Practical Resource Allocation for the Broadcast Phase of Three-Step Bidirectional Relaying

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Abstract—In this paper, we present a low-complexity algorithm for the broadcast of three-step bidirectional relaying that decides which resources should be used for a network coding transmission, and which resources should be used for a conventional transmission. The algorithm requires a simple comparison operation for every transmission resource, and is independent from one resource to another thus enabling a parallel processing per transmission resource if desired. The comparison metric is based on information readily available in today’s wireless networks, hence not requiring any additional overhead signaling. Simulation results show that the proposed algorithm provides comparable gains to the higher-complexity heuristic-based approaches found in the literature, rendering it a good candidate for practical implementation purposes.

Key Words: Network Coding, Bidirectional Relaying, Resource Allocation.

I. INTRODUCTION

A common trend characterizing the evolution of wireless networks is the increase in achievable bit rates and the provision of a continuously enhanced quality of service (QoS) for the end-user. Among other indicators, this trend is exemplified by the increase of bit rates from the order of Mbps to Gbps with 4G systems, and the further decrease in the user plane and control plane delays. In the industrial world, such improvements are enabled through the use of e.g. advanced radio resource management and antenna techniques [1]. Meanwhile, research in academia is already targeting beyond 4G systems to further improve capacity and throughput of wireless networks. One of the very promising techniques is network coding (NC) [2]. NC is a new area within networking, in which data is manipulated at intermediate nodes inside the network to improve throughput, delay, and/or robustness. In particular, NC allows the nodes to recombine several input packets into fewer output packets, hence decreasing the number of needed transmissions while delivering the same amount of information. Specifically, the area of bidirectional relaying (BR) has attracted a lot of interest [3], [4]. BR refers to the scenario where two end-nodes \( U_1 \) and \( U_2 \) want to exchange data through a third relaying node \( RN \). In this work, the main focus is on the resource allocation aspect for the broadcast phase of three-step bidirectional relaying.

A. Previous Work and Existing Solutions

Initial works in the area of (three-step) bidirectional relaying focused on the application of NC to pure time-division multiplexing systems where the transmission to/from a certain node occupies the whole operational bandwidth [3], [4]. However, recent studies are starting to focus on more advanced multiple access schemes by considering e.g. OFDMA-based systems as in [5], [6], [7]. In an OFDMA system, the transmission occurs on multiple distinct resources. So given two end-nodes that would like to exchange data through bidirectional relaying, an interesting question for the transmission from the relay node is: which resources should be used for transmission based on network coding, and which resources should be used for conventional transmission\(^1\)? This problem is referred to as the ‘resource allocation at the relay node’ and some methods to address it have been proposed in the literature [5], [6], [7]. In particular, the authors in [5], [6] aim at assigning appropriate resources in a way that optimizes the maxmin throughput of bidirectional flows encoded by the base station in a cell while avoiding session starvation. To reduce the complexity of the solution, an alternative heuristic algorithm is proposed and evaluated. The work in [7] examines the impacts of physical layer parameters on the throughput gain in OFDMA relay structure and formulates optimization frameworks to exploit the network capacity in slow frequency selective fading channels. The authors also examine how wireless nodes can select relay paths in the presence of NC and how network resources can be assigned for the transmitting nodes.

B. Scope, Motivation and Contribution of this Work

In this work, and similar to [5], [6], we confine ourselves to the study of resource allocation for the broadcast phase of three-step bidirectional relaying assuming an OFDMA system. By resource allocation we mean which resource blocks \(^2\) (RB) should be used for NC, and which RBs should be used for conventional (i.e. non-NC) transmission. In other words, we assume that the end-nodes that want to exchange information, and the relay node performing the relaying or NC operation, are already known. As such, the problem of identifying which users should be selected to perform NC on is outside the scope of this work, and interested readers are referred to [8], [9].

Although the algorithms presented in [5], [6], [7] are polynomial time in nature and quicker than the brute force exhaustive search algorithms, they are still too computationally exhaustive for practical implementations. Radio resource management (RRM) schemes such as scheduling and link adaptation in today’s wireless networks (e.g. LTE and WiMax)
should make decisions in less than a millisecond. These
decisions include e.g. which packets to transmit, on which part
of the spectrum, with how much power and utilizing which
modulation and coding scheme (MCS), just to mention a few
operations. With an already tight delay budget available to the
scheduler, it is of significant interest to propose extremely fast
resource allocation schemes for NC in order to be considered
as a candidate for practical implementation.

The main contribution of this work is a low-complexity
algorithm, in two variants, that is proposed for deciding
which resources should be used for NC, and which should
be used for conventional transmission. The algorithm requires
a simple comparison operation that is done per RB and
that is independent from one RB to another; as such, the
algorithm can be run in parallel for the different RBs in the
system. Equally important is that the variables needed for the
comparison operation are based on values readily available in
today’s systems: the channel quality indicator (CQI) and the
resultant modulation and coding scheme (MCS) of the links on
which transmissions will/might occur. As such, the proposed
algorithm does not require any extra overhead transmissions.
The proposed algorithm is compared to three different baseline
algorithms in terms of spectral efficiency at network level and
their effect on the fairness performance.

The rest of this paper is organized as follows. In Section II,
we present the studied system model. Section III contains the
baseline schemes used for comparison. The proposed method
is presented in Section IV. The simulation model and results
are included in Section V. We finally conclude in Section VI.

II. SYSTEM MODEL

In this work we study the broadcast phase of three-step
bidirectional relaying where RN serves two end-nodes \( U_1 \) and
\( U_2 \). We assume a backlogged traffic model (i.e. full
buffer) at RN. This means that RN always has enough data
to transmit, and all this data may be network coded. Furthermore,
we assume an OFDMA system with adaptive coding and
modulation (ACM). This means that different MCS can be
used on the different RBs based on the CQI reports of each
RB\(^3\). Given a certain MCS, RN is then able to estimate a
spectral efficiency measure \( \eta_i^k \) for RB \( k \) and for end-node \( i \)
as follows:

\[
\eta_i^k = r_i^k \times m_i^k; \quad i = 1, 2
\]

(1)

where \( r_i^k \) and \( m_i^k \) are the code rate and modulation order
obtained by mapping the CQI of RB \( k \) and end-node \( i \) onto
a certain MCS. For example, if for the same RB \( k \) the MCS
of \( U_1 \) consists of a coding rate of 1/2 along with a 64QAM
modulation (i.e. \( m_1^k = 6 \)), whereas that of \( U_2 \) consists of a
coding rate of 3/4 and a QPSK modulation (i.e. \( m_2^k = 2 \)), then
one would obtain the following spectral efficiency measures:
\( \eta_1^k = 3[b/s/Hz] \) and \( \eta_2^k = 1.75[b/s/Hz] \).

The proposed algorithm performs resource allocation by
simply basing its decision on the spectral efficiencies of the
different links that are available at RN prior to any trans-
mission, thus avoiding the more complicated maximization
problems. In this work, we consider two different values of
spectral efficiency: per end-node and per scheme. Spectral
efficiency per end-node is the spectral efficiency measured for
each end-node \( U_i \) separately, averaged over all the RBs in
the system. Spectral efficiency per scheme is the sum of the
spectral efficiencies of the two end-nodes for a given resource
allocation scheme at RN.

III. CONVENTIONAL RESOURCE ALLOCATION SCHEMES

In this part, we introduce the three conventional resource
allocation schemes that the proposed method will be compared
to. The first two schemes do not use NC at the relay, whereas
the third consist of the classical NC with bitwise XOR.
These schemes will be parameterized in terms of the spectral
efficiency measure \( \eta_i \) i.e. using the proposed method, in order
to provide a fair and straightforward comparison with the
proposed algorithm.

A. RR: Round Robin Allocation

Round Robin (RR) allocation with conventional relaying
represents the case where no NC is performed at RN and each
end-node is assigned every other RB. This means that half of
the RBs will be assigned to each end-node. The achievable
spectral efficiency per end-node for RB \( k \) is denoted by \( \eta_{i,RR}^k \)
and can be computed as:

\[
\eta_{i,RR}^k = \begin{cases} 
\eta_i^k & \text{if } k \in U_i \\
0 & \text{otherwise}
\end{cases}
\]

where \( \eta_i^k \) is the spectral efficiency for RB \( k \) obtained based
on the prevailing channel condition \( h_i \) over \( k \) as in (1), and
\( k \in U_i \) means that RB \( k \) is allocated to \( U_i \). The RBs that are
not assigned to a certain end-node (due to the Round Robin
allocation) will then yield a zero spectral efficiency, as shown
above. The average spectral efficiency per end-node \( \eta_{i,RR} \)
is then obtained by averaging \( \eta_{i,RR}^k \) over all RBs. Consequently,
the spectral efficiency per scheme of round robin allocation
\( \eta_{RR} \) which is computed as the sum of the two end-nodes’
average spectral efficiencies is given by:

\[
\eta_{RR} = \eta_{1,RR} + \eta_{2,RR}
\]

(2)

B. OPP: Opportunistic Allocation

Opportunistic allocation (OPP) represents the case where
no NC is performed at RN and each RB is assigned to
the end-node that has the highest spectral efficiency on that
RB. This would increase the throughput on the expense of
fairness, especially when one of the end-nodes always has
better channel quality compared to the other. For a given RB
\( k \), the resultant spectral efficiency per end-node \( \eta_{i,OPP}^k \)
will be given by:

\[
\eta_{i,OPP}^k = \begin{cases} 
\max(\eta_1^k, \eta_2^k) & \text{if } \eta_i^k = \max(\eta_1^k, \eta_2^k) \\
0 & \text{if } \eta_i^k = \min(\eta_1^k, \eta_2^k)
\end{cases}
\]

In today’s systems such as LTE, CQI reports are always available at the
base station prior to data transmission to users.
The average spectral efficiency per end-node $\eta_i,OPP$ is obtained by averaging $\eta_i^k,OPP$ over all RBs in the system. Consequently, the spectral efficiency per scheme of the opportunistic allocation $\eta_{OPP}$ can be computed as:

$$\eta_{OPP} = \eta_i,OPP + \eta_{2,OPP} \quad (3)$$

C. XOR: Conventional Network Coding

XOR represents the case of conventional bitwise XOR network coding where RN uses all the RBs for NC transmissions. As the network coded signal will be sent to both end-nodes, it has to be coded and modulated with an MCS that ensures a correct reception at the weakest receiver. For a given RB $k$, this means that the resultant spectral efficiency per end-node $\eta_i^k,XOR$ will be the same for both end-nodes and is given by:

$$\eta_i^k,XOR = \min\{\eta_1^k; \eta_2^k\}; \quad i = 1, 2 \quad (4)$$

The average spectral efficiency per end-node $\eta_i,XOR$ is obtained by averaging $\eta_i^k,XOR$ over all RBs. Consequently, the spectral efficiency per scheme of conventional network coding $\eta_{XOR}$ can be computed as:

$$\eta_{XOR} = 2 \times \eta_i,XOR \quad (5)$$

IV. PROPOSED RESOURCE ALLOCATION SCHEME

Here we introduce the proposed resource allocation algorithm in its two variants, ONC and RNC, which aim at identifying which resources to utilize for NC and which resources to utilize for conventional transmission. They are intended as low-complexity alternatives to the heuristic-based algorithms presented in e.g. [5], [6], [7]. The low-complexity stems from the fact that the proposed algorithm requires a simple comparison operation of two spectral efficiency measures to decide whether NC should be performed on a certain RB or not.

A. ONC: Opportunistic Network Coding Allocation

Opportunistic network coding (ONC) allocation is the first resource allocation variant of the proposed algorithm. In this variant, for every RB, NC is only applied when it results in a higher spectral efficiency compared to conventional opportunistic transmission. A flowchart description of this variant is shown in Fig. 1. Mathematically, the resource allocation choice is based on the following simple comparison that is done independently for every RB $k$:

$$2 \times \min\{\eta_1^k; \eta_2^k\} \leq \max\{\eta_1^k; \eta_2^k\} \quad (6)$$

where the left-hand side (LHS) represents the NC-favorable case and the right-hand side (RHS) represents the conventional opportunistic-favorable case. As long as a NC transmission provides a higher or equal spectral efficiency compared to opportunistic transmission over a certain RB, NC transmission will be chosen. As such, the resultant spectral efficiency per end-node $\eta_i^k,ONC$ over RB $k$ can be written as:

$$\eta_i^k,ONC = \begin{cases} \eta_i^k,XOR & \text{if } 2 \times \min\{\eta_1^k; \eta_2^k\} \geq \max\{\eta_1^k; \eta_2^k\} \\ \eta_i^k,OPP & \text{otherwise} \end{cases} \quad (7)$$

B. RNC: RR Network Coding Allocation

Round Robin network coding (RNC) allocation is the second resource allocation variant of the proposed algorithm. In this variant, for every RB, NC is only applied when it results in a higher spectral efficiency compared to conventional relaying with Round Robin resource allocation. A flowchart description of this variant is shown in Fig. 2. Mathematically, the resource allocation choice is based on the following simple comparison that is done independently for every RB $k$:

$$2 \times \min\{\eta_1^k; \eta_2^k\} \leq \eta_i^k \quad (8)$$

where $i$ is the index of the end-node to which RB $k$ was supposed to be allocated to in the REL scheme, LHS represents the NC-favorable case and RHS represents the conventional RR-favorable case. As long as a NC transmission provides a higher or equal spectral efficiency compared to RR allocation over a certain RB, NC transmission will be chosen. As such, the resultant spectral efficiency per end-node $\eta_i^k,RNC$ over RB $k$ can be written as:

$$\eta_i^k,RNC = \begin{cases} \eta_i^k,XOR & \text{if } 2 \times \min\{\eta_1^k; \eta_2^k\} \geq \eta_i^k \\ \eta_i^k,RR & \text{otherwise} \end{cases} \quad (9)$$

The average spectral efficiency per end-node $\eta_i,RNC$ is obtained by averaging $\eta_i^k,RNC$ over all RBs in the system.
TABLE I
MCS SWITCHING THRESHOLD AND DERIVED SPECTRAL EFFICIENCY

<table>
<thead>
<tr>
<th>Modulation</th>
<th>Code Rate</th>
<th>Min. SINR [dB]</th>
<th>η [b/s/Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>QPSK</td>
<td>1/2</td>
<td>5.0</td>
<td>1.0</td>
</tr>
<tr>
<td>QPSK</td>
<td>3/4</td>
<td>8.0</td>
<td>1.5</td>
</tr>
<tr>
<td>16QAM</td>
<td>1/2</td>
<td>10.5</td>
<td>2.0</td>
</tr>
<tr>
<td>16QAM</td>
<td>3/4</td>
<td>14.0</td>
<td>3.0</td>
</tr>
<tr>
<td>64QAM</td>
<td>2/3</td>
<td>18.0</td>
<td>4.0</td>
</tr>
<tr>
<td>64QAM</td>
<td>3/4</td>
<td>20.0</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Consequently, the spectral efficiency per scheme of the proposed RR network coding $\eta_{RNC}$ can be computed as:

$$\eta_{RNC} = \eta_{1,RNC} + \eta_{2,RNC}$$

Obtain $\eta_1^k$ and $\eta_2^k$  

No  

Yes  

$\eta_1^k > 2 \times \min\{\eta_1^k; \eta_2^k\}$  

XOR  

RR

V. SIMULATION SETUP AND RESULTS

In this section, we compare the performance of the proposed resource allocation schemes with the three different conventional ones by examining spectral efficiency and fairness measures. Two generic scenarios are studied: symmetric channels where the link between RN and $U_1$ and the link between RN and $U_2$ have an equal average SNR; and asymmetric channels where the link between RN and $U_2$ is fixed to an average of $20\,dB$ whereas the link between RN and $U_1$ varies between $0\,dB$ and $30\,dB$. The results are based on system-level simulations with adaptive coding and modulation. The MCS selection is based on Table I from [10] and ideal link adaptation is assumed. For simplicity, we assume equal transmit power for all RBs. The C2 metropolitan area path loss and channel model from [11] are used in the evaluations. Non-line-of-sight propagation is log-normally distributed with a standard deviation of $8\,dB$. We assume an LTE-like system where each RB consists of 12 sub-carriers with a sub-carrier spacing of $15\,KHz$, resulting in RBs of $180\,KHz$. With a bandwidth of $20\,MHz$, 110 RBs are obtained out of which 10 are used for control information and the remaining 100 are used for data transmission.

A. Spectral Efficiency Performance

The spectral efficiency performance of the different resource allocation schemes is compared by examining the relative gains that these schemes provide compared to the conventional relaying case with RR. The results for symmetric and asymmetric channels can be seen in Fig. 3. and Fig. 4, respectively.

Strictly examining the broadcast phase, XOR should provide 100% gain (i.e. ratio of 2) when compared to RR as it requires one transmission to deliver ideally the same amount of information that RR does with two transmissions instead. However, due to varying instantaneous channel conditions, XOR does not provide such gains even for symmetric channels. Although the two links could have an equal average SNR, instantaneously one of them would typically need a lower MCS compared to the other, thus limiting the gains achievable by XOR as could be expected from (4). At low SNR regions,
XOR would have a significantly worse performance as the worst link might not be able to support any data. OPP has its best performance at low SNR regions where one of the end-nodes is not able to receive any information, whereas the other end-node is instantaneously better. On the other hand, the proposed ONC would ensure to select the resource allocation that achieves the highest system spectral efficiency and it always operates at gains close to the 90% compared to RR. These gains are comparable to the ones obtained in [5], [6]; the main difference is the significantly lower complexity and implementation-friendliness of the proposed scheme. One should also keep in mind that resource allocation by itself is not able to always achieve the maximal gain of 100%; achieving such gains is also dependent on how the NC operation is performed at RN as a simple XOR will always be limited by the weakest link. The other proposed variant, RNC, achieves lower gains compared to ONC especially when one of the end-nodes has a poor link since neither RR nor XOR are able to provide a good performance in such cases. However, RNC performs as an upper bound to the XOR scheme. In general, the gains of the two proposed variants arise from the selective application of NC as a function of the instantaneous channel conditions of the two end-nodes, with ONC being the most selective of the two.

### B. Fairness Performance

One would expect that the large gains of ONC are caused by an imbalance in the number of assigned RBs to each end-node. As such, we investigate the fairness performance by examining the allocation symmetry factor (ASF) which is defined as the number of RBs allocated to the end-node that obtained less RBs, divided by the number of RBs allocated to the end-node that obtained more RBs. As such, $\text{ASF} \leq 1$ where 1 means complete symmetry in allocation to both end-nodes for a given scheme. The results for symmetric and asymmetric channels can be seen in Fig. 5 and Fig. 6, respectively.

In case of symmetric links, all schemes would allocate resources equally on average as all schemes have an ASF of about 1. This means that the gains shown in Fig. 5 are not obtained through over-allocating resources to one of the end-nodes but instead through a smarter non-random resource allocation, especially with ONC. However, the ASF performance looks different in case of asymmetric links where only RR, XOR and RNC have an ASF of 1. That is because RR and XOR will, regardless of instantaneous channel quality, allocate resources equally to both end-nodes. RNC would still allocate resources equally while still taking the channel quality into account as it effectively consists of a combined XOR and RR implementation. On the other hand, the resource allocation of OPP and ONC are strictly dictated by the channel conditions. With a large asymmetry between the different links, more resources are allocated to the end-node with better link. However, a very interesting observation can be made by comparing the results from spectral efficiency (i.e. Fig. 3 and Fig. 4) and from allocation symmetry (i.e. Fig. 5 and Fig. 6). The allocation symmetry is affected by the average channel quality whereas the spectral efficiency performance is affected by the instantaneous channel fluctuations. This is reflected by both ONC and OPP achieving an ASF of 1 for the case of symmetric channels, while at the same time being able to opportunistically allocate these resources to provide high gains compared to the non-opportunistic schemes (compare Fig. 3 and Fig. 5). This means that opportunistic allocation schemes can be safely used for the case of symmetric channels to achieve high spectral efficiency without compromising the symmetry of resource allocation between the end-nodes.

### C. End-Node Performance

Allocating more resources to one end-node compared to the other does not conclusively result in an unbalanced system. Particularly, if the two end-nodes are exchanging files of different sizes, an asymmetric distribution of resources is desirable so that a higher data rate is experienced by the end-
node requesting the larger file. As such, we examine the end-node performance for the different schemes by looking at the rate symmetry factor (RSF) which is defined as:

\[ RSF = \frac{\min\{\eta_1; \eta_2\}}{\max\{\eta_1; \eta_2\}} \]  \hspace{3cm} (10)

The RSF shows how much less bits the end-node with the weaker link will be able to receive compared to the end-node with the stronger link. The results for symmetric and asymmetric channels can be seen in Fig. 7 and Fig. 8, respectively.

\[ \text{Symmetric links} \]

![Symmetric links graph](image)

**Fig. 7.** Rate symmetry factor for symmetric channels.

\[ \Gamma_2 = 20\text{dB} \]

![Asymmetric links graph](image)

**Fig. 8.** Rate symmetry factor for asymmetric channels.

In case of symmetric links, and similar to ASF, both end-nodes will achieve the same rate for a given resource allocation scheme. However, one should keep in mind that in absolute value, the end-nodes using e.g. ONC will achieve higher rate than the ones using RR as shown in Fig. 3. Asymmetry in file sizes in case of symmetric channels will lead to an under-utilization of the stronger end-node’s link. In case of asymmetric links, only XOR will result in an RSF of 1 as both end-nodes will always have the same efficiency: that of the weaker end-node. Comparing the RSF of ONC and RNC to that of RR, one can see that ONC allocates more resources to the stronger end-node whereas RNC allocates more resources to the weaker end-node, making each more suitable to different asymmetric file sizes. However, for a given file size and operating channel conditions, a combination of the spectral efficiency per scheme and the RSF are needed to determine which resource allocation scheme is best.

**VI. CONCLUSION**

In this paper, we have presented a low-complexity method that determines which of a plurality of resources should be used for network coding at the broadcast phase of three-step relaying. The proposed method has two variants that trade-off spectral efficiency and fairness. In terms of achievable rate, the proposed method has a similar performance to heuristic-based maximization algorithms that are already proposed in the literature, rendering it a plausible candidate for practical implementation purposes. We have argued that spectral efficiency measures at both system level (i.e. per scheme) and end-node level are needed to select a suitable resource allocation scheme that takes the asymmetry in file sizes to be exchanged into account. This asymmetry in file size can be balanced through an asymmetry in resource allocation by selecting more opportunistic allocation schemes. It has also been shown that the resource allocation symmetry is affected by the average channel quality whereas the spectral efficiency performance, at scheme and end-node levels, is affected by the instantaneous channel fluctuations. As a future work, we will explicitly model files with specific sizes and examine their effect on the end-to-end performance.

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