Model Based Current Analysis of Electrical Machines to Detect Faults in Planetary Gearboxes

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Abstract—Vibration based diagnosis to detect damage in gearboxes has been studied to schedule maintenance and reduce possible capital losses from gearbox failures. However, such vibration based techniques are difficult to implement in planetary gearboxes due to the complex nature of measured vibration spectrum. Motor current signal analysis (MCSA) provides an alternative and non-intrusive way to detect mechanical faults through electrical signatures. However, literature still lacks reported investigation to monitor a planetary gearbox in an electro-mechanical drive-train using MCSA. A successful practical implementation of MCSA to planetary gearboxes is challenging because of the harmonics present in the measured current signal that arises from: 1) the structural properties of the electrical machines, and 2) the errors in manufacturing and assembly of large number of gears. This paper proposes a novel fault detection method that extends the resonance demodulation technique from vibration analysis to MCSA for detection of the gear faults where the measured stator current signal may contain high background noise. The capability of this approach is demonstrated through simulation. A nonlinear multi-DOF lumped parameter model is developed for an electro-mechanical drive-train consisting of a driving motor connected through a back-to-back planetary gearbox to a load generator. Afterwards, a seeded gear tooth fault is introduced in this electro-mechanical model to investigate its effect on the stator current. Simulation results result in a higher fault indicator under faulty gear conditions which demonstrate the feasibility of using resonance demodulation based MCSA for monitoring of mechanical faults such as those arising from gearboxes.

Index Terms—Demodulation technique, fault detection, MCSA, planetary gearbox

Nomenclature

- $i$: Subscripts for sun gear.
- $a$: Subscripts for annulus gear.
- $p$: Subscripts for planet gear.
- $x$: Subscripts for carrier of planetary gearbox.
- $i$: Subscript representing the $i$th planet gear (1 to $P$).
- $j$: Subscript representing the $j$th planetary gear set (1 or 2).
- $c$: Damping coefficient of the gear meshing or couplings.
- $k$: Stiffness of gear meshing or shaft couplings.
- $P$: Number of planets in the planetary gearbox.
- $f_{GM}$: Gear meshing frequency in the planetary gearbox.
- $f_{GF}$: Gear fault signature frequency in the planetary gearbox.
- $\lambda$: Stator winding flux linkage.

I. INTRODUCTION

Electrical machines connected to mechanical drive-trains containing planetary gearboxes are employed in several heavy-duty applications, such as wind turbines [1, 2], maritime vessels, helicopters and automobiles, as they permit a larger transmitted torque to weight ratio, easier control, high reliability and efficiency, and reduced maintenance costs. However, failure of the planetary gearbox can result in catastrophic accidents, long down-time and expensive maintenance. Therefore, detecting such failures at an early stage can significantly reduce the associated capital losses and down-time. The fault detection of gearboxes has traditionally been carried out using vibration and acoustic emission based methods [3, 4], but practical implementation of such approaches can encounter several challenges, such as, (a) vibration and acoustic signals often contain significant noise associated with external perturbations, (b) measurement is sensitive to the sensor location and the optimal sensor location may not be accessible, and (c) measurement can be affected by the operating environment for the system [5].

Motor current signal analysis (MCSA) provides an alternative and non-intrusive approach to detect the mechanical faults based on the electrical measurements from the electric machines. Compared to the vibration methods, an additional benefit of applying MCSA on planetary gearbox is that the modulation effect due to the planet motion does not appear, because the faulty signal transmitted along the torsional direction is utilized instead of the translation ones. The feasibility of fault diagnosis for fixed-axis gearboxes containing spur, helical, or worm gears through MCSA based on detecting the sidebands located at $f_c \pm m f_{GM} \pm n f_{GF}$ has been demonstrated in [5-10]. The lowest fault indicating frequency components by setting $m = 0$, that is $f_c \pm n f_{GF}$, was used as an indicator to detect the gear tooth fault in [6, 7]. The sidebands around the gear meshing frequency and its harmonics $f_c \pm m f_{GM} \pm n f_{GF} (m \neq 0)$ were proposed as an alternative indicator in [5, 6, 9]. However, for an incipient gear tooth faults, most of these sidebands have rather low amplitude because of which they can be hidden by the harmonics due to the motor structure and noise present in the stator current measurements. Therefore, it is necessary to find the sidebands having high amplitudes that can provide rich fault information, which has still not been investigated for MCSA applications. The sidebands amplitude at a given frequency is determined by the mechanical properties of the...
drive-trains. In case of the vibration based fault detection, the resonance demodulation technique has been used in the vibration signature analysis of the spur gearbox, which indicates that the fault induced sidebands show a high amplitude around the natural frequencies of the spur gearbox [11]. The work presented in this paper extends this approach to extract the fault signature of a planetary gearbox using measured stator current signals that may contain high level of background noise.

In this paper, a novel fault detection scheme to detect planetary gear teeth faults using MCSA is proposed and implemented on an electro-mechanical drive-train consisting of a driving motor (a Permanent Magnet Synchronous Machine, PMSM) connected to a load generator (another PMSM) through a back-to-back planetary gearbox (Fig. 1). A multi-DOF lumped model for the electro-mechanical drive-train is developed by combining models of the individual components together. And this dynamic model is numerically integrated using Newmark’s method to simulate the response of the overall system. Therefore, a seeded annulus gear tooth crack is modeled in this work through a local reduction in gear meshing stiffness function corresponding to each of the crack is modeled in this work through a local reduction in the overall system. Therefore, a seeded annulus gear tooth crack is modeled in this work through a local reduction in gear meshing stiffness function corresponding to each of the other gear set is connected to the load generator. The carriers of the planetary gear set are connected together through a steel spline, and the annulus gears are fixed to the housing. Each planetary gear set consists of four equally spaced planets as illustrated in Fig. 3.

![Fig. 1. Schematic of investigated electro-mechanical drive-train](image)

**Fig. 1. Schematic of investigated electro-mechanical drive-train**

A three phase PMSM comprises of (i) a fixed stator with three phase windings a, b and c, and (ii) a rotor with permanent magnet poles (Fig. 2). When a three phase AC power supply is applied to the stator windings, it produces a rotating magnetic field that interacts with the permanent magnet poles and pushes the rotor forwards. Park’s transmission to describe PMSM dynamics has been described in [13]. It can be rearranged in a state-space equation representation in the rotor reference frame (d-q frame) as

\[
\begin{bmatrix}
\frac{\dot{\lambda}_q}{\lambda_{dq}} \\
\frac{\dot{\lambda}_d}{\lambda_{dq}}
\end{bmatrix} =
\begin{bmatrix}
-R_y/L_y & -\omega_y \\
\omega_y & -R_y/L_y
\end{bmatrix}
\begin{bmatrix}
\lambda_q \\
\lambda_d
\end{bmatrix} +
\begin{bmatrix}
0 \\
-I_d
\end{bmatrix}
\begin{bmatrix}
v_q \\
v_d
\end{bmatrix} +
\begin{bmatrix}
\lambda_f' R_y/L_y
\end{bmatrix}
\]

where \(A\) is the magnetic flux linkage vector with \(\lambda_q\) and \(\lambda_d\) as the corresponding magnetic flux linkages. \(\lambda'_f\) is the permanent magnet flux. \(i_q\) and \(i_d\) are the line currents in \(q\) and \(d\) axes. \(v_q\) and \(v_d\) are the terminal voltages in \(q\) and \(d\) axes. \(R_y\) is the resistance per phase. \(L_y\) and \(L_d\) are the inductance in \(q\) and \(d\) axes. The matrix \(A(\omega_y)\) is a function of electrical rotor speed \(\omega_y\), \(B\) is the input vector comprising of the terminal voltages and a constant term from the permanent magnet flux linkage. The stator current \(I\) can be evaluated using \(A\) as

\[
[I_q] = \begin{bmatrix} 1/L_y & 0 \\ 0 & 1/L_d \end{bmatrix} \begin{bmatrix} \lambda_q \\ \lambda_d \end{bmatrix} + \begin{bmatrix} 0 \\ -\lambda' \end{bmatrix}/L_d
\]

where \(L\) is the inductance matrix, and \(G\) is a constant vector. The electromagnetic torque \(T_e\) developed by the PMSM acting as an input to rest of the mechanical drive-train is

\[
T_e = (3/2)(W/2)(\lambda_d i_q - \lambda_q i_d)
\]

where \(W\) is the number of the poles of the PMSM.

**B. Lumped Parameter Model for Planetary Gearbox**

The planetary gearbox used in this work has two identical planetary gear sets in a back-to-back configuration such that the overall speed ratio is \(1/4\times4 = 1.0\), where \(4\) is the speed ratio for each gear set. The sun gear of one gear set is connected to the driving motor, while the sun gear of the other gear set is connected to the load generator. The carriers of the planetary gear set are connected together through a steel spline, and the annulus gears are fixed to the housing. Each planetary gear set consists of four equally spaced planets as illustrated in Fig. 3.

![Fig. 2. Magnetic axes of a PMSM](image)

**Fig. 2. Magnetic axes of a PMSM**

A multi-DOF lumped parameter model for a planetary gear set is developed and can be expressed in matrix form [14] as

\[
M \dot{Q}_j + C \dot{Q}_j + K Q_j = T_j
\]

where \(j = 1\) or 2 represents one of the two gear sets, \(M_j\) is the mass matrix, \(C_j\) is the damping matrix, \(K_j\) is the stiffness matrix, and \(T_j\) is the external applied torques vector. Since only the torsional DOF is significant in modeling the gearbox coupling with the motor, the vector \(Q_j\) is chosen to contain all the rotational DOFs of the gear set as.
Fig. 3. A planetary gear set with four planets

![Planetary Gear Set](image)

Fig. 4. Gear meshing stiffness under healthy and faulty conditions

\[
Q_j = \begin{bmatrix} \theta_j & \theta_{p1,j} & \theta_{p3,j} & \theta_{p4,j} & \theta_{s,j} \end{bmatrix} \tag{5}
\]

where \( s \) represents the sun gear, \( x \) represents the carrier and \( p_1 \) to \( p_4 \) represent planet gears 1 to 4. When the gearbox is free of any defects, a square waveform can be used to describe the angle-varying gear meshing stiffness [14]. The gear meshing stiffness of one pair of teeth is determined by the teeth bending stiffness, fillet-foundation stiffness, and the Hertzian contact stiffness [15]. If there is a tooth defect, a partial loss in contact occurs each time the defective tooth passes through the gear meshing, which leads to a local reduction of the meshing stiffness function [15]. Figure 4 describes the meshing stiffness functions used in simulation for healthy and faulty conditions of the gear set. The local reduction in the meshing stiffness can introduce a local reduction in the meshing stiffness function [15]. These sidebands can further be transferred from the gear torsional vibration to the current flowing in the electric machines.

The governing equation of motion of the electro-mechanical drive-train model can be described as

\[
M\ddot{Q} + C\dot{Q} + KQ = T \tag{8}
\]

where the details of the matrices and vectors are provided in the Appendix.

### III. FAULT DETECTION ALGORITHM

The gear teeth fault can result in amplitude and phase modulation of the healthy vibration signal which can be detected by observing the sideband patterns in measured spectra. This vibration signal can also be transmitted from the gearbox to the load generator where the dominant sideband amplitudes can be expected around the resonance frequency of the drive-train. Finally, the sidebands can be reflected onto...
the stator current, enabling detection of the mechanical gear fault by MCSA. This is the basis of the proposed resonance demodulation based method utilizing the sidebands in the current spectra around the resonance frequency to carry out the fault detection of the planetary gearbox. The algorithm for the proposed resonance demodulation method is summarized in Fig. 5. First, the stator current is transformed from stationary reference frame (abc) to rotor reference frame (d-q) to remove the line fundamental frequency \( f_e \) which otherwise may mask the weaker fault signatures. Then, the signal is resampled from the time domain to the angle domain/shaft order domain to remove the influence of small speed/load fluctuations that often blur the obtained current spectra and thus improve signal to noise characteristics of the measurements [16]. Fast Fourier Transform is applied to find the spectrum of the current signal with respect to the shaft orders. However, for presenting the results the time domain frequency representation is chosen using the transformation: frequency = shaft order \( \times \) nominal operation speed for easing the reader understanding regarding the resonance region in the presented spectra. Afterwards, the gear meshing frequency and its harmonics, as well as the identified harmonics due to the electrical machine structure are set to zero. The harmonics resulting from the electrical machine structure can be predetermined using the measured stator current spectrum. The natural frequency \( f_c \) of the drivetrain corresponding to the gearbox-generator coupling is set to be the central frequency of a band-pass filter used to extract the current response within the resonance band. Then, Inverse Fourier Transform is used to obtain the residual signal \( r(t) \). Followed by rectifier, low pass filter, and removing the DC component, the demodulated signal \( sd(t) \) can be calculated. Finally, as proposed in [17], a fault indicator \( FI \) is defined to evaluate the health condition of the gearbox as

\[
FI = PP(\text{sd}(t))/\sum A_{\text{GM}},
\]

where \( PP(\text{sd}(t)) \) is the peak to peak value of the demodulated signal \( sd(t) \) to measure the amplitude fluctuation, and

\[
\sum A_{\text{GM}}
\]

is the summation of the amplitude of the meshing frequency and its harmonics and is determined by the operation speed and load condition.

IV. SIMULATION RESULTS ANALYSIS

The parameters for the three-phase PMSM and the planetary gear set used in this study are shown in Table I and II respectively. The stiffness of the two flexible couplings is \( k_{cpl1} = 1.29 \text{kN}\cdot\text{m/rad} \) and \( k_{cpl2} = 150 \text{kN}\cdot\text{m/rad} \), while the rotational gear meshing stiffness is estimated to be significantly higher. The mass and stiffness matrices of electromechanical drive-train in (8) can be used to determine the first three natural frequencies as 90, 449, and 2530 Hz. The first natural frequency is thus related to the coupling between the driving motor and gearbox which has the lowest corresponding stiffness. Similarly, the second one is related to the coupling between the load generator and the gearbox, and the third one is related to the meshing among the gears.

![Fig. 5. Resonance demodulation based fault detection method](image-url)
which is significantly stiffer. For a healthy planetary gearbox, oscillations can be excited in stator current due to the variation of the gear meshing stiffness function excites a periodic impulse in the finite element analysis-ANSYS. This local change in the gear teeth fault in the planetary gearbox can be detected effectively by stator current using the proposed method.

A local drop around 0.3×10^8 N/(rad.m) in gear meshing stiffness function is obtained through the finite element analysis-ANSYS. This local change in the gear meshing stiffness function excites a periodic impulse in the torsional vibration which is reflected in the stator current signal as shown in Fig. 6(b). From the corresponding spectra of the stator current, it can be seen that only the gear meshing frequency and its harmonics appear under healthy condition (Fig. 7(a)) while sidebands are excited in the evaluated spectrum under faulty conditions (Fig. 7(b)). For the given operating speed and gearbox parameters, the gear meshing frequency is evaluated to be 210Hz and the sideband spacing of 10Hz. A peak in the sidebands amplitude can be found around 449Hz which is one of the natural frequencies of the drive-train and determined by the coupling between the load generator and gearbox. It can be observed from Fig. 7(b) that the amplitude of the sidebands at the low frequencies and around the resonance frequency is comparable. However the latter has more sidebands. The demodulated signals are obtained in Fig. 7(c) using the proposed fault detection algorithm based on resonance demodulation technique. The signal from the faulty gearbox shows a high peak in each revolution of the load generator, indicating the healthy issues of the annulus gear teeth in the planetary gearbox, with evaluated value of fault indicator as 0.0034. While the demodulated signal for healthy gearbox has significantly lower value, with the evaluated value of fault indicator as 0.0010. Thus, the simulation results indicate the capability of the proposed algorithm to detect planetary gearbox faults effectively.

V. CONCLUSION

In this paper, a novel fault detection scheme to detect the planetary gear teeth faults using MCSA is developed and implemented to an electro-mechanical drive-train consisting of a driving motor connected to a load generator through a back-to-back planetary gearbox, where the resonance demodulation technique is extended from the vibration analysis to MCSA to detect the planetary gearbox fault by utilizing the sidebands around the natural frequency of the drive-train to cope with the harmonics and noise in the stator current spectra. A dynamic model is developed for an electro-mechanical drive-train to assist in the scheme of tooth fault detection in the planetary gearbox using stator current and is numerically solved by Newmark’s integration method. Through the simulation method, it is shown that the gear teeth fault in the planetary gearbox can be detected effectively by stator current using the proposed method.

APPENDIX

Mass matrix \(\mathbf{M}\) (10) is a diagonal with non-zero elements except the diagonals, where \(M_p\) is the mass of each planetary gear, \(r_{bs}\) is the base radius of the sun gear, \(r_{bp}\) is the base radius of the planet gear, \(r_s\) is the radius of the carrier, \(J_s\), \(J_p\), and \(J_c\) are the gears’ inertias, and \(J_i\) is the PMSM inertia.

Damping matrix:

\[
\mathbf{C} = \begin{bmatrix}
A_{bs} & B_{bs} & O_{bs} \\
D_{bs} & E_{bs} & F_{bs} \\
G_{bs} & H_{bs} & I_{bs}
\end{bmatrix}
\]  \hspace{1cm} (11)

where sub-matrix \(\mathbf{B}\) is a zero matrix except the element \((-c_{sp2}/r_s\) ) at the bottom left corner of matrix, and sub-matrix \(\mathbf{D}\) is also a zero matrix except the element \((-c_{sp2}/r_s\) ) at the top right corner of matrix.

\[
\mathbf{O}_{bs} = \begin{bmatrix}0 & 0 & 0 & 0 & 0\end{bmatrix}
\]  \hspace{1cm} (14)
where $c_{sjp}$ is the damping coefficient between the sun gear $j$ and the planet gear $i$ of the gear set $j$ and $c_{spl}$ is the damping coefficient of the spline.

$$F_{s06} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -c_{eq}/r_{sn} \end{bmatrix}^T$$ (15)

$$T_{s06} = c_{eq}$$ (16)

$$G_{s06} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ (17)

$$H_{s06} = \begin{bmatrix} 0 & 0 & 0 & 0 & -c_{eq} \end{bmatrix}$$ (18)

where $c_{eq}$ is the equivalent damping coefficient between the #2 sun gear and the load generator. $A$ and $D$ are shown in next page. The stiffness matrix $K$ shares the same form as damping matrix and can be obtained by replacing $c$ which denotes damping with $K$ which denotes stiffness.

**External torque vector $T$**

$$T = \begin{bmatrix} T_{dr} \\ r_{sn} \end{bmatrix}$$ (19)

where $T_{dr} = k_{p1}(\theta_{d1})$ is the driving torque, and $T_{s0}$ is the load torque, $k_{p1}$ is the stiffness of #1 flexible coupling, $\theta_{d1}$ is the rotation angle of the driving motor.

$$Q = \begin{bmatrix} \theta_{d1} \\ \theta_{p1} \\ \theta_{p2} \\ \theta_{p3} \\ \theta_{d4} \\ \theta_{d5} \\ \theta_{d2} \\ \theta_{p2} \\ \theta_{d3} \\ \theta_{p2} \end{bmatrix}$$ (20)

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