Abstract—The paper evaluates the error performance of three random finite set based multi-object trackers in the context of pedestrian video tracking. The evaluation is carried out using a publicly available video dataset of 4500 frames (town centre street) for which the ground truth is available. The input to all pedestrian tracking algorithms is an identical set of head and body detections, obtained using the Histogram of Oriented Gradients (HOG) detector. The tracking error is measured using the recently proposed OSPA metric for tracks, adopted as the only known mathematically rigorous metric for measuring the distance between two sets of tracks. A comparative analysis is presented under various conditions.

I. INTRODUCTION

Random set theory has recently been proposed as a mathematically elegant framework for Bayesian multi-object filtering [1]. Research within this theoretical framework has resulted in new multi-object filtering algorithms, such as the probability density hypothesis (PHD) filter [2], Cardinalised PHD filter [3], and Multi-Bernoulli filter [1], [4]. The main feature of multi-object filters is that they estimate sequentially the number of objects in the surveillance volume of interest (the so-called cardinality) and their states in the state space. Formulation of multi-object trackers from random-set based multi-object filters has attracted a lot of interest recently, see e.g. [5], [6], [7], [8]. The output of a tracker is a set of tracks, that is, labeled temporal sequences of state estimates associated with the same object.

In this paper we adopt the “tracking-by-detection” approach to pedestrian tracking, which has become very popular in computer vision due to its applicability to moving un-calibrated cameras [9], [10]. Typically pedestrian detections are obtained using the Histogram of Oriented Gradients (HOG) detector [11], trained using either head images (for head detections) or body images (for body detections). An example of head and body detections is shown in Fig.1.(a). Head and body detections are unreliable in the sense that: (1) not all pedestrians are detected in every frame; (2) the chance of false detections is quite real, with the spatial density of false detections typically non-uniform. This is evident in Fig.1.(a).

If the state vector of each object (pedestrian) contains the position (e.g. the head centroid), but not the size of the object, then both the head and body detections are instances of imprecise measurements: they represent rectangular regions (two-dimensional intervals) within which the true object is possibly located. As such, they can be modelled as random closed sets (rather than random variables). The first tracking algorithm considered in the paper is designed specifically for imprecise measurements: it represents a multi-object tracker built from the Bernoulli filter described in [12]. We will refer to this tracker as Algorithm 1.

An imprecise measurement (e.g. an interval in the measurement space) can always be converted to a precise but random measurement (e.g. a point in the measurement space which is affected by additive noise). The two remaining algorithms considered in the paper assume precise random detections of heads/bodies for pedestrian tracking. Algorithm 2 is the same as Algorithm 1, but using the precise random (Gaussian) measurements model. Algorithm 3 is based on the Cardinalised PHD filter with data association [13].

Evaluation of multi-object tracking performance has been one of the main stumbling blocks in advancing the scientific field of target tracking. A large number of evaluation measures have been proposed, both in the general context (e.g. [14], [15]) and specifically for video surveillance (e.g. [16], [17], [18], [19]). At present there is no consensus in sight on the preferred common approach. In this paper tracking error will be measured using the recently formulated Optimal sub-pattern assignment (OSPA) metric for tracks (or OSPA-T) [20]. The OSPA-T metric has an important advantage over all above mentioned performance metrics: it is a mathematically rigorous metric (it satisfies the axioms of a metric) for measuring the distance between two sets of tracks (i.e. between the ground truth and the tracker output). OSPA-T is also consistent with intuition, as discussed in [21].

The remainder of the paper is organised as follows. Section II describes the performance evaluation framework: the video dataset, the method of pre-processing detections and the OSPA-T metric. Section III reviews the three random-set based tracking algorithms. Section IV presents the experimental results under various conditions, while the conclusions of this study are drawn in Section V.
II. PERFORMANCE EVALUATION FRAMEWORK

A. Video dataset and detections

The video dataset and the (hand labelled) ground truth are downloaded from the website [22]. The dataset is a video of an occasionally busy town centre street (the average number of pedestrians visible at any time is 16). The video was recorded in high definition (1920 × 1080 pixels at 25 fps). Only the first 4500 frames of the video are used in the performance evaluation. Frame number 320 of the dataset is shown in Fig. 1. Head detection and pedestrian body detection algorithms were applied to each frame in the video sequence. The fastHOG GPU implementation [23] of the Histogram of Oriented Gradients (HOG) detection algorithm [11] was used for both detectors. The HOG detector applies a sliding window to the image at a regular grid of locations and scales, and classifies each sub-window as containing or not containing an object (head or pedestrian). Classification is performed using a pre-trained linear Support Vector Machine, the input to which is a set of block-wise histograms of image gradient orientation. A classification threshold of 0.75 was used for both detectors.
Sliding window detectors tend to give multiple detections for one object due to their tolerance to shift and scale, so a post-processing step groups overlapping detections.

The head and pedestrian (whole body) detections have some complementary characteristics. The detector is only partially tolerant to occlusions, so the head detector tends to have a higher false alarm rate, pedestrian textures are more distinctive than head textures, more visible than whole bodies in surveillance video. However, the pedestrian detector is more tolerant to occlusions, so the head detector tends to have complementary characteristics. The detector is only partially tolerant to shift and scale, so a post-processing step groups overlapping detections.

One object due to their tolerance to shift and scale, so a post-processing step groups overlapping detections.

For the case \( m > n \), the definition is \( D_{p,\infty}(X, Y) = D_{p,\infty}(\mathbb{E}_k, X_k) \). If both \( X_k \) and \( Y_k \) are empty sets (i.e. \( m = n = 0 \)), the distance is zero.

\[
\begin{align*}
1: & \quad \text{function OSPA-T(} \{X^{(1)}, \ldots, X^{(L)}\}, \{Y^{(1)}, \ldots, Y^{(R)}\}\}) \\
2: & \quad \text{\% Label the estimated tracks} \\
3: & \quad \text{For } j = 1, \ldots, R, \text{ Label}[Y^{(j)}] = I \text{ (where } I > L) \\
4: & \quad \text{Find } \lambda^* \text{, the globally best assignment of tracks } \{X^{(1)}, \ldots, X^{(L)}\} \rightarrow \{Y^{(1)}, \ldots, Y^{(R)}\} \\
5: & \quad \text{For } i = 1, \ldots, L, \text{ Label}[X^{(i)}] = \text{Label}[X^{(i)}] \\
6: & \quad \text{\% Compute the distance} \\
7: & \quad \text{For } k = 1, \ldots, K \\
8: & \quad \text{Form the labeled sets at } t_k: \\
9: & \quad \text{Ground truth: } X_k = \{(l_1, x_{k,1}), \ldots, (l_m, x_{k,m})\} \\
10: & \quad \text{Estimated: } Y_k = \{(s_1, y_{k,1}), \ldots, (s_n, y_{k,n})\} \\
11: & \quad \text{Compute the OSPA distance between } X_k \text{ and } Y_k \\
12: & \quad \text{end function}
\end{align*}
\]

The base distance \( d(\bar{x}, \bar{y}) \) is defined as:

\[
d(\bar{x}, \bar{y}) = \left( d(x, y)^p + d(\ell, \ell)^p \right)^{\frac{1}{p}},
\]

where \( p' \in [1, \infty) \) is the base distance order parameter; \( d(x, y) \) is the localisation base distance, typically adopted as the \( p' \)-norm: \( d(x, y) = \|x - y\|_{p'} \); \( d(\ell, \ell) \) is the labelling error, adopted as: \( d(\ell, \ell) = \alpha d[s, t] \), where \( d[i, j] \) is the complement of the Kroneker delta, that is \( d[i, j] = 0 \) if \( i = j \), and \( d[i, j] = 1 \) otherwise. Parameter \( \alpha \in [0, c] \) here controls the penalty assigned to the labelling error \( d(s, t) \) interpreted.
relative to the localisation distance \( d_l(x, y) \). The case \( \alpha = 0 \) assigns no penalty, and \( \alpha = c \) assigns the maximum penalty.

Since in this paper we consider a sequence of a large number of frames \( (K = 4500) \), the OSPA-T is applied over non-overlapping segments (blocks) of frames.\(^{[1]}\)

### C. Base distance is a metric

The base distance \( d(\tilde{x}, \tilde{y}) \), defined in \([4]\), satisfies the three axioms of a metric: identity, symmetry and triangle inequality. To prove identity and symmetry is trivial. The proof of triangle inequality, presented in \([20, \text{Sec III.A}]\), is wrong and this section presents the correct proof.

Let \( \tilde{x} = (l, x), \tilde{y} = (s, y), \tilde{z} = (u, z) \). The following proof for the triangle inequality is given in \([20, \text{Sec III.A}]\)

\[
d(\tilde{x}, \tilde{y})^p \leq d(\tilde{x}, \tilde{z})^p + d(\tilde{z}, \tilde{y})^p
\]

where in Sec 1-B and 20 notation \( p' \) was used instead of \( p \). Equation (5) is wrong and this can be seen for example by adopting: \( p = 2, \tilde{x} = (1, 0), \tilde{y} = (1, 5), \tilde{z} = (1, 4.999) \). Then \( d(\tilde{x}, \tilde{y})^p = 25 \) and \( d(\tilde{x}, \tilde{z})^p + d(\tilde{z}, \tilde{y})^p \approx 24.90. \) Moreover, (5) does not prove the triangle inequality.

We want to prove that

\[
d(\tilde{x}, \tilde{y}) \leq d(\tilde{x}, \tilde{z}) + d(\tilde{z}, \tilde{y})
\]

where according to \([4]\)

\[
d(\tilde{x}, \tilde{y})^p = d_l(x, y)^p + \alpha p \delta[l, s].
\]

As \( d_l(\cdot, \cdot) \) is a metric, it meets the triangle inequality

\[
d_l(x, y) \leq d_l(x, z) + d_l(z, y)
\]

As both sides of the inequality are positive numbers and \( p \geq 1 \)

\[
d_l(x, y)^p \leq \left( d_l(x, z) + d_l(z, y) \right)^p
\]

We also have that

\[
\alpha \delta[l, s] \leq \alpha \delta[l, u] + \alpha \delta[u, s]
\]

As both sides of inequality \([10]\) are positive and \( p \geq 1 \)

\[
\left( \alpha \delta[l, s] \right)^p \leq \left( \alpha \delta[l, u] + \alpha \delta[u, s] \right)^p
\]

Using \([9]\) and \([11]\)

\[
d_l(x, y)^p + \left( \alpha \delta[l, s] \right)^p \leq \left( d_l(x, z) + d_l(z, y) \right)^p + \left( \alpha \delta[l, u] + \alpha \delta[u, s] \right)^p
\]

that is

\[
\sqrt[p]{d_l(x, y)^p + \left( \alpha \delta[l, s] \right)^p} \leq \sqrt[p]{(d_l(x, z) + d_l(z, y))^p + \left( \alpha \delta[l, u] + \alpha \delta[u, s] \right)^p}
\]

As \( p \geq 1 \), using the Minkowski inequality \([25]\) on the right hand side of \([13]\)

\[
\sqrt[p]{d_l(x, z) + d_l(z, y)}^p + \left( \alpha \delta[l, u] + \alpha \delta[u, s] \right)^p \leq \left( d_l(x, z) + d_l(z, y) \right)^p + \left( \alpha \delta[l, u] + \alpha \delta[u, s] \right)^p
\]

Finally, using \([13]\) and \([14]\), we get

\[
\sqrt[p]{d_l(x, y)^p + \left( \alpha \delta[l, s] \right)^p} \leq \sqrt[p]{d_l(x, z)^p + \alpha \delta[l, u] + \left( d_l(z, y) \right)^p + \alpha \delta[u, s]}
\]

The proof is finished using \([7]\):

\[
d(\tilde{x}, \tilde{y}) \leq d(\tilde{x}, \tilde{z}) + d(\tilde{z}, \tilde{y}).
\]

### III. DESCRIPTION OF ALGORITHMS

The state vector of a single object is adopted for all algorithms as \( x = [x, \dot{x}, y, \dot{y}]^\top \), where \( (x, y) \) is the position (in pixels) of the pedestrian head centroid and \( (\dot{x}, \dot{y}) \) is its velocity vector (in pixels/s). The number of objects from frame to frame varies. The random finite set of head detections at frame \( k \) is denoted \( Z^{(1)}_k \). Accordingly, the random set of head-like body detections (see the explanation in the last sentence of Sec II-A) is \( Z^{(2)}_k \).

Algorithms 1 and 2 are based on the multi-sensor Bernoulli filter \([26]\), where the “sensors” are the two types of pedestrian head detections. Separate and independent Bernoulli filters are run for each target. Target interactions are taken care of by the appropriately increased clutter intensity, as in \([27]\). This multi-object tracking algorithm has been described in some detail in \([28]\). The difference between Algorithms 1 and 2 is in the model of the single-object likelihood function. Let \( \zeta \in Z^{(1)}_k \), for \( i = 1, 2 \), be a detection resulting from an object (i.e. a pedestrian head) in the state \( x \). A head detection is a rectangle, thus \( \zeta \) is specified by a tuple \((\chi, \eta, w, h)\), where \((\chi, \eta)\) determines its upper-left corner, while \( w \) and \( h \) are the width and height, respectively.

The single-object likelihood function used in Algorithm 1 treats the detection \( \zeta \) as an imprecise measurement and is defined as in \([12]\):

\[
g_k^{(i)}(\zeta|x) = \varphi(\mathbf{H}x; \zeta, \Sigma^{(i)}) - \varphi(\mathbf{H}x; \zeta, \Sigma^{(i)})
\]

where \( \varphi(z; \mu, \Sigma^{(i)}) \) is the Gaussian cumulative distribution function with mean \( \mu \) and covariance \( \Sigma^{(i)} = \text{diag}[\sigma^2_x, \sigma^2_y] \); \( \zeta \) and \( \zeta \) are the lower and upper bound of the rectangle, and \( \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \). If \( \sigma^2_x = \sigma^2_y = 0 \), then \([17]\) simply states that \( g_k^{(i)}(\zeta|x) = 1 \) if \((x, y)\) is inside the rectangle \( \zeta \), and zero otherwise. The algorithm is applied to the video dataset using \( \sigma^2_x^{(1)} = 1 \) and \( \sigma^2_x^{(2)} = 25 \).

Algorithms 2 and 3 first convert the rectangular detection \( \zeta \) into a point measurement \( z = [\chi + w/2, \eta + h/2]^\top \), with the associated covariance matrix \( \mathbf{R} = \text{diag}[(w/6)^2, (h/6)^2] \). Then the single-object likelihood function of \( z \) is adopted as:

\[
g_k^{(i)}(z|x) = N(z; \mathbf{H}x, \mathbf{R})
\]
where $\mathcal{N}(m; \mu, P)$ is the Gaussian probability density function with mean $\mu$ and covariance $P$.

Algorithm 3 is based on the Cardinalised PHD (CPHD) filter \cite{13}, but with additional logic to deal with track labeling. The key idea of \cite{13} is to form the clusters of targets, and to apply the CPHD filter update to each cluster separately. The update uses every available detection (measurement) to calculate the weight of the track-to-measurement association. The weight of no-measurement association is also computed.

Finally these weights are used to form an association matrix. Since we have at our disposal two types of detections ($Z_k^{(1)}$ are head detections, and $Z_k^{(2)}$ are head-like body detections), the update step in Algorithm 3 is applied twice, first using $Z_k^{(1)}$, and then using $Z_k^{(2)}$. Although this is not an optimal approach \cite{29}, it has been suggested as a reasonable approximation.

### Table II

<table>
<thead>
<tr>
<th>Alg.</th>
<th>Likelihood function</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Eq. (17)</td>
<td>Multi-Bernoulli Tracker of \cite{28}</td>
</tr>
<tr>
<td>2.</td>
<td>Eq. (18)</td>
<td>Multi-Bernoulli Tracker of \cite{28}</td>
</tr>
<tr>
<td>3.</td>
<td>Eq. (18)</td>
<td>CPHD based tracker \cite{13}</td>
</tr>
</tbody>
</table>

All three algorithms used the same clutter maps (one map for heads, the other for body detections). The probability of detection was set to $P_D^{(1)} = 0.58$ and $P_D^{(2)} = 0.52$. A short summary of algorithms is given in Table II.

### IV. Numerical results

The localisation base distance of the OSPA-T error $d(x, y) = \|x - y\|_{\rho'}$ only takes into account the positional error (i.e. neglecting the velocity error). Fig.2 shows the resulting OSPA-T error for the three random-set based tracking algorithms. The parameters of the OSPA-T metric used in evaluation: $p = p' = 1$, $c = 100$ and $\alpha = 75$. Identical head detections and body-to-head converted detections, from every frame, have been used by all three algorithms. Fig.2 also shows, as a guideline, the OSPA-T error of the Benfold-Reid algorithm \cite{10}, whose tracking results are available online \cite{22}. We point out that the comparison between the BR algorithm and the three random-set based trackers is not fair because the BR algorithm is a smoother (operates as a batch algorithm over a sliding window of image frames) and does not use body/head detections in every frame. From Fig.2 one can observe that ranking of the algorithms according to OSPA-T varies with time. For example, from frame number 800 to 1100, the BR algorithm is far superior than the random-set based trackers, but the opposite is true from frame 1400 to 1600. In order to obtain an overall ranking, the time averaged OSPA-T error has been computed: its value for Algorithms 1, 2, 3 and the BR algorithm is 45.2, 42.8, 40.7 and 40.4, respectively.

The conclusion is that the most accurate of the three random-set tracking algorithms is Algorithm 3. Furthermore, it appears that the imprecise measurement model is not justified in the adopted context: the transformation of head and body-to-head rectangles (imprecise detections) into random precise measurement points provides better tracking results. This can be explained by the nature of head and body-to-head rectangular detections; it has been observed that if a detection is not false, then its rectangular centre is a very accurate estimate of the centre of a pedestrian head. Thus the likelihood \cite{17}, which is based on the interpretation that the true head centroid is somewhere inside the rectangle, appears to be too cautious and consequently does not use the full information content of a measurement.

We repeated the OSPA-T error computations for $\alpha = 0$ (no penalty for the labeling error). This case corresponds to the original OSPA error proposed in \cite{21}. The obtained time averaged OSPA-T error for Algorithms 1, 2, 3 and the Benfold-Reid algorithm in this case were 34.1, 29.5, 27.4, and 30.2, respectively. Again Algorithm 3 performs the best among the random-set based trackers, and even outperforms the Benfold-Reid algorithm. This result reveals that the major problem with Algorithm 3 is the lack of track consistency (too many broken tracks), which by adopting $\alpha = 0$ is not penalised. Track consistency can be improved by smoothing over multiple image frames (to be considered in the future work).

Head and body detection algorithms are very computationally intensive and consequently in real-time applications it may not be possible to provide them at every image frame. Next we compare the OSPA-T error performance of Algorithm 3 for the situations where head and body detections are available for: (1) every frame, (2) every 2nd frame, (3) every 4th frame and (4) every 8th frame. The results are shown in Fig.3. We note that the error performance does not change dramatically with the reduced frequency of head and body detections. The time averaged OSPA-T error for the four cases are: 40.7, 37.9, 39.2, and 42.7. Somewhat surprisingly, using body/head detections every 2nd and every 4th frame, reduces the number of false tracks and overall improves the accuracy. Only when body/head detections become available only every 8th frame, some of the true tracks start to be missing occasionally and consequently the OSPA-T error performance deteriorates.

### V. Conclusions

The paper presented a framework for performance evaluation of multi-object trackers. The framework is illustrated in the context of video tracking by comparison of three random-set based pedestrian tracking algorithms, using a video data set of a busy town centre. The multi-object tracking error was evaluated using the “OSPA for tracks” (OSPA-T) metric. The OSPA-T metric has an important property that it satisfies the axioms of a metric. The mathematical proof the triangle inequality axiom is presented in the paper.

The results of performance evaluation indicate that the CPHD based tracker of \cite{13} performs the best. Although
this is a single-frame recursive algorithm, its performance is comparable to that of [10] (which operates over a sliding window of frames). Future work will consider a smoothing version of the algorithm in [13] since a delay by a few frames in reporting the tracks is tolerable and has the potential to further improve the tracking accuracy.

REFERENCES


Fig. 3. The influence of detection frequency on Algorithm 3: OSPA-T error for the sequence of 4500 frames; OSPA-T metric parameters: $p = p' = 1$, $c = 100$ and $\alpha = 75$.


