Complex Banks’ Networks, Cascades and Systemic Risks

Marcel Bluhm
Frankfurt University and CFS

Ester Faia
Frankfurt University, CFS and Kiel IfW*

Jan Pieter Krahnen
Frankfurt University, CFS, CEPR

First draft: March 2011.

Abstract

The monitoring and assessment of systemic risk in complex financial systems has become paramount in the design of reforms devoted to safeguard financial stability. Against this background stands the lack of proper models devoted to such a task and used to analyze the optimal design of financial regulation. In this paper we propose an agent based model of banks featuring network externalities, endogenous asset markets and resale. Systemic risk, measured by the Shapley value, is reduced with lower balance sheet inter-linkages and higher capital requirements. Risk charges, a form of Pigouvian taxation, help to mitigate network externalities and contagion.

Keywords: networks, complexity, tatonnement, contagion, marked to market.

*Correspondence to: Department of Money and Macro, Goethe University Frankfurt, House of Finance, office 3.47, Grueneburgplatz 1, 60323, Frankfurt am Main, Germany. E-mail: faia@wiwi.uni-frankfurt.de. Webpage: www.wiwi.uni-frankfurt.de/profs/faia.
1 Introduction

One of the most important legacies of the 2007-2008 crisis has been the creation and development of a number of institutions whose mission is that of measuring systemic risk, monitoring financial vulnerabilities and safeguarding the financial system. In the U.S. the Dodd–Frank Wall Street Reform and Consumer Protection Act\(^1\) had created the Financial Stability Oversight Council, whose statute states in Title 1 that the primary objective of this institute is that of monitoring systemic risk. The main mission of the European Systemic Risk Board, established 16 December 2010, is the prevention or mitigation of systemic risks to financial stability in the Union that arise from developments within the financial system. The Financial Stability Board (FSB) has been established to coordinate, at the international level, the work of national financial authorities in addressing vulnerabilities and to develop and implement strong regulatory and supervisory policies. The common denominator behind such reforms is the necessity to correctly assess and monitor systemic risk, with the aim of designing regulatory policies that could help in preventing crises. Against this background, the events of the recent crisis have also highlighted the lack of proper macro and financial models, particularly in terms of their ability to quantify systemic risk and to forecast the development of financial contagion. Such tasks have also been complicated by the number of intricate inter-linkages that characterize modern financial systems and by the fast developments of financial innovation. For the construction of models apt to perform such tasks, a promising direction is represented by the category of agent based models with network externalities. One key element in understanding systemic risk is indeed the analysis of shock transmission in systems in which banks have heterogenous portfolio positions and inter-linkages in asset markets.

The aim of this paper is twofold. First, we provide a first step toward the creation of a modeling framework for assessing systemic risk in financial markets. Second, we use their model to analyze the effectiveness of regulatory policies such as the risk charges recently proposed in the Basel III accords.

The financial system featured by our model consists of a network of \(n\) financial institutions, which are linked to each other through cross asset investment and which endogenously determine their optimal portfolio allocation by maximizing profits subject to liquidity and capital constraints.

Banks have deposits, lend to each other and hold liquid assets, such as cash, and non-liquid assets, such as bonds or collateralized debt obligations. Non-liquid assets are marked-to-market, while liquid assets are referred to as cash on banks’ balance sheets. Banks differ at time zero for the returns on non liquid assets due to different information and administrative cost. When the interbank rate rises above an individual bank’s return, the latter automatically releases more capital to the interbank market. Such differences in returns gives raise to heterogenous optimal portfolio allocation. The optimal portfolio allocation of the banks in the system can be mapped into a matrix of assets and liabilities. Due to pervasive heterogeneity the equilibrium in the interbank market is reached through a tatonnement process. Individual optimization indeed result in excess demand or supply of bank capital. Equilibrium is reached through an iterative process in which equilibrium in bank capital is obtained by netting out lending in the interbank market. In each step of the iterative process banks re-optimize their portfolio position. The equilibrium outcome determines the final asset positions, including the aggregate allocation of non liquid assets. At this stage the market price of non liquid asset is determined endogenously as a function of aggregate non liquid assets. Such function, obtained through an interpolation procedure, decreases with the amount of non-liquid assets sold on the market.

Contagion in this model occurs through the transmission of shocks to non-liquid assets. Shocks are generated from a multivariate lognormal distribution and are randomly drawn for a certain number of periods. Contagion manifests itself through direct and indirect effects. The direct effects emerge from the network inter-linkages. First, if banks invest in the same financial products their balance sheets are correlated. Second, as banks are interlinked through counterpart exposure, a defaulting bank transmits losses to creditor banks. There are also indirect contagion effects, which manifest through firesales. A negative shock in the value of non liquid assets induces several banks to deleverage, an event that produces a fall in the market price and a cascade of losses in the balance sheet of all other banks. In this respect the model allows to analyze credit risk, namely contagion effects via direct interbank linkages, and market risk, namely contagion effects through marking-to-market the non-liquid assets.

We use the model outlined to compute aggregate systemic risk and the contribution to it of

\[2\text{ See Cifuentes, Ferucci and Shin [9].}\]
each individual bank. Systemic risk is computed through the Shapley value\(^3\). Through numerical analysis we verify that our model replicates a number of stylized facts, which characterize financial crisis and flight to quality events. For instance we show that an increase in the share of short term investment, a fall in the default probability or an increase in the equity ratio all produce a decrease in the aggregate systemic risk and in the banks’ contribution to it. Generally speaking financial systems with a lower degree of interconnection and higher capital requirement feature lower systemic risk.

At last we analyze the design of an optimal risk charge from a policy regulator. By contributing to aggregate risk, banks produce a negative network externality, which could be offset through a form of Pigouvian taxation. By evaluating the contribution to risk of each individual bank, the regulator can design heterogenous optimal risk charges, whose proceeds can be reallocated in a fund devoted to insure against the risks of contagion. Banks internalizing such risk charges will include them as costs into their profit function and choose an optimal portfolio allocation that results in lower aggregate systemic risk.

This paper is related to several strands of literature. There is an extensive literature that highlights the possibility of network failures and contagion in financial markets. An incomplete list includes Allen and Gale [2], Lagunof and Schreft [?]1, Rochet and Tirole [19], Freixas, Parigi and Rochet [12], Leitner [18], Eisenberg and Noe [10], Cifuentes, Ferucci and Shin [9] (see Allen and Babus [3] for a recent survey). The paper is also related to the literature which aims at measuring systemic risk in models with interconnected banks and complex inter-linkages: see Billio, Getmansky, Lo and Pelizon [18], Geanakoplos [14], Gai and Kapadia [13] all study network based vulnerabilities. Caballero and Simsek [8] analyse complexity, namely uncertainty about the network links, while Krahnen and Bluhm [15] analyze the diffusion of systemic risk in a network model of interconnected banks. Finally our paper is related to the literature studying the design of regulations aimed at abating systemic risk (see for instance Allen and Gale [4]).

The rest of the paper is structured as follows. Section 2 describes the model, the equilibrium formation process, the shock transmission and the measure of systemic risk. Section 3 describes the numerical results and the ability of the model to replicate stylized facts characterizing financial

\(^3\)See Bluhm and Krahnen [15] and Borio, C., N. Tarashev and K. Tsatsaronis [6].
contagion. Section 4 analyzes the design of optimal regulations, in the form of risk charges. Section 5 concludes. Appendices describe the optimal portfolio problem and the algorithm used to solve our agent based model. Tables and figures follow.

2 The Model

The financial system is made up of $n$ banks which undertake an optimal portfolio allocation by maximizing profits subject to liquidity and capital requirement constraints and a non zero non liquid asset constraint. The profit function includes penalty for deviation form an optimal value at risk parameter. Crucially banks feature balance sheet interlinkages. Banks have deposits, lend to each other and hold liquid assets and non-liquid assets, such as bonds or collateralized debt obligations. Non-liquid assets are marked-to-market, while liquid assets are referred to as cash on banks’ balance sheets. Banks differ at time zero for their equity allocation, which results in heterogenous optimal portfolio allocation. The final portfolio allocation can be mapped into a matrix. The model is solved in two steps. First, each bank undertakes the optimal portfolio allocation, leading to heterogenous balance sheets and bank capital positions. Second, excess demand and supply of bank capital are net out through an allotment process in bank lending: iterations allow excess demand or supply of bank capital to be re-absorbed through changes in the lending rate.

Contagion occurs when the financial system is subject to shocks to non liquid assets. Initial shocks to non liquid assets are distributed according to a multivariate lognormal distribution and are transmitted through the balance sheet interlinkages. and through the endogenous price formation process. In response to a negative shocks, banks have to re-adjust their capital position. They do so by selling non liquid assets in the interbank market. The final equilibrium determines the aggregate amounts of non liquid assets and their price. This transmission channel, which works through firesales, implies that sales of non liquid assets induce downward pressure on the market value of banks’ balance sheets.

2.1 Banks’ Portfolio Allocation

Banks portfolio allocations are endogenously determined. As banks are heterogenous, their asset allocation carries an index $i$. Aggregate variables are instead denoted without the index. Banks in
the model start with a certain amount of deposits, \( d^i \), and equity, \( e^i \). Banks choose the optimal amounts of cash, \( c^i \), lending in the interbank market, \( bl^i \), borrowing in the interbank market, \( bb^i \), and investment in non liquid asset, \( nla^i \). Cash pays no interest but has no probability of default. Bank lendings yield an interest \( r_2(bl) \) and have a subjective probability of default, \( p_2(ER) \). The interest rate on lending is endogenously determined by the aggregate amount of lending, while the probability of default, \( p_2 \), is endogenously determined by the aggregate equity ratio, \( ER \), through an interpolation process as outlined in the next section. Bank borrowings cost an interest of \( r_2(bl) + \Delta(bl) \), where \( \Delta(bl) \) is a small spread between borrowing and lending. Finally, non-liquid assets yield an interest of \( r_3 \) and have a probability of default of \( p_3 \). Banks differ with respect to the interest return on their non-liquid asset investments, reflecting different information costs and efficiency. They thus face individual yields, \( r^i_3 \). For simplicity and without loss of generality we assume that both, interest rates on non liquid assets and the default probability, \( p_3 \), are exogenous parameters. The underlying assumption, which complies with realism, is that banks are competitive, hence atomistic and take both those parameters as given and determined from the market. At last notice that at time zero banks differ for the returns on non liquid assets: this is due to different information and administrative cost. When the interbank rate rises above an individual bank’s return, the latter automatically releases more capital to the interbank market. Such differences in returns gives raise to heterogenous optimal portfolio allocation. A bank’s balance sheet thus consists of the elements displayed on Table 1.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>Deposits</td>
</tr>
<tr>
<td>Bank lendings</td>
<td>Bank borrowings</td>
</tr>
<tr>
<td>Non-liquid assets</td>
<td>Equity</td>
</tr>
</tbody>
</table>

Table 1: Banks’ Balance Sheets

Banks’ objective function is given by:

\[
\pi = r_2(bl) \cdot bl^i - (r_2(bl) + \Delta(bl)) \cdot bb^i + r^i_3 \cdot nla^i
\]

(1)

Banks face a liquidity constraint, as they have to hold at least a percentage, \( \alpha \), of their deposits
in cash\textsuperscript{4}:

\[ c^i \geq \alpha \cdot d^i \]  

Furthermore banks face a capital requirement constraint, as they must maintain an equity ratio $\tau$:

\[ ER = \frac{c^i + p(nla) \cdot nla^i + bl^i - d^i - bb^i}{\chi_1 \cdot p(nla) \cdot nla^i + \chi_2 bl^i} \geq \tau \]  

where $\chi_1$ and $\chi_2$ are risk weights and $p(nla)$ is the aggregate price of non liquid assets and is determined endogenously through an interpolating procedure, as outlined later. If banks’ equity ratio, $ER$, is lower than the capital requirement, $\tau$, bank begin to net their interbank lending, in which case the denominator in equation 3 sinks relatively to the numerator. If a bank cannot fulfill the capital requirement it defaults. The higher the risk factors, $\chi_1$ and $\chi_2$, the larger is the extent to which banks have to net their interbank lending. Two further observations are worth. First, note that liquid asset do not appear in the denominator of equation 3, because banks do not have to hold capital for their liquid asset holdings. Second, note that the non-liquid assets are marked to market, which gives the potential for resale spirals in the model. Realistically we assume that banks need to hold less capital for bank lendings than for investments in non-liquid assets, i.e. $\chi_1 \succ \chi_2$. At last, banks face a no-short sales constraint:

\[ nla^i \geq 0. \]  

In Appendix A banks’ maximization problem is reformulated as a minimization problem subject to smaller-equal constraints. As banks differ in terms of their initial equity allocation, the individual optimization gives raise to heterogenous portfolio allocations. The next section shows how individual portfolio positions are allotted in the financial market giving raise to an equilibrium price and asset allocation.

2.2 Tatonnement Equilibrium in the Interbank Market

The optimal individual portfolio allocation results in a number $n$ of excess demand and supplies for bank capital. Banks enter the interbank market and engage in an allotment process of their individual positions. Banks match their capital positions through netting out interbank lending.\footnote{For simplicity this fraction is assumed constant, though in future modifications it might be made dependent on the non-liquid asset position. This is the assumption envisaged in Basel III.}
This implies, for instance, that an excess supply of bank capital is mirrored in excess supply of lending in the interbank market. In this case the lending rate will adjust to re-equilibrate excess supply. The allotment takes place through an iterative process which is determined as follows.

First notice that allocation of interbank lendings is affected by two endogenous parameters: (i) banks perceived stability of the financial system proxied by a subjective probability of default, $p_2(ER)$, and, (ii) $r_2(bl)$, the price for interbank lending. In turn the perceived or subjective probability of default is obtained through an interpolation process based on the observed aggregate equity ratio, $ER$. The aggregate equity ratio as well as the aggregate amount of lending are indeed assumed to be public information. If the aggregate equity ratio fulfills the microprudential capital requirement, as set by the parameter $\tau$, the perceived riskiness of a counterpart default, $p_2(ER)$, is zero. If the system equity ratio is zero or smaller, $p_2(ER)$ equals one. All intermediate values of $p_2(ER)$ between a system equity ratio of 0 and 8 are obtained by linear interpolation. The perceived stability determines the overall amount banks are willing to provide to the interbank market. Note that $p_2(ER)$ affects banks’ lending choice through equation 3.

The iterative process of netting out bank lending is obtained by adjusting $r_2(bl)$ in an iterative procedure. In this process banks set three reference points: an upper interest bound, $r^u_2(bl)$, a lower interest bound, $r^l_2(bl)$, and the actual lending rate, $r_2(bl)$. Initially, $r^l_2(bl) = 0$ and $r^u_2(bl) = r^i_3$. Recall that $r^l_2(bl) \leq r_2(bl) \leq r^u_2(bl)$. To fix ideas, suppose that the initial portfolio optimization results in an excess supply of bank capital, which in turn implies an excess supply of bank lending. In this case the lending rate will adjust downwards to re-equilibrate bank lending. The new lending rate is adjusted by to $r'_2(bl) = \frac{r_2(bl) - r^u_2(bl)}{2}$. Given this new lending rate, banks re-optimize their portfolio allocation, which result in a new bank capital position. Gradually the excess supply of bank capital, hence bank lending, is absorbed through adjustment of the lending rate. The opposite adjustment takes place if capital demand exceeds supply. The process has converged if the relative matching error, defined as $\frac{|\text{capital supply} - \text{capital demand}|}{\text{capital supply} + \text{capital demand}}$, is smaller than some specified tolerance value.

### 2.3 Shock Transmission in the Financial System

An additional iterative process through allotment takes place in the event in which non liquid assets of financial intermediaries are hit by a shock\(^5\). Shocks to the financial system consist of a loss of

\(^5\)We follow Bluhm and Krahnen [15] to model the shock transmission process.
banks’ assets. The loss can take the form of banks’ liquid assets as well as non-liquid assets. In response to the shock banks start to sell non liquid assets in the interbank market with the aim of re-equilibrating their bank capital positions. A loss in the value of non liquid assets determines an excess supply of non liquid assets. As a consequence the price of non liquid assets falls. Again a tatonnement process takes place such that in each iterations banks re-optimize their portfolio and excess demand and supply of non liquid assets are reabsorbed through price adjustment. Figure 1 outlines the transmission routine which is taken out along equations 1 to 4, for given bank lendings and borrowings.

Analytically the market price resulting from the tatonnement process, takes the form of the following inverse demand function:

\[ p(nla) = \exp(-\mu \sum_i s_i) \]  

(5)

where \( s_i \) are the amount of non liquid assets sold to fulfill the capital requirements. The excess demand/supply of non-liquid assets can therefore be inferred by the inverse of equation 3 evaluated at the optimal portfolio allocation.

Numerically the process can be detailed as follows. Prior to any shock the market price equals 1, which is the price when all banks initially fulfill the capital requirement and sales of the non-liquid asset are zero. A shock chosen from a multivariate lognormal distribution hits a cluster of banks. As a consequence the same cluster of banks begins selling non-liquid assets to fulfill capital ratios and the supply curve shifts upwards, resulting in \( s(1) = s_i > 0 \). In correspondence of the excess demand \( s(1) \), a discrepancy between the offered price, which is \( p(s(1))^{bid} \), and the demanded price, which is equal to one, arises. The resulting market price, which is labeled \( p(s(1))^{mid} \), is the average between the prices offered and demanded. The ensuing fall in the market price depresses further the value of non liquid assets, which is marked to market in the banks’ balance sheet. This results in further sales of non-liquid asset, however at a decreasing rate so that convergence can be reached after a limited number of iterations.

Importantly, the changes of market prices in response to non liquid asset sales, are the driver of the firesale effects and the indirect cascades channels. As explained earlier indeed, falls in

\(^{6}\)Other shocks are possible, for example a sudden drop in non-liquid asset prices or the default of a bank in the system.
the market price depress the balance sheet values of other banks, potentially resulting in further defaults.

2.4 Calibration

The model parameters are chosen to match values observed in the financial system and/or imposed by supervisory policy. The parameter $\alpha$, the amount of liquid assets banks have to hold as a function of the amount of deposits, is set to 0.1, thus being equivalent to the cash reserve ratio in the U.S. The parameter $\chi_1$, the risk weight for non liquid assets, is set to 1: this value reflects the risk weight applied in Basel II to commercial bank loans. The parameter $\chi_2$, the weight for interbank lending, is set to 0.2, which is also the risk weight actually applied to interbank deposits between banks in OECD countries. The amount of equities and deposits that banks have initially on their balance sheets is set to 40 billions and 400 billions, respectively, which is the amount of Deutsche Bank’s respective positions in U.S. Dollars on its consolidated balance sheet from 31 March 2010. Finally, following federal reserve bank regulatory agency definitions, banks must hold a capital ratio of at least 8%.

2.5 Systemic Risk Measure

Generally speaking systemic risk occurs in the event of a collapse of the entire financial system, rather than simply the default of individual banks or of a limited group of financial intermediaries. A prerequisite for the emergence of systemic risk is the presence of interlinkages and interdependencies in the market, so that the default (or a run) on a single intermediary or on a cluster of them leads to a cascade of failures, which could potentially undermine the functioning of the entire financial system. While there is much agreement about the general definition of systemic risk, there is much less agreement upon its quantitative measure. The traditional analysis for measuring systemic risk was based upon the judgement on whether the defaulting bank or group of intermediary was too big to fail: such an assessment is based on indicators such as the institution’s size relative to the system, market share concentration, based for instance on the Herfindahl-Hirschman Index, the oligopolistic structure of the market and the presence of barriers to entries. Recently and due to the emergence of complex financial relations, the focus of systemic risk measures has been shifted toward an assessment of the too interconnected to fail. It is on this last concept that we focus.
One measure which has been recently proposed to determine the link between systemic risk and interconnection is the Shapley value\(^7\), an indicator which allows us to determine the contribution of individual banks to aggregate risk. In game theory this value is used to find the fair allocation of gains obtained by cooperation among players. For a game consisting of \(n\) players the Shapley value is defined as:

\[
\xi_i(v) = \frac{1}{n!} \sum_{K \ni i, K \subseteq N} v(K) - (v(K) - \{i\})
\]  

(6)

where \(N\) is the set of all players, \(v(K)\) is the value obtained by coalition \(K\), including player \(i\) and \((v(K) - \{i\})\) is the value of coalition \(K\) without player \(i\). The Shapley value for player \(i\) is the average contribution to the gain of the coalition over all permutations in which players can form a coalition. The Shapley value has the following properties:

1. **Pareto efficiency.** The total gain of a coalition is distributed.
2. **Symmetry.** Players with equivalent marginal contributions obtain the same Shapley value.
3. **Additivity.** If one coalition can be split into two sub-coalitions then the pay-off of each player in the composite game is equal to the sum of the sub-coalition games.
4. **Zero player.** A player that has no marginal contribution to any coalition has a Shapley value of zero.

Since the number of permutations involved in calculating the Shapley value increases strongly with the number of banks, the analysis is subject to the curse of dimensionality. However, following Stanojevic, Laoutaris and Rodriguez [21] and as displayed in equation (7) the Shapley value can be approximated by the average contribution of banks to systemic risk over \(l\) randomly sampled permutations:

\[
\hat{\phi}_i(v) = \frac{1}{l} \sum_{K_i \ni i, K_i \subseteq N} v(K) - v(K - \{i\})
\]  

(7)

The parameter \(l\) determines the discrepancy between the real Shapley value and its estimate, that is, the error. It can be shown that this estimator is unbiased and efficient.

Generally speaking the Shapley value is affected by the degree of bank interconnections. In our model interconnection occurs through both, direct and indirect links. Direct links are given by the correlations of shocks to non liquid assets and the exposure to others’ banks balance sheet.

\(^7\)See Shapley [20]. See also Tarashev, Borio, and Tsatsaronis [6] and Bluhm and Krahnen [15]. Alternative measures of systemic risks are proposed for instance in Adrian and Brunnermeier [1] through a CoVaR methodology.
Indirect links are given by the effects that a fall in the market price of non-liquid assets has on the balance sheet of the entire system. The overall degree of interconnections in our model is affected by a number of parameters. For instance, an increase in the initial perceived default probability, \( p_2(ER_0) \), induces banks to lend less in the interbank market. This endogenously reduces the extent of interconnections in the system. Similarly an increase in the share of liquid investment, \( \alpha \), abates the detrimental effects of shocks to non liquid assets on the aggregate market price, hence on the system balance sheet.

Given the link between the Shapley value and the degree of interconnections, we can trace in our model a number of links between the Shapley value and some of parameters which affect the endogenous determination of market interlinkages. This is the purpose of the next section.

### 3 Cascades and Financial Fragility

In this section we analyze the transmission of shocks and the contribution of individual banks to aggregate risks under different parameters characterizing the financial system. Those parameters are chosen to identify different degrees of fragility. For instance, a lower fraction of investment in liquid assets, \( \alpha \), a lower regulatory equity ratio, \( \tau \), a higher initial default probabilities, \( p_2, p_3 \), or higher risk factors, \( \chi_1 \) and \( \chi_2 \), all identify more fragile financial systems. Fragile financial systems are more prone to amplification of systemic risk, as each banks contributes more heavily to it. Consider, for instance, the mechanics of a financial system characterized by a lower fraction of liquid investment, \( \alpha \), and subject to an adverse shock to non liquid assets. In this case, banks start to sell their non liquid assets to fulfill capital ratios. The ensuing fall in the aggregate price, \( p(nla) \), induces a cascade due to the firesale effect. However, the size of the cascade is larger in financial systems in which the fraction of non liquid assets holdings is larger than the fraction of liquid assets holdings. In sum, systems which rely more on short term investment are less fragile since they are less prone to contagion. Consider now the mechanics of financial systems characterized by large perceived probabilities of defaults. In this case portfolio optimization induces banks to reduce lending and counterpart exposure in the interbank market. In equilibrium the degree of interconnectedness is lower. This reduces systemic risk as well as the risk contribution of each individual bank. Similarly larger regulatory capital requirements induce banks to hold less
counterpart exposure, thereby reduces the extent of interconnection and the size of the systemic risk. At last consider the mechanics of systems in which banks equity ratios features lower risk factors, $\chi_1$ and $\chi_2$. In face of adverse shocks, banks start selling non liquid assets. The higher are the risk factors, the larger is the amount of assets sale and the ensuing fall in aggregate price. The downward pressure on banks’ balance sheet and the ensuing cascade are therefore larger in the systems characterized by higher risk factors.

Table 3 displays a random individually optimal financial system, consisting of 30 banks. The parameters are calibrated as outlined in Subsection 2.4. Table 3 displays banks’ contribution to systemic risk which amounts to 0.387 in the baseline calibration. This value means that given the shock distribution the expected proportion of the financial system that will default is 38.7%. Matching the results in Table 2 and 3 shows that banks that borrow and invest heavily into non liquid assets tend to contribute more to systemic risk than those that lend and do not invest. Also the former have a larger balance sheet size.

In the following this system will be subject to comparative-static analyses. In a first analysis we investigate how systemic risk changes in the model when banks’ liquidity requirement (parameter $\alpha$) is increased from 0 to 0.2. Figure 2 displays that systemic risk first decreases between a level of 0 to 0.08, then increases slightly until 0.12 and for the remainder range decreases. Figure 3 displays all banks’ contributions to systemic risk over the varying levels of liquididty requirements analyzed. When raising the required liquidity ratio there are two important effects. First, banks try to compensate the higher required cash holdings via borrowing from other banks and investing the borrowed money into non-liquid assets. This increases the interest rate on the interbank market, $r_2$. With an increasing interbank rate, banks that have a relatively low return on their investment in non-liquid assets, $r_3$, find it more profitable to lend their money on the interbank market if $r_2(bl) \geq r_3$. Hence, increasing the liquidity requirement increases the exchange of interbank funds and the interest rate on the interbank market. This first effect increases interconnectedness as well as interbank exposure and makes the financial system more vulnerable to contagion via the interbank markets. The second effect goes in the opposite direction. Higher required levels of liquid assets make banks individually safer because they cannot be subject to resales. Given the opposite directions of both effects, it can happen over some ranges of that systemic risk increases, when the
former effect prevails. However, at high levels of , the latter effect clearly prevails.

In a second analysis we investigate how systemic risk changes in the model when banks’ capital requirement ratio (parameter ) is increased from 0.02 to 0.14. Figure 3 displays that systemic risk initially decreases when banks’ capital requirement is increased from 0.02 to 0.04, then increases again at a level of 0.06 and then decreases monotonically. Figure 4 displays all banks’ contributions to systemic risk over the varying level of capital requirement. When the capital requirement is low, the interest rate on the interbank market is relatively high because at low capital requirements, banks with high returns on non liquid assets, try to leverage themselves up to the limit. With increasing capital requirement however, this demand for capital goes down, lowering the interest rate on the interbank market, , and thus causing banks to switch from lenders to investors in non-liquid assets as soon as . Hence, with increasing capital requirements, the amount exchanged on the interbank market goes down. The financial system thus changes from (a: very low ) a system in which there are very few highly leveraged monoliths to (b: medium ) a system where banks are interlinked and have a relatively equal size to (c: high ) no interlinks at all. All banks have the same size and invest what they are allowed according to in non-liquid assets. These dynamics can also cause the systemic risk increasing over some ranges of capital requirements, even when the capital requirement is increased, as depicted on figure 3. The spike at a capital requirement ratio of 0.06 comes up because from a systemic risk point of view the interlinked system (medium ) is most dangerous for contagion. Clearly there are also few banks which have borrowed from almost all other banks in a system with very low capital requirement. But these too-big-to-fail banks need to be hit by a large shock to wreak havoc. However, since there are few of them this does not happen very often. Effectively the nancial system emerging at low values of capital requirement thus display the ’robust-yet-fragile’ property which emerges in many networks. By contrast, at low values of capital requirement one observes a relatively balanced financial system with a large number of contagious interbank links.

\[ \text{Robust-yet-fragile means that a system is more robust to medium shocks but more vulnerable from a certain point of shock size onwards.} \]
4 The Optimal Design of Systemic Risk Charge (to be completed)

By contributing to systemic risk banks give raise to a negative network externality, which has to be corrected via a system of Pigouvian taxes\(^9\). Such taxes can take the form of risk charges. Once systemic risk has been measured through the Shapley value, a regulator can tax banks proportionally to their individual contributions. Total revenues collected are determined based on an estimate of the systemic value at risk. The crucial aspect of those charges is that banks can internalize such costs in their profit functions, thereby giving raise to portfolio allocations that result in lower aggregate systemic risk. The goal of such taxation systems should be that of fostering incentives toward lowering systemic risk.

The optimal risk charges, which take the form of additional capital injection, are determined based on the following mechanism design. Given the optimal portfolio allocation at time zero, the regulator measures individual banks contribution to risk based on the Shapley value. The total amount of additional capital injection is determined by ensuring that systemic risk, proxied through a systemic value at risk (SVaR), (see also Bluhm and Krahnen\(^15\)), does not exceed a 95% probability. Given the total amount of capital injection needed to ensure stability, the regulator then determines the individual charges proportionally to the risk contributions of the Shapley value. At this stage the regulator offers to the banks the possibility of avoiding the additional capital injection by adjusting their portfolio position, so as to comply with the desired level of systemic risk. Banks can do so by internalizing the risk charges as costs in their profit function. Internalizing those costs induces banks to manœuvre the components that determine their contribution to systemic risk, namely the portfolios’ size\(^10\), the degree of interconnectedness and the leverage, and to re-position their portfolios. A crucial aspect, for the re-balancing to be effective, lies in the possibility for banks of gathering the correct estimate of the relation between the regulatory risk charge and the above mentioned components determining individual contribution to aggregate risk.

\(^9\) See also Korinek [16].
\(^10\) See also Korinek [16].
5 Conclusions

One of the major legacies of the recent financial crisis is the quest for measuring, assessing and monitoring systemic risk. So far, this task was made difficult by the mounting complexity of the modern financial systems, all characterized by extensive degrees of interconnections, and the lack of models apt to perform such tasks. We laid down an agent based model of banks, in which heterogeneity, network externalities and firesale effects associated with endogeneity of portfolio decisions and aggregate prices of non liquid assets, all produce cascades. We have shown that a number of institutional features characterizing financial systems can exacerbate the extent of systemic risk and cascades. A normative analysis shows how to design an optimal risk charge, which takes the form of a Pigouvian tax on the risk externality.

In future work we aim at extending the model along the following dimensions. First, the micro-foundations of the banking problem can be explored further, by introducing maturity mismatch or asymmetric information. Second, adding dynamic could enrich the model results along several dimensions. Third, we aim at elaborating more on the role of expectations and learning in determining financial vulnerability.
6 Appendix A. Banks Optimization Problem.

Reformulating the maximization problem as a minimization problem for the $i$'th bank subject to smaller equal constraints yields:

$$\min_{bl_i, bb_i, nla_i} \pi = -r_2 \cdot bl_i + (r_2 + \Delta) \cdot bb_i - r_3 \cdot \frac{nla_i}{P}$$

s.t.

$$-c_i \leq -\alpha \cdot d_i$$

$$-c_i - nla_i (1 - \chi_1 (\tau + \zeta_i (P))) - bl_i (1 - \chi_2 (\tau + \zeta_i (P))) + bb_i \leq -d_i$$

$$-nla_i \leq 0$$

Reformulating the constraints in matrix notation $A \cdot x \leq b$ yields

- $A = \begin{pmatrix} -1 & 0 & 0 & 0 \\ -1 & -(1 - \chi_1 (\tau + \zeta)) & -(1 - \chi_2 (\tau + \zeta)) & 1 \\ 0 & -1 & 0 & 0 \end{pmatrix}$
- $x = (c_i \ nla_i \ bl_i \ bb_i)'$
- $b = (-\alpha \cdot d \ -d \ 0)'$

7 Appendix B. The Algorithm

As outlined in Subsection 2.3, a shock to the financial system consists of a random percentage loss of all banks’ non liquid assets (Step A on Figure 1). In Step B), banks re-optimize their holdings of cash and non-liquid assets subject to the constraints outlined in Equations (3) to (5). Note that in this step interbank lendings are given and not considered as choice variables. In Step C), bankrupt banks are identified (that is, those that violate one of the constraints in the optimization routine) and a shock to interbank lendings is set up to those banks of which the creditor banks have a negative net value (with the net value being the difference between a bank’s assets and liabilities). Banks with a negative net value subtract the difference between their assets and liabilities, first proportionally from their interbank lendings, and if there are no interbank lendings left, from their deposits (Step D)). After this shock has been assigned, banks again re-optimize their portfolio (Step
B). If there are no interbank shocks to assign and banks do not desire to change their holdings of non liquid assets on their balance sheet, the shock has been transmitted. Systemic risk given the shock is then calculated as the proportion of banks that default in the financial system. Expected systemic risk is obtained via computing the average systemic risk resulting from a large number of random shocks to the financial system, drawn from a multivariate normal distribution which is centered at a loss of 4% and features a variance of 20, for each bank, respectively.

8 Appendix C. Banks Contribution to Systemic Risk

Table 3 displays a randomly generated financial system subject to the parameter calibrations from Subsection 2.4. The rows display banks’ assets and the columns banks’ liabilities. The last row contains each bank’s deposits and the last two columns contain banks’ portfolio holdings of non-liquid and liquid assets, respectively. For example, in the field where the fifth row and third column intersect can be seen that in the individual optimal nancial system bank 5 lends 17.5 to bank 2. The equilibrium interest rate on the interbank market equals 6% in the baseline setting. Figure 2 displays bank’s contribution to systemic risk (y-axis) as a function of varying levels of liquidity requirement (x-axis), in the calibrated baseline model.

References


<table>
<thead>
<tr>
<th>Bank 1</th>
<th>Bank 2</th>
<th>Bank 3</th>
<th>Bank 4</th>
<th>Bank 5</th>
<th>Bank 6</th>
<th>Bank 7</th>
<th>Bank 8</th>
<th>Bank 9</th>
<th>Bank 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.014</td>
<td>0.017</td>
<td>0.003</td>
<td>0.017</td>
<td>0.013</td>
<td>0.017</td>
<td>0.018</td>
<td>0.004</td>
<td>0.013</td>
<td>0.012</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>0.007</td>
<td>0.017</td>
<td>0.018</td>
<td>0.013</td>
<td>0.011</td>
<td>0.013</td>
<td>0.007</td>
<td>0.007</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>0.018</td>
<td>0.010</td>
<td>0.017</td>
<td>0.017</td>
<td>0.016</td>
<td>0.015</td>
<td>0.010</td>
<td>0.004</td>
<td>0.013</td>
<td>0.018</td>
</tr>
</tbody>
</table>

*Table 2: Banks’ Contribution to Systemic Risk in the Baseline Scenario*
Figure 1: Algorithm for Shock Transmission

A) Assign initial shock

B) Banks choose their holdings of cash and non-liquid assets to optimize their profit under constraints (1) to (4)

D) Assign shock in the financial system matrix

C) Identify bankrupt banks and update shock to interbank lendings

E) Exit after shocks to solvent banks are fully assigned
Figure 2: Systemic Risk at Varying Levels of $\alpha$.

Figure 3: Systemic Risk at Varying Levels of $\tau$. 
Figure 4: Effect of changing the parameter $\alpha$ from 0 to 1.
Table 3: Randomly generated financial system subject to comparative-static analysis
Figure 3 displays bank’s contribution to systemic risk (y-axis) as a function of varying levels of capital requirement (x-axis), \( \tau \), in the calibrated baseline model.

Figure 5: Effect of changing the parameter \( \tau \) from 0.02 to 0.14.