Probabilistic equalizer for Ultra-Wideband energy detection

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Abstract—This study proposes an efficient way of interference mitigation for ultra-wideband energy detection. A receiver for pulse position modulation systems is investigated. The inter-slot (i.e., intra-symbol) and inter-symbol interferences are studied and a probabilistic equalizer is derived. This energy equalizer is embedded into the loop of an iterative channel decoder. Computer simulations are performed on the channel models from the IEEE 802.15.3a task group.

I. INTRODUCTION

Low-cost Ultra-wideband (UWB) impulse radio (IR) is a promising technology for a broad range of applications [9]. UWB can be employed in personal applications like the wireless personal area networks (WPAN), in civilian applications such as sensor networks or radio frequency identification (RFID), etc. The successful future of this technology is related to its efficiency and its low cost feasibility. An UWB energy detection receiver offers a low cost analog front end with a low power consumption.

This paper considers an impulse radio UWB system with energy detection (non-coherent demodulation) based on Schottky diode and capacitor circuit. A pulse position modulation with $M$ positions per signal (M-PPM) is adopted for simplicity and low cost. However, in highly dispersive channels (e.g., CM3 and CM4 defined in [6]) M-PPM modulation may cause inter-slot interference (IStI) and inter-symbol interference (ISI) at high data rates. Recent research [4] treated the inter-symbol interference in UWB system, but the obtained results can be further improved in highly dispersive channels at high data transmission rates.

One of the keypoints in energy detection is how to define a well matched energy model that takes the interference into account. The dispersion analysis of UWB channel models shows that the maximum excess delay is about 120ns and 250ns for CM3 and CM4 respectively. The number of interfering symbols depends on both the PPM symbol data rate and the channel model.

In this work, we derive a probabilistic energy equalizer that helps the receiver in reducing the interference (both IStI and ISI). In order to meet the low-complexity low-cost constraint, the energy receiver will handle a fixed finite number of interfering symbols while performing energy detection. The energy equalization complexity depends on the considered number of interfering symbols/slots.

The equalizer is referred to as probabilistic, also known as soft-input soft-output, since it applies modern a posteriori probability detection/decoding techniques as described in [2][10]. It can be naturally cascaded with an error-correcting code for iterative processing in the receiver in order to boost up the error rate performance [5][7]. Channel estimation is out of the scope of this paper. The channel state information (CSI) is assumed to be perfectly known at the receiver side.

The paper is organized as follows: Section II describes the system model with interference. The conditional distribution of the received energy is established from that model. The probabilistic energy equalizer is derived in Section III. An example of energy patterns due to interference between two consecutive PPM symbols is also given. In Section IV computer simulation results show the effect of the equalizer on the system performance. The simulations are carried out with and without channel coding for different channels and equalization complexity. Finally, conclusions are given in Section V.

II. SYSTEM MODEL

The system under consideration consists of pulse-based UWB transmission sent over a noisy channel. An M-PPM modulation is adopted with $M$ slots per symbol. As illustrated in Figure 1, the encoded data is mapped into channel symbols suitable for modulation. Then the pulse generator transmits symbols over the channel. This type of modulation, although simple and low cost, can cause inter-slot interference and inter-symbol interference at high data rates. An example of transmitted symbols is depicted in Figure 2(a), where three symbols are sent over the channel model CM4 at a data rate of 100Mbps with a 4-PPM. The channel response for each symbol is given by Figure 2(b) and the output of the channel filter $H$ is displayed in Figure 2(c) which is the sum of the three channel responses.

At the receiver side, we assume that interference will not exceed $K$ symbols, or equivalently, the number of
interfering slots is \( P = (K - 1)M + 1 \). The receiver will decode according to a fixed value for \( P \), i.e., the size of observation window.

The signal at the output of channel filter can be written as follows:

\[
\mathbf{s}_n(t) = \sum_{k=0}^{K-1} x_{n-k}(t)
\]

where \( x_{n-k}(t) \) is the \((n-k)\)th channel response for the \((n-k)\) transmitted symbol, i.e., \( x_{n-k}(t) = e_{n-k}(t) \otimes h(t) \) where \( \otimes \) denotes the convolution product and \( e_{n-k}(t) = p(t - A_{n-k}T_{\text{slot}}) \) where \( p(t) \) is the pulse shape, \( A_{n-k} \) takes value in \( \{0, 1, 2, 3\} \) according to transmitted symbol and \( T_{\text{slot}} \) is the time slot duration for an M-PPM modulation. For simulation reasons, the considered pulse, in this study, is the Dirac delta function, thus \( x_{n-k}(t) = h(t - A_{n-k}T_{\text{slot}}) \). This assumption will not affect our reasoning.

The receiver detector consists of a square operation followed by a finite time integrator as is displayed on Figure 1. Hence the energy per slot duration for the \( m \)th slot in \( n \)th symbol is equal to:

\[
\mathcal{E}_{n,m} = \int_{nT_s + (m-1)T_{\text{slot}}}^{nT_s + (m)T_{\text{slot}}} (s_n(t) + z_n(t))^2 \; dt
\]

where \( T_s = M T_{\text{slot}} \) is the symbol duration and \( z_n(t) \) is an additive white Gaussian noise with mean zero and variance \( \sigma^2 \).

Urkowitz [11] showed that the energy, of duration \( T_{\text{slot}} \), of a process which has a bandwidth \( W \) (negligible energy outside this band) is approximated by a set of sample values \( 2T_{\text{slot}}W \) in number. So we can rewrite (2) as follows:

\[
\mathcal{E}_{n,m} = \sum_{\ell=1}^{2L} (s_{n,m}^\ell + z_{n,m}^\ell)^2
\]

where \( 2L = 2T_{\text{slot}}W \) is the number of freedom degrees over the interval \( T_{\text{slot}} \), and \( s_{n,m}^\ell \) and \( z_{n,m}^\ell \) are respectively the \( \ell \)th sample of \( s_n(t) \) and \( z_n(t) \) in \( m \)th slot of \( n \)th symbol.

We note that the energy per slot is defined as \( \mathcal{E}_{n,m} = \sum_{\ell=1}^{2L} X_\ell^2 \), where \( X_\ell = (s_{n,m}^\ell + z_{n,m}^\ell) \) is a Gaussian random variable with mean \( s_{n,m}^\ell \) and variance \( \sigma^2 \). Hence, if \( \sum_{\ell}(s_{n,m}^\ell)^2 \neq 0 \), \( \mathcal{E}_{n,m} \) follows a non-central chi-square \((\chi^2)\) distribution with \( 2L \) degrees of freedom as defined in [8]. Its probability density function, also referred to as the
channel observation for probabilistic detection, is

\[
p(E_{n,m}|s_{n,m}) = \frac{1}{2\sigma^2} \left( \frac{E_{n,m}}{B_{n,m}} \right)^{\frac{L-1}{2}} e^{-\left(\frac{E_{n,m}+s_{n,m}}{2\sigma^2}\right)}
\]

where, by definition

\[
B_{n,m} = \sum_{l=1}^{2L} (s_{n,m}^l)^2 = (s_{n,m})^2
\]

is the resultant energy on the slot \( m \) of the \( n^{th} \) symbol.

\( I_{L-1}(u) \) is the \((L-1)^{th}\)-order modified Bessel function of the first kind [1]. If \( \sum (s_{n,m}^l)^2 = 0 \), the energy distribution has the following form [8]:

\[
p(E_{n,m}|0) = \frac{1}{\pi^{\frac{L-1}{2}}(L)} \pi_{L-1}^{\frac{1}{2}} e^{-\frac{E_{n,m}}{2\sigma^2}}
\]

where \( \Gamma(z) \) is the gamma function [1].

### III. Energy Equalization

The optimal receiver computes the a posteriori probability (APP) for each \( x_n \) in order to use the SISO decoder. The a posteriori probability of a received symbol \( x_n \) is defined by \( APP(x_n) = p(x_n|\mathcal{E}) \) where \( \mathcal{E} = (\mathcal{E}_1, \ldots, \mathcal{E}_n) \) is the total energy of the transmitted frame for \( N \) symbols and \( \mathcal{E}_n = (\mathcal{E}_{n,1}, \ldots, \mathcal{E}_{n,M}) \). Since the exact computation of \( APP(x_n) \) renders an extremely complex receiver it is important to derive an efficient sub-optimal detection.

The energy detector computes the extrinsic probability of \( x_n \) that is employed for the APP computation. So the detector computes the conditional probability density function \( p(\mathcal{E}_n|x_n) \) to get the possible transmitted symbol. Using the marginalization over the interfering symbols, the conditional density can be rewritten according to the channel observation (4) and (6) defined in Section II as follows:

\[
p(\mathcal{E}_n|x_n) = \sum_{x_{n-1}} \cdots \sum_{x_{n-K+1}} p(\mathcal{E}_n, x_{n-1}, \ldots, x_{n-K+1}|x_n)
\]

Then, applying the Bayes’ law with the knowledge that the set \( \{x_{n-i}\}_{i=0}^{K-1} \) are independent, we get:

\[
p(\mathcal{E}_n|x_n) = \sum_{x_{n-1} \ldots x_{n-K+1}} p(\mathcal{E}_n|x_n, x_{n-1}, \ldots, x_{n-K+1}) \prod_{k=1}^{K-1} \pi(x_{n-k})
\]

where \( \pi(x_{n-k}) \) is the a priori probability of \( x_{n-k} \) provided by the decoder. The first part of equation (8) can be simplified even more. In fact using (1) it follows that:

\[
p(\mathcal{E}_n|x_n, x_{n-1}, \ldots, x_{n-K+1}) = p(\mathcal{E}_n|s_n)
\]

moreover \( \mathcal{E}_n = (\mathcal{E}_{n,1}, \ldots, \mathcal{E}_{n,M}) \), \( s_n = (s_{n,1}, \ldots, s_{n,M}) \), and each energy slot \( \mathcal{E}_{n,m} \) depends only on the received signal \( s_{n,m} \), then

\[
p(\mathcal{E}_n|s_n) = \prod_{m=1}^{M} p(\mathcal{E}_{n,m}|s_{n,m})
\]

this means that if we neglect the noise and we know the sent symbols \( (x_n, x_{n-1}, \ldots, x_{n-K+1}) \), we could know the received signal at slot \( m \) in the \( n^{th} \) symbol, hence, we can determine the energy \( B_{n,m} \) as defined by (5). So this leads to:

\[
p(\mathcal{E}_{n,m}|s_{n,m}) = p(\mathcal{E}_{n,m}|B_{n,m})
\]

From the above equality, equalization is reduced to the energy per slot defined as follows:

\[
p(\mathcal{E}_{n,m}|x_n) = \sum_{x_{n-1} \ldots x_{n-K+1}} \left( \prod_{m=1}^{M} p(\mathcal{E}_{n,m}|B_{n,m}) \prod_{k=1}^{K-1} p(x_{n-k}) \right)
\]

The complexity of the equalizer depends on the number of \( B_{n,m} \) that it needs. As an example we consider \( K = 2 \), this includes that \( P = 5 \), i.e. we suppose that the excess delay for a certain channel does not exceed 5 slots. Then we try to determine all possible cases of interference regardless of the additive noise. This leads to Table I whose notations has been changed for a better comprehension of the interference. In fact, the index of energies \( B \) represent the position of the pulses according to the considered slot. As an example, we consider \( B_{3,4} \), it means that the pulse for symbol \( x_n \) is transmitted three slots behind, and the pulse for \( x_{n-1} \) is transmitted four slot behind.

<table>
<thead>
<tr>
<th>( x_{n-1} )</th>
<th>00</th>
<th>01</th>
<th>10</th>
<th>11</th>
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<tbody>
<tr>
<td>00</td>
<td>( B_{0,4} )</td>
<td>( B_1 )</td>
<td>( B_2 )</td>
<td>( B_3 )</td>
</tr>
<tr>
<td>01</td>
<td>( B_{0,3} )</td>
<td>( B_{1,4} )</td>
<td>( B_2 )</td>
<td>( B_3 )</td>
</tr>
<tr>
<td>10</td>
<td>( B_{0,2} )</td>
<td>( B_{1,3} )</td>
<td>( B_{2,4} )</td>
<td>( B_3 )</td>
</tr>
<tr>
<td>11</td>
<td>( B_{0,1} )</td>
<td>( B_{1,2} )</td>
<td>( B_{2,3} )</td>
<td>( B_{3,4} )</td>
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<thead>
<tr>
<th>( x_n = )</th>
<th>00</th>
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<td>( B_1 )</td>
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<td>( B_3 )</td>
</tr>
<tr>
<td>00</td>
<td>( B_4 )</td>
<td>( B_0 )</td>
<td>( B_1 )</td>
<td>( B_2 )</td>
</tr>
<tr>
<td>01</td>
<td>( B_4 )</td>
<td>( B_0 )</td>
<td>( B_1 )</td>
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</tr>
<tr>
<td>10</td>
<td>( B_4 )</td>
<td>( B_0 )</td>
<td>( B_1 )</td>
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</tr>
<tr>
<td>11</td>
<td>( B_4 )</td>
<td>( B_0 )</td>
<td>( B_1 )</td>
<td>( B_2 )</td>
</tr>
</tbody>
</table>

Table I: Exhaustive listing of energy patterns \( B_{n,m} \) for two consecutive 4-PPM interfering symbols, \( P = 5 \).
before, the parameter $P$ represents the presumed number of interfering slots at the receiver side, so the index in Table I will not exceed $(P - 1)$. An element like $B_1$ means that the energy comes from only one pulse $x_n$ or $x_{n-1}$. One notices, that for a certain combination of symbols the receiver supposes that there is not interference at certain slots. For instance if $x_n = 11$ and $x_{n-1} = 00$ are sent, the equalizer presumes that there is not interference in slot 2 and slot 3 of symbol $x_n$.

We notice that the number of energies $B_{n,m}$ is finite, which is equal 15 for this example. So the receiver will compute equalization over a finite number of energies $\{B_{n,m}\}$. And this set grows according to the channel dispersivity. Let $|\{B_{n,m}\}|$ denotes the number of elements in the set $\{B_{n,m}\}$, table II shows the relation between $|\{B_{n,m}\}|$ and the parameters $K$ and $P$:

| $K$ | $P$ | $|\{B_{n,m}\}|$ |
|-----|-----|----------------|
| 2   | 5   | 15            |
| 3   | 9   | 88            |
| 4   | 13  | 424           |

We remark that $|\{B_{n,m}\}|$ grows greatly if $K$ increases slightly. This involves a higher complexity at the equalizer level, since it has to go through all the possible values of $B_{n,m}$.

IV. Simulation in a Perfect CSI

The simulations in this paper are done for perfect CSI. CIR is required to determine the energy channel parameter according to the number of interfered symbols. 4-PPM modulation is assumed. First simulations are performed without channel coding for different number of interfered symbols and for different channel models (CM1, CM2, CM3 and CM4). We simulated the bit error rate (BER) at 100 Mbps. Then a channel encoder is added to study the performance of the receiver.

A. Simulation without channel coding

The simulations are performed with and without equalizer for $K = 2, 3$ and 4. This means that the number of presumed interfering slots is $P = 5, 9$ and 13 respectively. In fact the number of interfering slots could be much bigger than the supposed one by the receiver. We notice from Figure 3 that the probabilistic equalizer improves the receiver for both channel model CM1 and CM2 with $P = 5$.

However, for highly dispersive channels such as CM3 and CM4, the receiver performances are not exploitable. This is due to the low considered value of $P$ compared to the real number of interfering slots. We extended our investigation to $P = 9$ and $P = 13$ as shown on Figure 4 and Figure 5 respectively. The performances are better for high dispersive channel such as CM3 and CM4 and could be improved by channel encoder. A channel encoder is then
added with a SISO decoder in order to exploit the output of the probabilistic equalizer at best.

B. Simulation with channel coding

In this part, simulations are computed with channel encoder. A bit interleaved coded modulation (BICM) [3] is used. A convolutional channel encoder at rate $1/2$ with octal generator $(23, 35)$ followed by a pseudo-random bit-inter-leaver is implemented. The frame has a length of 1024 bits and the SISO decoder computes 10 iterations. At each SISO iteration the equalizer computes an update and forwards the probabilities to the decoder. Figure (6) shows the results for different channel models with $P = 5$. The receiver is improved for channel models CM1 and CM2, but for CM3 and CM4 the receiver is slightly improved when $P = 5$.

Figure 6. BER for different channel models using BICM(23,35) at rate 1/2 with $P = 5$.

Figure 7. BER for different channel models using BICM(23,35) at rate 1/2 with $P = 9$.

Figures 7 and 8 illustrate the bit error rate, for different channel models, with $P = 9$ and $P = 13$ respectively. For $P = 9$ and $P = 13$, in Figures 7 and 8, the receiver is more efficient for CM3 and CM4, although it requires a great number of calculation. One notices from Figure 8 compared to Figure 5 that the result obtained for CM3 is well satisfying since the signal-to-noise ratio gain is about $3$dB around $10^{-2}$.

V. Conclusion

A probabilistic equalizer for energy detection based UWB system using the IEEE 802.15.3a channel models have been derived. Our equalizer takes into account the channel energy profile with a few number of parameters in order to simplify the calculations. The results show that this energy equalizer improves the system performance, especially when an iterative decoder is added. This allows to get high data rates with a mere receiver based on energy detection.

References