Newton’s Identities Once Again!

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ABSTRACT

1. INTRODUCTION. Newton’s identities allow one to calculate a sum of $d$th powers of the roots of a given polynomial, if one already knows the sums of $c$th powers of the roots for $c < d$ and the coefficients of the polynomial. These identities were first stated by Newton around the year 1666, but were not published until 1707 (see [2, p. 32]). The Newton identities are now part of the standard course on polynomials and Galois theory. Many proofs have already appeared, yet these curious identities still keep on attracting attention. (See, for example, [2], [3], [4], [5], and [6].) Although most proofs of Newton’s identities are not complicated, neither are they trivial. If $n$ is the degree of our polynomial $f(X) = (X - a_1) \cdots (X - a_n)$ and $d = n$, there is a proof that is both transparent and immediate. If $d > n$, one quickly realizes that essentially the same idea works, and again one obtains an instant proof of Newton’s identities. This part of the proof of the identities is well known. However, when $d < n$ one uses different ideas to complete this proof (see [1] or [5]). The goal of this note is to show that this change of method is unnecessary, and that one can prove Newton’s identities essentially in the same way for $d < n$ as in the case $d \geq n$. 