A Robust Higher Order Sliding Mode Control Of Rigid Robotic Manipulator

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Abstract—The control of a robotic manipulator has always been a difficult task due to the presence of nonlinearity, uncertainties and external perturbations. In this work, the concept of higher order sliding mode is applied to control a robotic manipulator and it is observed that system sensitivity to uncertainties is lowered.

Keywords- Sliding mode control, Robot Manipulator, Uncertainties, parameter variations.

I. INTRODUCTION

A robot [1],[2],[3] is a multi-input/multi-output, highly coupled, nonlinear mechanical system which is reprogrammable and multi-functional. It is designed to move material, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks. In such fields, the usage of robotic system cannot be avoided. Due to the complexity of the model and the added uncertainty due to parameter variations, any model based control approach for the robotic manipulator, tends to be very tedious. A robust control design approach which is not much dependent on the system model has to be adopted. Sliding mode control[4], and in particular, higher order sliding mode[5] is a control technique wherein the controller parameters are not dependent on the exact values of system parameters. Hence, it would be a viable option as a solution for the problem under consideration.

In this work, two link and three link articulated manipulators are considered for controller design using the concept of higher order sliding mode. It is assumed that all the joints are actuated.

II. THE DYNAMIC MODEL OF THE TWO AND THREE JOINT ROBOTIC MANIPULATOR

The dynamics[1] of any n link robot manipulator can be described in joint space, by using Lagrangian approach as:

\[ M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau \]  

(1)

Here, \( \theta = [\theta_1, \theta_2, ..., \theta_n]' \) is the vector of joint angle variables and \( \omega = [\omega_1, \omega_2, ..., \omega_n]' \) is the vector of joint velocities with \( \theta_i \in [0, 2\pi) \), \( \tau = [\tau_1, \tau_2, ..., \tau_n] \) is the controlling torque vector , \( M(\theta) \) is the inertia matrix, \( C(\theta, \dot{\theta}) \) is vector of centripetal and coriolis terms and \( G(\theta) \) is vector of gravity terms respectively.

Defining \( \omega = \dot{\theta} \), we get,

\[ \dot{\theta} = \omega \]

\[ \dot{\omega} = -M^{-1}(\theta)C(\theta, \omega) + M^{-1}(\theta)(\tau - G(\theta)) \]

A. Dynamic equations for two link Robot manipulator

For the two link robotic manipulator considered here, the joint positions are \( \theta_1 \) and \( \theta_2 \) and \( \omega_1 \) and \( \omega_2 \) are velocities of joints 1 and 2 respectively. The lengths and masses of the two links are given as \( L_1, L_2 \) and \( m_1, m_2 \) respectively.

\[
\theta = \begin{bmatrix}
\theta_1 \\
\theta_2
\end{bmatrix}
\]

\[
\omega = \begin{bmatrix}
\omega_1 \\
\omega_2
\end{bmatrix}
\]

\[
M(\theta) = \begin{bmatrix}
M_1 & M_2 \\
M_3 & M_4
\end{bmatrix}
\]

\[
C(\theta, \dot{\theta}) = \begin{bmatrix}
c_1 & c_2 \\
c_3 & c_4
\end{bmatrix}
\]

\[
G(\theta) = \begin{bmatrix}
G_1 \\
G_2
\end{bmatrix}
\]

B. Dynamic equations for three link Robot manipulator

In the present work, the dynamic equations for three link manipulator[2] have been derived using Lagrangian technique. The joint positions are \( \theta_1, \theta_2, \theta_3 \) and \( \omega_1, \omega_2, \omega_3 \) are velocities of joints 1, 2 and 3 respectively. The lengths of the three links are \( L_1, L_2, L_3 \) and \( m_1, m_2, m_3 \) are the masses of the three links. It is assumed that the mass of each link is concentrated at the mid point of the respective link. The lengths of the links where masses of the respective links 1, 2 and 3 are concentrated are \( L_{c1}, L_{c2}, L_{c3} \).

\[
\theta = \begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3
\end{bmatrix}
\]

\[
\omega = \begin{bmatrix}
\omega_1 \\
\omega_2 \\
\omega_3
\end{bmatrix}
\]

\[
M(\theta) = \begin{bmatrix}
M_1 & M_2 & M_3 \\
M_4 & M_5 & M_6 \\
M_7 & M_8 & M_9
\end{bmatrix}
\]

\[
C(\theta, \dot{\theta}) = \begin{bmatrix}
c_1 & c_2 & c_3 \\
c_4 & c_5 & c_6 \\
c_7 & c_8 & c_9
\end{bmatrix}
\]

\[
G(\theta) = \begin{bmatrix}
G_1 \\
G_2 \\
G_3
\end{bmatrix}
\]
### III. Sliding Mode Controller

Sliding mode control (SMC) [4],[6],[7],[8] is a nonlinear, robust control method that alters the dynamics of a system. In SMC, motion of the system trajectory remains along a chosen surface of the state space. Prior to reaching sliding surface, the system gets affected by disturbances and uncertainties in the parameters. When system is in sliding mode, entire system dynamics is governed by sliding surface parameters and not original system parameters and the system behavior becomes invariant [9] to any disturbance or change in parameters. Therefore, sliding mode control is designed with the aim to achieve sliding mode in finite time. In the simplest form of sliding mode control, termed as first-order sliding mode, the control signal is made to switch between two chosen structures about the sliding surface. Theoretically, this switching happens at an infinite frequency. Practically, actuators cannot switch at infinite frequency. Therefore, high frequency oscillations of non-zero magnitude are generated in system state. This undesirable phenomenon is called chattering. The problem of chattering can be reduced by using Higher order sliding mode(HOSM)[5]. In HOSM, the control is designed in such a manner that the higher order derivatives of the sliding function also reach zero and remain equivalently zero within a finite time.

#### A. Higher order sliding mode control (HOSM):

HOSM is a movement on a discontinuity set of dynamic system. The task is to keep sliding function zero in the vicinity of sliding mode. The sliding order characterizes the dynamic smoothness degree in the vicinity of the mode. The $r^{th}$ order sliding mode is determined by the equalities $s = s = s = .... = s^{r-1} = 0$. The $r$-sliding mode realization can provide for up to the $r^{th}$ order precision. In HOSM, the control can be made continuous.

1) **Second order sliding mode:** In second order SMC, the derivative of system control is considered as the actual control variable. A discontinuous control (derivative of system control) steers the sliding function $s$ and first derivative of sliding function $\dot{s}$ to zero in finite time, so that system control is continuous and chattering is avoided. Depending upon the choice of sliding surface, control can be made continuous or discontinuous. The control signal is simple and does not depend on system parameters or dynamics.

$$\tau = -20 \text{sgn}(\dot{s}) + 2 \text{abs}(s^{1/2}) \text{sgn}(s)$$

2) **Third order sliding mode:** In third order SMC, sliding function $s$, first derivative of sliding function $\dot{s}$ and second derivative $\ddot{s}$ of sliding function are made to reach zero in finite time. The control is invariant to system uncertainties and dynamics.

$$\dot{\tau} = -20 \text{sgn}(\dot{s} + 2e_1 \text{sgn}(e_2))$$

$$e_1 = (|\ddot{s}| + |\dot{s}|^2)^{1/6}$$

$$e_2 = \ddot{s} + |\dot{s}|^{2/3} \text{sgn}(s)$$

### IV. Simulation Results

Simulation in MatLab has been carried out to test the controller for the two link and three link systems under consideration and simulation results are shown in present section. Here, $s = \theta - \theta_{refi}$, $\dot{s} = \dot{\theta} - \dot{\theta}_{refi}$ and $\ddot{s} = \ddot{\theta} - \ddot{\theta}_{refi}$.

#### A. Two link planar manipulator ($R||R$)

Here, a planar robot manipulator is considered. The path given to end effector of planar manipulator is given by sets of $x$ and $y$ coordinates as $X=[1 0 -0.5 -0.5 0 0.5 1]$ and $Y=[0 1 0.5 -0.5 -1 -0.5 0]$ respectively. For the desired path to be followed by end effector, inverse kinematics problem is solved and trajectory for the joint angles is obtained. In SMC design, a sliding mode surface is first defined and then a sliding mode controller is designed to drive the system state variables to sliding mode surface. The control problem is to make the joints to track that specific trajectory. The lengths of links 1 and 2 are taken as 1 and 0.5 respectively and the masses of two links are taken as 1 each. Mass and length of the links considered are prone to change. Using sliding mode control it is observed that the control becomes invariant to any changes in the system parameters.

1) **Second order sliding mode:** In this graph of fig 1, it is clearly seen that the error in positions of joints 1 and 2 which is also the sliding surface reduces to zero in finite time. For two link manipulator by using second order SMC the error in tracking for joint angle 1 is 0.1411 and for joint angle 2 is 0.0712.

In second order SMC the control torque is discontinuous.

2) **Third order sliding mode controller:** In this graph in fig 2 of third order sliding mode control, it is observed that $s$ reduces to zero in finite time. There is no chattering in the system. It can be appreciated that by using third order SMC the tracking error reduces to zero rather slowly as compared to second order SMC but there is no chattering. In third order SMC more information is needed i.e. both $s$ and $\dot{s}$ are needed. When third order SMC is used for two link manipulator, then the error in tracking for joint angle 1 is 0.2149 and for joint angle 2 is 0.2835.
In the present graph of fig 3, it is clearly seen that \( \dot{s} \) also becomes zero in finite time. 

The graph in fig 4 shows the control inputs for two joints 1 and 2. In third order SMC, the control is continuous as compared to second order SMC, but the control action required is quite more.

It is observed that although the tracking error is more in case of third order SMC as compared to second order SMC the chattering is removed completely in third order SMC.

B. Three link manipulator \((R \perp R// R)\)

The path to be followed by end effector of three link manipulator is given by the sets of X, Y and Z coordinates

\[
X = 0.35 \sin(2t) + 0.01, \quad Y = 0.7 \sin^2(t) + 0.01 \quad \text{and} \quad Z = 1 + 0.7 \cos(t) \quad \text{varying with} \quad t \in (0, 3).
\]

Inverse kinematics problem is solved and trajectories for joints 1, 2 and 3 to be followed are obtained. The masses of links considered are \( m_1 = 1, m_2 = 1 \) and \( m_3 = 1 \) and length of links considered are \( L_1 = 1, L_2 = 0.5 \) and \( L_3 = 0.5 \).

1) third order sliding mode controller: The sliding function \( s \), its first derivative \( \dot{s} \) and its second derivative \( \ddot{s} \) are continuous and reach zero in finite time. The control for all the three joints is continuous. In fig 5, it is visible that the error of position of joints i.e. \( s \) become zero in finite time. For three link manipulator as the system dynamics are very complicated and as the number of links and joints is more it is more prone to parameter uncertainties and dynamic uncertainties than two link manipulator. The second order SMC is not sufficient to control the system. Hence third order SMC is used. In third order sliding mode control, the performance index of integral absolute error (IAE) is used. The errors in position tracking of joint 1, joint 2 and joint 3 are obtained as 4.6724, 1.1485, 14.7198 respectively and the finite reaching time is 2.3 seconds.

In fig 6, it is observed that the derivative of sliding surface \( \dot{s} \) becomes zero in finite time.

In fig 7, it can be seen that the second derivative \( \ddot{s} \) of sliding surface reduces to zero in finite time. As \( s, \dot{s} \) and \( \ddot{s} \) all reduce to zero in finite time, the system states slide on the surface smoothly and the chattering problem associated with SMC is removed.

In fig 8, it can be seen that the by using HOSM control

In fig 9, it can be seen that the by using HOSM control
V. Conclusion

SMC is a non-linear controller which gives robust control for very complicated, nonlinear and highly coupled system like robotic manipulator. By using proper sliding function the system motion can be brought to sliding mode in finite time. Once in sliding motion, the system becomes invariant to any change in parameters or uncertainties. In first order SMC, the tracking error will come to zero in finite time but the control of chattering will be large. By using HOSM, the control chattering is removed and the system moves on sliding surface smoothly. The problem with HOSM is that more information is needed. In case of second order SMC $s$ and $\dot{s}$ should be known while in third order SMC $s, \dot{s}$ and $\ddot{s}$ should be known. If the robotic manipulator is used for an operation where chattering can not be tolerated then third order SMC is the best choice. It is observed that by using higher order SMC the system becomes invariant to any parametric uncertainties and disturbances. In the present work, higher order sliding mode controller has been successfully applied for the motion control for two and three link manipulators. The simulation results show that higher order sliding mode control gives better performance in controlling robotic manipulator.

REFERENCES


VI. APPENDIX

For two link robotic manipulator:

\[
\begin{align*}
c_1 &= -2m_2L_1L_2\sin(\dot{\theta}_1)\dot{\theta}_1\dot{\theta}_2 \\
c_2 &= -m_2L_1L_2\sin(\theta_2)\dot{\theta}_2^2 \\
c_3 &= m_2L_1L_2\sin(\theta_2)\dot{\theta}_1^2 \\
c_4 &= 0 \\
G_1 &= m_2L_2\cos(\theta_1 + \theta_2) + (m_1 + m_2)L_1g\cos(\theta_1) \\
G_2 &= m_2L_2\cos(\theta_1 + \theta_2) \\
M_1 &= m_2L_2^2 + (m_1 + m_2)L_1^2 + 2L_1L_2m_2\cos(\theta_2) \\
M_2 &= L_2^2m_2 + L_1L_2m_2\cos(\theta_2) \\
M_3 &= L_2^2m_2 + L_1L_2m_2\cos(\theta_2) \\
M_4 &= L_2^2m_2
\end{align*}
\]
For three link robotic manipulator:

\[ G_1 = 0 \]
\[ G_2 = g_m L_{c2} \cos(\theta_2) + g_m L_2 \cos(\theta_2) + g_m L_{c3} \cos(\theta_2 + \theta_3) \]
\[ G_3 = g_m L_{c3} \cos(\theta_2 + \theta_3) \]
\[ M_1 = 0.5m_2 L_{c2}^2 \cos(\theta_2) + L_2^2 \cos(\theta_2) + L_{c3}^2 \cos(\theta_2 + \theta_3) \]
\[ M_2 = 2 \]
\[ M_3 = 0 \]
\[ M_4 = 0 \]
\[ M_5 = 0.5m_2 L_{c2}^2 + L_{c2}^2 + L_{c3}^2 + 2L_2 L_{c3} \cos(\theta_2) \cos(\theta_2 + \theta_3) \]
\[ M_6 = L_{c3}^2 + L_2 L_{c3} \cos(\theta_2) \cos(\theta_2 + \theta_3) \]
\[ M_7 = 0 \]
\[ M_8 = L_{c3}^2 + L_2 L_{c3} \cos(\theta_2) \cos(\theta_2 + \theta_3) \]
\[ M_9 = L_{c3}^2 \]
\[ c_1 = -0.25m_2 L_{c2}^2 \sin(2\theta_2) - 0.5L_2^2 \sin(2\theta_2) - L_{c3}^2 \sin(2\theta_2 + 2\theta_3) \]
\[ c_2 = -0.25m_2 L_{c2}^2 \sin(2\theta_2) - 0.5L_2^2 \sin(2\theta_2) - 0.5L_{c3}^2 \sin(2\theta_2 + 2\theta_3) \]
\[ c_3 = -0.5L_{c3}^2 \sin(2\theta_2 + 2\theta_3) \]
\[ c_4 = 0.25m_2 L_{c2}^2 \sin(2\theta_2) + 0.5L_2^2 \sin(2\theta_2) + 0.5L_{c3}^2 \sin(2\theta_2 + 2\theta_3) \]
\[ c_5 = -L_2 L_{c3} \sin(2\theta_2 + 2\theta_3) - L_2 L_{c3} \cos(2\theta_2 + 2\theta_3) \]
\[ c_6 = L_2 L_{c3} \sin(2\theta_2) \cos(2\theta_2 + 2\theta_3) \]
\[ c_7 = -0.5L_{c3}^2 \sin(2\theta_2 + 2\theta_3) \]
\[ c_8 = -L_2 L_{c3} \cos(2\theta_2) \sin(2\theta_2 + 2\theta_3) \]
\[ c_9 = 0 \]