor-to-effector conversion is fast with respect to the receptor response, that it presents a small amplification factor, contrary to the photoreceptor, and that most of the amplification is achieved in the post-effector processes involving the second messenger and ionic channels.

© 2006 Elsevier Ireland Ltd. All rights reserved.

Keywords: Olfaction; Neuron model; Sexual pheromone; Transduction; Lateral diffusion

1. Introduction

Olfaction is involved in food and mate location, social interactions, and interspecific communication and is thus of major importance for the survival of most animal species. The olfactory systems have evolved to discriminate a large number of odor molecules. This discrimination is performed by several hundreds of odorant receptor proteins in vertebrates and several tens in insects, each olfactory receptor neuron (ORN) expressing a single type of odorant receptor (OR). This discovery (Buck and Axel, 1991) earned their authors the Nobel Prize in 2004. The reliance on the sense of smell is especially conspicuous in moths whose males can sense minute amounts of the sexual pheromone emitted by conspecific females. Usually a moth pheromone is a blend in a specific ratio of 2 to 3 hydrocarbon molecules with 12 to 16 carbon atoms. The male pheromone system is uniquely devoted to the detection of the pheromone. It is composed of several thousand neurons typically associated in pairs within specialized hair-like units, the sensilla, each neuron expressing one of the 2 or 3 pheromone receptor proteins. This specialization is a major difference with other olfactory subsystems for which a given olfactory receptor can respond to several
(usually unknown) related odorant molecules. The fact that the ligands of receptors are known and the sensilla can be morphologically recognized and individually recorded from, explain why the pheromone system is an excellent preparation for the experimental study of olfactory processes (see reviews by Stengl et al., 1999; Kaissling, 2004; Vogt, 2004; Jacquin-Joly and Merlin, 2004).

The pheromone molecules enter the hair lumen through multiple pores. In the sensillum lymph they are transported by pheromone binding proteins (PBP, Vogt and Riddiford, 1981) to the neuron membrane and metabolized by pheromone degrading enzymes (Kasang, 1971; Ishida and Leal, 2005). The pheromone receptor neuron is a typical ORN. Its outer dendritic membrane contains the transduction machinery, which yields a membrane depolarization in the presence of pheromone molecules. The early steps of transduction involve the sequential interaction of three types of proteins—pheromone receptors, G-proteins, and effector enzymes. The effector catalyzes the formation of second-messenger molecules open ion channels and depolarize the membrane, yielding the receptor potential. The pheromone receptor of the silkmoth Bombyx mori has recently been identified (Krieger et al., 2004; Nakagawa et al., 2005). The effector is phospholipase C (Breer et al., 1990; Maida et al., 2000) which catalyzes the cleavage of phosphatidylinositol biphosphate PI(2) in inositol triphosphate IP(3) and diacylglycerol DAG, whose respective roles have not yet been completely clarified (Stengl, 1994; Zufall and Hatt, 1991; Pophof and van der Goes van Naters, 2002; Pézier et al., 2007).

Some modelling studies are available on olfactory transduction in vertebrates (Lamb and Pugh, 1992b; Lindeman, 2001; Suzuki et al., 2002; Rospars et al., 2003a; Dougherty et al., 2005; Reidl et al., 2006). In insects, models of the reactions involved in perireception (PBP and lymph enzymes) and reception in moth pheromone sensilla (Kaissling, 1998a; Kaissling, 1998b; Kaissling, 2001; Minor and Kaissling, 2003; Rospars et al., 2003b; Kaissling and Rospars, 2004) have been developed. However, no modelling study is yet available for post-receptor transduction events in insects. The present paper is an attempt to fill this gap. First, a simplified model of perireceptor, receptor, and post-receptor events is proposed. Second, this model is utilized to determine quantitatively the kinetics of activated proteins (receptors, G-proteins, and effector enzymes) in the dendritic membrane as a function of the concentration of pheromone in the air delivered to the sensillum. Third, the model output is discussed by comparing the time course of the calculated protein densities to the experimentally measured time course of the receptor potential (Zack, 1979; see also Dolzer et al., 2003).

### 2. Model of the cascade

The reactions involved are shown in Table 1. The model studied involves 16 parameter values of two types, rate constants, and initial densities (Table 2). The perireceptor reactions take place in the lymph volume (in 3D) whereas the reactions involving receptor, G- and effector proteins take place within the membrane (in 2D). The model describing the fate of the pheromone molecules in the sensillum lymph is simplified since it considers only fast pheromone deactivation (hypothetical enzyme N, Kaissling, 2001) but no PBP. This simplification has no effect on the kinetics of activated receptors (Kaissling and Rospars, 2004). The parameter values for the perireceptor reactions and the pheromone-receptor interactions were estimated initially by Kaissling (2001) and Minor and Kaissling (2003).

#### Table 1

<table>
<thead>
<tr>
<th>Reaction network describing the uptake, perireception, reception and early amplification in the moth pheromone receptor neuron, showing the reaction steps and species formed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uptake</strong>: I&lt;sub&gt;air&lt;/sub&gt; + k&lt;sub&gt;1&lt;/sub&gt; L&lt;sup&gt;0&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>Deactivation</strong>: L + N ⇌ NL + k&lt;sub&gt;1&lt;/sub&gt; P + N</td>
</tr>
<tr>
<td><strong>Activation</strong>: R + L + k&lt;sub&gt;1&lt;/sub&gt; RL</td>
</tr>
<tr>
<td><strong>Inactivation</strong>: R* + G&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>Other reactions</strong>: G* + E&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

\( ^a \) Notations for rate constants: see Table 2.

\( ^b \) Notations for species: L = main pheromone component, P = deactived pheromone, N = deactivating enzyme, R = pheromone receptor, G<sub>a</sub> = GTP, G<sub>b</sub> = GDP, E = effector enzyme (phospholipase C), E<sup>a</sup> = G<sub>a</sub>-GTP, E<sup>i</sup> = G<sub>i</sub>-GDP, E<sup>b</sup> = G<sub>b</sub>-GDP.

\( ^c \) Translocation reaction for pheromone entering the sensillar lymph from the surrounding air.
Table 2
Values of parameters used in numerical simulations of the neuron type sensitive to the main pheromone component

<table>
<thead>
<tr>
<th>Protein concentrations</th>
<th>Reaction rate constants</th>
<th>Reaction rate constants limited by diffusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_0 = 3700$ molec $\mu m^{-2}$</td>
<td>$k_i = 2.9 \times 10^4 s^{-1}$</td>
<td>$\epsilon_{RG} = 0.75 \mu m^2 s^{-1}$</td>
</tr>
<tr>
<td>$R_0 = 6000$ molec $\mu m^{-2}$</td>
<td>$k_a = 30 s^{-1}$</td>
<td>$\epsilon_G = 2 \mu m^2 s^{-1}$</td>
</tr>
<tr>
<td>$G_0 = 1000$ molec $\mu m^{-2}$</td>
<td>$k_{LN} = 1.1 \times 10^{-3}$ molec$^{-1} \mu m^{-2}$</td>
<td>$\epsilon_{GE} = 0.3 \mu m^2 s^{-1}$</td>
</tr>
<tr>
<td>$E_0 = 500$ molec $\mu m^{-2}$</td>
<td>$k_{-LN} = 100 s^{-1}$</td>
<td>$\epsilon_G = 2 \mu m^2 s^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$k_1 = 5.6 \times 10^{-5}$ molec$^{-1} \mu m^{-2}$</td>
<td>$\epsilon_{GE} = 0.3 \mu m^2 s^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$k_{-1} = 8 s^{-1}$</td>
<td>$\epsilon_{RG} = 0.75 \mu m^2 s^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$k_2 = 17 s^{-1}$</td>
<td>$\epsilon_G = 2 \mu m^2 s^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$k_{-2} = 100 s^{-1}$</td>
<td>$\epsilon_{GE} = 0.3 \mu m^2 s^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$k_{Ga} = 0.05 s^{-1}$</td>
<td>$\epsilon_G = 2 \mu m^2 s^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$k_{Ga} = 2 s^{-1}$</td>
<td>$\epsilon_{RG} = 0.75 \mu m^2 s^{-1}$</td>
</tr>
</tbody>
</table>

a Concentration of $N$ in lymph (in $\mu M$) was converted to an equivalent density on membrane with $V = 2600 \mu m^3$ (volume of the hair) and $S = 426 \mu m^2$ (surface of outer dendrite) from Keil, 1984. The conversion factor from $\mu M$ to molec/$\mu m^2$ is $N_A \times (V/S) \times 10^{-21}$, with $N_A$ Avogadro’s number.

b Similarly, the rate constants of the bimolecular reactions ($k_{Ga}$ and $k_i$ in $M^{-1} s^{-1}$) were converted to $s^{-1}$ molec$^{-1}\mu m^2$.

c Modified values $k_{EA} = 20 s^{-1}$ and $\epsilon_{RG} = 0.15 \mu m^2 s^{-1}$ were also tested.

Lateral diffusion in the membrane and, thus, depend on their sizes. Their diffusion-limited rate constants ($\epsilon_{RG}$, $\epsilon_{GE}$, $\epsilon_G$), as well as the decomposition reactions of the activated $G$-proteins ($k_{Ga}$) and activated effectors ($k_{EA}$), can be tentatively derived from the corresponding reactions in the vertebrate rod photoreceptor (see references in Table 2). For the densities (molecules $\mu m^{-2}$) of $G$-proteins ($G_0$) and effector enzymes ($E_0$), which are also unknown, the proposed values are conjectures.

The standard set of reaction rates (Table 2) was derived from these assumptions. In order to account for experimental data, we examined also a different set with two modified rates (see Table 2, note c). These modifications concern the activation of $G$-proteins by receptors ($\epsilon_{RG}$), which is slower than in the photoreceptor, and the decomposition of the activated effector ($k_{EA}$), which is faster than in the photoreceptor.

The reactions were expressed as a set of ordinary differential equations (see Appendix A). This means that all calculations were done assuming a relatively large number of molecules of each species, ignoring stochastic fluctuations, spatial effects and time-variation of the diffusion-limited rates.

The reaction system was subjected to pheromone square impulse of constant duration (2 s) and variable height (stimulus intensity). As shown by Kaissling (1998a), the intensity of stimulation is best expressed as a pheromone uptake, that is “the number of pheromone molecules adsorbed per unit volume of the perireceptor compartment and per unit time”. In practice we expressed this uptake $U$ in $\mu M s^{-1}$. Both volume and surface reactions being involved, concentrations and densities were all converted in molecules $\mu m^{-2}$ for calculations (see Table 2).

3. Results

Given the initial concentrations, the reaction rates (Table 2) and the system of ordinary differential equations (in Appendix A) representing the reaction scheme of Table 1, the time courses of all species concentrations shown in Table 1 can be computed at any time for any given stimulation. These calculations were done for the same stimulation conditions as used by Zack (1979) (see also Kaissling, 2001) in her experiments. For the sake of simplicity, in the present account, we consider only the activated form of the receptors and effector enzymes.

3.1. Time course of the response (Fig. 1)

With single pulses of 2 s duration and various uptakes, expressed in $\mu M$ of pheromone per second entering the
sensillum, the calculated densities of all activated proteins rise from zero at stimulus onset, reach a maximum (e.g. $E_{\text{peak}}^*$), then return to zero after stimulus offset, like the receptor potential. Starting from the lowest intensity of the stimulation, the height of the response increases with the stimulus intensity. However, beyond a certain intensity, the shape of the $E^*$ responses is no longer a sharp peak but presents a plateau. Thus, the system shows two qualitatively distinct regimes, one where the height varies and the other where it remains constant while the duration increases. The duration of the plateau is longer for the standard values of the parameters (Fig. 1B) than for the modified values of $e_{\text{RG}}$ and $k_{Ea}$ (Fig. 1D). A more quantitative description can be achieved using the height, half-rise time $\tau_{\text{rise}}$, and half-fall time $\tau_{\text{fall}}$.

3.2. Heights

The heights $R_{\text{peak}}^*$ and $E_{\text{peak}}^*$ of the responses increase with the stimulus uptake $\dot{U}$. As shown in Fig. 2A and D, these are sigmoid curves in semilog scale, which increase from zero at low uptake to maximum values at high uptake. At the peak, whereas 88% or more of the effectors are activated (total density $E_0$ being 100%), only 12% of receptors are activated. These curves can be characterized by their rising point, saturation point, and dynamic range.

Taking the asymptotic maxima as references, the sigmoid curves can be characterized by the uptakes at which 1% (rising) and 99% (saturation) of proteins are activated. With the standard values of the parameters the corresponding uptakes are $10^{-0.25}$ (1%) and 100 $\mu$M s$^{-1}$ (99%) for $R^*$, $10^{-3}$ and $10^{-0.5}$ $\mu$M s$^{-1}$ for $E^*$ (Fig. 2A). The dynamic ranges, which are the ratios expressed in log units of the uptakes at saturation and at rising, are ca. 1.75 and 2.5 log units respectively, indicating that both curves are steep and almost parallel. The shift of the 1% point towards smaller uptakes along the transduction cascades, shows that the initial $R^*$ response is about 500 times less sensitive than the $E^*$ response. With the modified parameters ($e_{\text{RG}}$ and $k_{Ea}$) the range becomes $10^{-1.5}$.
Fig. 2. Response characteristics of activated proteins $R^*$ (dash-dot), $G^*$ (dash), $E^*$ (solid) and experimentally observed receptor potential $V$ (dot) as a function of stimulus uptake. The characteristics are defined in Fig. 1C. Relative heights (A, D), half-rise times $\tau_{\text{rise}}$ (B, E), and half-fall times $\tau_{\text{fall}}$ (C, F) shown for the standard values of the parameters (left column) and the modified values of $e_{RG}$ and $k_{Ea}$ (right column; see Table 2, note c). In A (respect. D), the relative heights $X_r = X_{\max}(X)$ where maximum heights, $\max(X)$, are 720 $R^*$, 504 (respect. 475) $G^*$, 493 (respect. 438) $E^*$ molec/$\mu$m$^2$ and 30 mV ($V$). $V$ data by courtesy of K.-E. Kaissling (see Zack, 1979 and Kaissling, 2001).

to $10^{1.75}$ $\mu$M s$^{-1}$ for $E^*$, so that the increase in sensitivity is much smaller (18 times); the $E^*$ curve remains close to the $R^*$ curve (Fig. 2D). Although the dynamic range of $E^*$ widens by 0.25 log units, it extends on only a fraction of the ca. 7.5 range found experimentally for the receptor potential.

3.3. Half-rise and half-fall times

The time needed by the activated proteins to reach their peak value depends on the stimulus uptake. However, the activated molecules tend asymptotically to the steady state, so that a precise time to peak is difficult to
define. For this reason, in accordance with experimental measurements, we preferred to use the half-rise time, which is the time elapsed between stimulation onset and 50% of the maximum response (Fig. 1C). In this part of the temporal response the slope of the curve is relatively steep, so that the half-rise time is always well defined. A similar approach was followed to analyze the decline of the responses, the half-fall time being defined as the time elapsed between stimulus offset and the decline of the response to 50% of its maximum value (Fig. 1C).

The calculated half-rise and half-fall times of the activated proteins present the same global behavior. At low uptakes they are constant (or slowly changing), then beyond a certain point they respectively decrease (Fig. 2B and E) or increase (Fig. 2C and F). The transitions take place close to the uptake yielding the half-maximum peak response so that the range of uptakes for which times are constant is wider for R* than for E* and with the modified constants than with the standard ones. Interestingly the times for E* are practically equal to those for R* in this constant part with the modified constants. At most uptakes the calculated times are either greater (rising time) or smaller (falling time) than those measured on the receptor potential, the difference being smaller at high uptake for rise and at low uptake for fall.

4. Discussion

4.1. Modelling olfactory transduction

Although much experimental work has been devoted to investigating the various properties of pheromone reception in moths, no complete model has yet been proposed to account quantitatively for these properties. Building upon the pioneering modelling work of Kaissling et al. on the perireceptor and receptor events (Kaissling, 1998a,b; Kaissling, 2001; Minor and Kaissling, 2003), we have concentrated on the postreceptor events. These events involve the interactions of receptor, G-protein, and phospholipase C molecules within the dendritic membrane. In the model studied, the G-protein cascade obeys the concept of the random walk amplifier (Liebman and Pugh, 1979) in which the proteins freely diffuse on the two-dimensional dendritic membrane, so that each activated receptor can sequentially encounter and activate several molecules of G-protein. Then, each of the active G-protein molecules can in turn bind and activate an effector molecule. In this model, the reactions are limited by diffusion, not by direct interaction of the partners. The responses are turned off by deactivation reactions acting on the receptor, the G-protein, and the effector.

4.2. Limitations

Before interpreting the results obtained it must be pointed out that they are subject to four kinds of limitations.

The first one comes from the uncertainties on the numerical parameters. Values based either on direct experimental measurements in moth pheromone ORNs or derived from models integrating such measurements, were used when available. However, to our knowledge, no values were estimated for the densities and reaction rates of the G-protein and the effector in the ORN. The diffusion-limited rate for the G-protein transducin and the phosphodiesterase effector in the rod photoreceptor (Lamb and Pugh, 1992a; Felber et al., 1996) were used as replacements. It is expected that their properties are similar in both the photoreceptor neuron and the ORN. However, there is no reason to believe that either the densities or even the ratios of the proteins are the same in both neuron types and therefore the densities we have chosen for these proteins are conjectural. Also, the activation of a G-protein when reacting with an activated receptor (controlled by rate constant $e_{RG}$) and the deactivation of the activated effector (controlled by rate constant $k_{EA}$) might differ in the ORN. For this reason we studied modified values of the latter two parameters.

The second limitation comes from the fact that the model studied is described by a set of ordinary differential equations (ODEs). This means that the numbers of molecules involved must be large enough to be considered as continuous variables. This condition is met for uptakes greater than ca. $10^{-2.5} \mu M s^{-1}$ when the number of activated molecules R* per ORN exceeds 30. In the present ODE model the uptake which yields a single R* per ORN is $10^{-4} \mu M s^{-1}$. Kaissling and Priesner (1970) showed that, at the behavioral threshold ca. $10^{-6} \mu M s^{-1}$, a single receptor per ORN is activated and can trigger a spike. Preliminary numerical experiments indicate that such a low threshold will be obtained using a stochastic approach (which is valid for a single R*).

A third limitation comes from the uncertainties on the exact mechanism of pheromone deactivation in the perireceptor space. The deactivating enzyme N considered here is hypothetical and was postulated by Kaissling (1972) because the pheromone adsorbed on the antenna has a much longer half-life than the cell response. Its action is dynamically equivalent to the more complex perireception system proposed by Kaissling (2001), both systems yielding exactly the same kinetics of activated receptors.

The fourth limitation arises from the fact that the most precise data currently available on transduction
result from the measurement of the receptor potential, whereas the present model does not take into account the post-effector steps, that is the second messenger and the various channels (including calcium channels), that are involved in its generation. Thus, assuming the parameter values of the present model are well chosen, any difference in height and temporal properties between the effector response and the receptor potential must be explained by these post-effector steps.

4.3. Small amplification factor and small delays in the RGE cascade

If the constants describing the perireceptor and receptor events are essentially correct, then the reaction rates describing the post-receptor events in the photoreceptor cannot apply to the ORN. The main reason for this conclusion comes from the height of the effector response as a function of pheromone uptake (Figs. 1B) and its comparison to the corresponding curve measured on the receptor potential (Fig. 2A). The two curves expressed in relative values, $E^\ast_r$ and $V_r$ (see legend of Fig. 2) cross one another and although they reach half-maximal response at the same uptake (ca. $10^{-1.5}$ $\mu$M s$^{-1}$), they have significantly different rising points, saturations points, and dynamic ranges (i.e. slopes). To account for these differences we would have to postulate that the amplification factor $V_r/E^\ast_r$ of the post-effector steps is larger than 1 up to $10^{-1.5}$ $\mu$M s$^{-1}$ and smaller than 1 beyond, which seems difficult to explain.

Decreasing $e_{rg}$ five times and increasing $k_{el}$ ten times with respect to their values in the photoreceptor, slows down the activation of G-proteins and accelerates the deactivation of effectors, resulting in two significant effects:

First, the amplification factor $E^\ast_r/R^\ast_r$ of the cascade falls from 330 to 7.5 (Fig. 2A and D). Consequently the effector curve is shifted towards high uptakes and saturates close to the uptake at which the receptor potential $V$ saturates, so that the $E^\ast_r$ and $V_r$ curves no longer cross. Now the amplification factor $V_r/E^\ast_r$ of the post-effector steps is always greater than 1 for all uptakes, although it varies, being maximum at small uptakes then progressively tending to one at high uptake. The small value of the amplification factor found (7.5) is in agreement with the recent finding by Takeuchi and Kurahashi (2005) that the second-messenger (cAMP) production in the frog ORN cilia is extremely small with respect to cGMP hydrolysis in the rod photoreceptor.

Second, the rise (Fig. 2B and E) and fall (Fig. 2C and F) times of $E^\ast_r$ become very similar to those of $R^\ast_r$, especially at low uptake. Although $E^\ast_r$ follows causally $R^\ast$, so that its transients must be delayed and proceed more slowly, this slowing down is very small (rise) or negligible (fall). Therefore, the production and removal of $G^\ast$ and $E^\ast$ must be much faster than those of $R^\ast$. This interpretation is in agreement with Kaisling’s (2001) conclusion that post-receptor processes must be fast enough to follow quickly any change of $R^\ast$ upon a change of stimulus intensity.

4.4. Role of the post-effector steps

If the differences between the model output and the receptor-potential measurements result from the post-effector steps, several consequences follow. The first one is that these steps can significantly increase the dynamic range of the response, which means that the 1% rising point of the electrical response is much lower than that of the effector. As a consequence, most of the amplification in the cascade is expected to arise from the second messenger(s) and the ionic channels. Secondly, the electrical response rises faster than the effector response, so decreasing the half-rise time. This effect may seem paradoxical but can result from the amplification factor just mentioned. Thirdly, possibly for the same reason, the electrical response falls more slowly than the effector response suggesting a significant contribution of the post-effector steps to the decline of the receptor potential.

Acknowledgements

The authors thank Janos Toth for helpful discussions, Karl-Ernst Kaisling for critical comments on successive versions of this paper, and Christine Young for linguistic corrections. This work was supported by ANR Aroma-lim (05-PNRA-002).

Appendix A. System of differential equations

\[ U = k_1 L_{air} \]  
\[ \frac{dL}{dr} = U - k_{LN} L \cdot N + k_{-LN}(LN) - k_a R \cdot L \]
\[ + k_{-a}(RL) \]  
\[ \frac{d(LN)}{dr} = -k_{LN} L \cdot N - (k_{-LN} + k_o)(LN) \]  
\[ \frac{dN}{dr} = -k_{LN} L \cdot N - (k_{-LN} + k_o)(LN) \]  
\[ \frac{dR}{dr} = -k_1 R \cdot L + k_{-1}(RL) \]
\[
\begin{align*}
\frac{d(RL)}{dt} &= k_1 R \cdot L - (k_2 + k_{-1})(RL) + k_{-2} R^* \\
\frac{dR^*}{dt} &= k_2 (RL) - k_{-2} R^* \\
\frac{dG}{dt} &= -e_{RG} R^* \cdot G + e_G G_x G_b \\
\frac{dG^*}{dt} &= e_{RG} R^* \cdot G - e_{GE} G^* \cdot E - k_{Ga} G^*_a \\
\frac{dG_b}{dt} &= e_{RG} R^* \cdot G - e_G G_x \cdot G_b \\
\frac{dG_x}{dt} &= k_{GA} G^*_a + k_G E^* - e_{GE} G_x \cdot G_b \\
\frac{dE}{dt} &= -e_{GE} G^* \cdot E + k_G E^* \\
\frac{dE^*}{dt} &= -e_{GE} G^* \cdot E - k_G E^*
\end{align*}
\]

References


