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THE ESTIMATION OF INCOME AND SUBSTITUTION EFFECTS IN A MODEL OF FAMILY LABOR SUPPLY

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This paper contains the formulation of the theoretical restrictions on the labor supply functions of the husband and wife in a model of family labor supply in a manner that makes them readily amenable to test and, if they are not rejected, to imposition onto the data. It also contains an application of the proposed empirical counterparts of the theoretical equations to cross-sectional data from the 1960 U.S. Census of Population.

RECENT PROPOSALS for a negative income tax have stimulated considerable interest in the question of the actual size of the income and substitution effects in the labor supply of household members.[2] The answer to this question is of obvious importance in determining the impact of alternative combinations of negative income tax rates and benefit levels chosen by policy makers. If, for example, the income compensated substitution effect of a wage change on labor supplied is small, then little attention need be paid to the choice of the optimal incentive tax rate. If, on the other hand, the income effect is small in absolute value, then high initial subsidy levels will have little effect on the supply of labor to unskilled labor markets.[3]

The basic theoretical apparatus underlying the flurry of recent empirical work is the application of the classical theory of consumer behavior to the demand for leisure. Mincer [**24**] fruitfully applied this framework with the family as the decision-making unit, and the subsequent literature has grown rapidly. Although the volume of empirical research has been substantial, there has been surprisingly little variation in estimation procedures. The work by Kosters [**20**] remains the standard framework of analysis for almost all studies.[4] This situation stands in dramatic contrast to the recent research in the estimation of the parameters of conventional consumer demand functions. On the one hand, in this latter work considerable attention has been paid to testing the full set of empirical implications

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[2] Though undoubtedly incomplete, the list of available empirical studies includes Ashenfelter and Heckman [**2**], Boskin [**6**], Cain [**9**], Cohen, Rea, and Lerman [**10**], Greenberg and Kosters [**14**], Hall [**15**], Heckman [**16**], Hill [**18**], Kalacheck and Raines [**19**], Kosters [**21**], Leuthold [**22**], Mincer [**24**], and Rosen and Welch [**30**]. Partial surveys are contained in Ashenfelter [**1**] and Parnes [**27**]. Moreover, the importance of the issue has resulted in governmentally sponsored social experiments that seek to estimate these effects by actual cash transfers. See, for example, the description in Watts [**36**].

[3] For an exposition of how estimates of these effects may be used to predict the effects of an income maintenance program, see Ashenfelter [**1**].

[4] The exception is Leuthold [**22**], who uses the framework of the linear expenditure system approach of Stone [**32**].

of the utility maximization hypothesis.[5] On the other hand, there has been substantial interest in imposing the constraints deduced from the classical theory onto the estimated demand systems in order to improve the efficiency of estimation.[6] Accordingly, the purpose of this paper is to formulate the theoretical restrictions on the labor supply functions of the husband and wife in a model of family labor supply in a manner that makes them readily amenable to test and, if they are not rejected, to imposition onto the data.

In Section 1 we first set out the predictions of the classical theory when applied to the supply of labor. We then formulate these predictions in terms of the differentials of the labor supply functions of the family members in a manner similar, but by no means identical, to the procedures proposed by Barten [**3**] and Theil [**33**] in a different context. In Section 2 we propose empirical counterparts to the theoretical equations and a discussion of their application to cross-sectional data. The section concludes with the results of an empirical test based on data from the 1960 Census of Population.

1. LABOR SUPPLY EQUATIONS

The force of the classical utility theory in a model of family labor supply resides in the assertion that the family acts as if it possesses and maximizes a twice continuously differentiable utility function

$$U = U(L_m, L_f, X). \tag{1}$$

L_m and L_f are the amounts of time spent in non-market activity during some interval by the male and female members of the household, respectively.[7] X is a Hicksian composite of all consumption goods [**17**, p. 312] since we will assume that the relative prices of goods within this consumption bundle do not change. The decision variables in (1) must satisfy the budget constraint

$$W_mT + W_fT + Y = W_mL_m + W_fL_f + PX, \tag{2}$$

where W_m and W_f are the wage rates confronted by the family members, P is the price of consumption goods, Y is non-labor income, and T is the fixed amount of total time that each family member has to allocate. The left-hand side of (2) is called full income [**5**], but it is sometimes more instructive to write (2) in the equivalent form

$$W_m(T - L_m) + W_f(T - L_f) + Y = PX$$

so as to bring out the fact that it implies nothing more than the equality of total income and expenditures. Assuming interior solutions, the maximization of (1)

[5] See, for example, the papers by Barten [**3**], Byron [**8**], Court [**11**], and Parks [**26**]. A remarkable survey and addition to this literature is the unpublished paper by Goldberger [**13**].

[6] See, for example, the papers by Frisch [**12**], Barten [**4**], Stone [**32**], Powell [**29**], and the references in the preceding note.

[7] As Mincer [**24**], Becker [**5**], and others have stressed, leisure is only one component of non-market or "non-work" activity. The crucial point is that the opportunity costs of L_m and L_f are the wage rates W_m and W_f, so that under the assumption of interior maxima the manner of use of the quantities of L_m and L_f is of no empirical consequence.

subject to (2) leads to the familiar conditions $\partial U/\partial L_i = \lambda W_i$ $(i = m, f)$ and $\partial U/\partial X = \lambda P$, where λ is a Lagrange multiplier interpreted as the marginal utility of income. For a given set of values of W_m, W_f, P, and Y these maximization conditions along with (2) are four equations in the four variables L_m, L_f, X, and λ. Assuming the second order conditions for a maximum are satisfied, these four equations may be solved for the latter as functions of the former. The resulting demand for leisure functions are

$$(3) \qquad L_i = L_i(W_m, W_f, P, Y) \qquad (i = m, f).$$

Since $R_i \equiv T - L_i$ is the labor supply of a family member, we may thus write the corresponding supply of labor functions in which we are primarily interested as

$$(4) \qquad R_i = R_i(W_m, W_f, P, Y) \qquad (i = m, f).$$

The signs of the partial derivatives of (4) will of course be equal and opposite in sign to those of (3) since $dR_i = -dL_i$.

The importance of the classical theory for our purposes resides mainly in the restrictions on the partial derivatives of the labor supply functions (4) with respect to wage rates and non-labor income. These restrictions are based on the famous Slutsky decomposition

$$(5) \qquad \frac{\partial R_i}{\partial W_j} = S_{ij} + R_j \frac{\partial R_i}{\partial Y},$$

where S_{ij} is the substitution effect, and the second term corresponds to the income effect.[8] First, we have the restriction that own substitution effects must be positive:

$$(6) \qquad S_{ii} > 0 \qquad (i = m, f),$$

so that an income compensated increase in a family member's wage rate results in an increase in that family member's work effort. Second, we have the restriction that cross-substitution effects must be equal:

$$(7) \qquad S_{mf} = S_{fm},$$

so that an income compensated change in the husband's wage rate has the same effect on the wife's work effort as an income compensated change in the wife's wage rate has on the husband's work effort. Finally, we have the restriction

$$(8) \qquad \begin{vmatrix} S_{mm} & S_{mf} \\ S_{fm} & S_{ff} \end{vmatrix} > 0,$$

[8] The decomposition in (5) differs slightly from the conventional budget allocation case because instead of coming to the market with a given quantity of money, which does not vary when prices vary, the family members come with a certain quantity of time for sale, so that the amount they have available for expenditure is affected by market wage rates. This point, and the decomposition (5), is discussed in Hicks' classic work [**17**, p. 313]. As Kosters [**20**] points out, this difference has not always been appreciated, with the result that the point of compensation, R_j in (5), has been incorrectly overstated in some empirical work.

with a strict inequality because we are dealing with only a subset of the family consumption bundle.[9]

Although they cannot be rigorously established, there are also presumptions regarding the signs of the income derivatives in (5). First, it seems implausible that non-market activity would be an inferior good.

Thus we expect

$$\frac{\partial R_i}{\partial Y} < 0 \qquad (i = m, f), \tag{9}$$

a negative effect of increases in non-labor income on the work effort of family members. A consequence of the combination of (6) with (9) is that the uncompensated or "Cournot" wage effect, $\partial R_i/\partial W_i$, may be of either sign. If $\partial R_i/\partial W_i < 0$, we have the famous backward bending labor supply function. Second, it also seems implausible that consumption goods would be inferior. This implies that the sum of wage-rate weighted income effects, $\Sigma\, W_i \partial R_i/\partial Y$, should be greater than -1.[10] Alternatively, the sum of the Cournot wage elasticities, $(W_m/R_m)\, \partial R_m/\partial W_m + (W_f/R_f)\, \partial R_f/\partial W_f$, must not be less than -1.[11]

Finally, if we suppose $dP = 0$, the total differentials of the labor supply functions (4) are

$$dR_i = (\partial R_i/\partial W_m)\, dW_m + (\partial R_i/\partial W_f)\, dW_f + (\partial R_i/\partial Y)\, dY \qquad (i = m, f).$$

After the substitution of (5), these become

$$dR_i = S_{im}\, dW_m + S_{if}\, dW_f + B_i[R_m(dW_m) + R_f(dW_f) + dY] \qquad (i = m, f), \tag{10}$$

where we have set $\partial R_i/\partial Y \equiv B_i$ for notational convenience. The equations (10) will form the basis for estimation and testing in the next section.

2. AN EMPIRICAL TEST

Equations (10) are derived in terms of the unobservable infinitesimal changes dW_i and dR_i. Prior to estimation, therefore, these must be replaced by observable finite differences.[12] In the case of time-series data, it is natural to replace these

[9] See Samuelson [**31**, p. 115].

[10] According to the budget constraint (2), we have

$$\partial PX/\partial Y = P(\partial X/\partial Y) = W_m(\partial R_m/\partial Y) + W_f(\partial R_f/\partial Y) + 1.$$

Since $P > 0$, $\partial X/\partial Y$ will be negative if $(W_m \partial R_m/\partial Y + W_f \partial R_f/\partial Y) < -1$.

[11] Substituting (5) into the expression in the preceding footnote we have

$$P(\partial X/\partial Y) = \sum_{i=m,f} (W_i/R_i)(\partial R_i/\partial W_i) - \sum_{i=m,f} (W_i/R_i)S_{ii} + 1.$$

Since $S_{ii} > 0$, $\partial X/\partial Y$ will be negative if $\Sigma_{i=m,f}(W_i/R_i)(\partial P_i/\partial W_i) < -1$.

[12] This procedure is common to all empirical studies of consumer demand not based on explicit direct or indirect utility functions. Using arguments of revealed preference for finite differences, the inequalities (6) remain rigorous predictions even for finite changes (Samuelson [**31**, pp. 107–116]), while equations (7) and (8) become hypotheses that may be contradicted by the data due to approximation errors. However, if these errors are randomly distributed, equations (13) below may be treated as a model of random parameters with the consequence that the predictions (7) and (8) may be expected to hold on the average (Zellner [**37**]). The presence of approximation error does dictate the strategy, followed below, of first testing the hypothesis (7) before imposing it in estimation.

differentials with first differences of the variables, as Barten [3] and others have done. In the case of the cross-sectional data that we will use, some alternative method is obviously required, and a natural procedure is to replace differentials with deviations of the variables from their mean values. We will thus make use of the definition $\Delta Z_k \equiv Z_k - \bar{Z}$, the deviation of the kth observation of any variable Z from its mean.

We must also specify the empirical counterparts of the points of compensation, R_i, in (10). The two obvious choices are the endpoints in the finite differences that we use to approximate the dR_i. These endpoints are the mean values, $\bar{R}_i$, and the observed values of the R_i themselves. A consequence of the first choice would be that the points of compensation would be the same for all observations, which unduly strains the interpretation of the equations (10) as first-order approximations. A consequence of either choice would be the asymmetry accorded the treatment of changes from $\bar{R}_i$ to R_i and vice versa. A natural solution to these problems would be to use a simple average of the points R_i and $\bar{R}_i$.[13] In practice this approach is not available to us because of the poor quality of the data available on Y and the fact that we must use data on market-wide averages. Both of these factors make it impossible to calculate directly the average value of the discrete approximation to the term $[R_m(dW_m) + R_f(dW_f) + dY]$ in (10) for the kth area. We proceed therefore by defining:

$$\Delta F \equiv \Delta(R_m W_m) + \Delta(R_f W_f) + \Delta Y \tag{11}$$

and using ΔF as an approximation to this term. The advantage of (11) from our point of view is that it may be calculated directly from data on average family income alone, and the latter is readily available and well measured.[14]

Shifting over to discrete differences in (10), using (11), and adding the disturbance term ε_{ik}, we have for the kth observation:

$$\Delta R_{ik} = S_{im}\Delta W_{mk} + S_{if}\Delta W_{fk} + B_i\Delta F_k + \varepsilon_{ik} \qquad (i = m, f). \tag{12}$$

We propose to treat the S_{ij} and the B_i as parameters for purposes of estimation. This choice is dictated entirely by its convenience for testing the restrictions (6)–(8) derived from utility maximization.[15] Assuming the exogeneity of the ΔW_{ik}, ordinary least squares is still not, of course, an appropriate estimator for (12) because the presence of the identity (11) guarantees that ΔF_k will be correlated with both ε_{mk} and ε_{fk}.[16] In addition, there are reasons noted below for supposing

[13] This is the procedure adopted by Theil [33] and Barten [3] with time-series data.

[14] Using (12) in this way is tantamount to using the actual values R_i as points of compensation and omitting the variable $-B_i(\bar{W}_m R_m + \bar{W}_f R_f)$ from the resulting approximation to (10). The bias in parameter estimates that results from this omission depends on the unknown magnitudes of the uncompensated labor supply elasticities of the husband and wife. Since the former turns out to be very small, the resulting bias from this source should also be small.

[15] Very little systematic discussion of the choice of parameterization in demand analysis seems available. For an exception, see Goldberger [13].

[16] Intuitively, the least squares estimator of the coefficient of ΔF will be biased upward because part of the "credit" for the definitional effect of increased labor supply on money income will be given to the effect of money income on labor supply. Or, as Mincer remarked in his well-known paper [24, p. 69], "Instead of serving as a determinant of labor force behavior, it [money income] already reflects such decisions."

that the disturbance terms ε_{mk} and ε_{fk} will be correlated. Taken together these conditions imply that three-stage least squares is an asymptotically efficient estimator for the unrestricted versions of the equations (12), where ΔF_k is treated as a right-hand endogenous variable.

TABLE I

THREE-STAGE LEAST SQUARES ESTIMATES OF (12) FOR 100 STATISTICAL AREAS, 1960[a]

Labor supply of:	Coefficients (and standard errors) of: ΔW_m	ΔW_f	ΔF
Husbands aged 25 to 54	.110	.139	−.112
	(.0564)	(.0895)	(.0886)
Wives aged 25 to 54	−.0810	.972	−.594
	(.255)	(.385)	(.387)

[a] The additional exogenous variables in the husband's equation are: proportion of households headed by a non-white, median years of schooling of males, aggregate unemployment rate, and net migration rate. The additional exogenous variables in the wife's equation are: number of children under 6 per household, median years of schooling of females, proportion of households headed by a non-white, average wage of domestic servants, and aggregate unemployment rate. An estimate of non-labor income, Y, is also used as an "outside" exogenous variable. These data are described in detail by Bowen and Finegan [7, Tables B-1 and B-10], to whom we are indebted for supplying them, and may be obtained from the present authors upon request. It is worthwhile noting in this connection that the results reported by Bowen and Finegan for married men [7, Table B-5] were inadvertently computed from an incorrect data set, and that a corrected version of that table may be obtained from the Industrial Relations Section, Princeton University. Finally, the dependent variables R_m and R_f are expressed as proportions, with means of $\bar{R}_m = .979$ and $\bar{R}_f = .341$; W_m, W_f, and F are expressed in \$10,000/year, with means for the former of $\bar{W}_m = .555$ and $\bar{W}_f = .320$.

Table I contains estimates of the parameters in (12) from data on the labor force participation rates of married males and females. Following Mincer [**25**], we assume that the reference period for the household utility maximization is the "lifetime" of the family and that the timing of hours worked during this lifetime is irrelevant. Under these assumptions the proportion of available lifetime hours worked may be estimated by the probability of being in the labor force at a point in time, so that aggregating over all households we reach the market labor force participation rate for each spouse.[17] Since these data refer to market-wide aggregates, we are also assuming that each of these labor force groups faces a very elastic demand curve.[18] Some tests of the possible bias that might result from the failure of this assumption are reported below. These equations also contain variables designed to measure differences in tastes and transitory life cycle and labor market conditions.[19] Although their coefficients are not reported in the table

[17] The limited evidence available on the veracity of this interpretation of labor force rates tends to support it. See Cain [**9**, pp. 89–100].

[18] The use of aggregate data to test hypotheses formulated at the household level raises the specter of the aggregation problem. Even if it is assumed that the labor supply parameters differ among households, however, so long as their deviations from mean values are distributed independently of the right-hand variables, the macro parameters in these functions may be shown to satisfy the underlying micro theory. For a discussion of this "convergence approach" to aggregation in demand functions, see Theil [**35**, pp. 570–573].

[19] For an extensive discussion of these variables, see Bowen and Finegan [**7**].

because they differ only slightly from those reported by others, adding these variables to the equations causes no additional complications for (12) because they may simply be thought of as a systematic component of the disturbance term ε_i. Since some of these variables are deleted from each of the equations, they also tend to overidentify (12). In any event, we have adopted the list of variables to be used in this way directly from Bowen and Finegan [**7**] with no further experimentation, and the list is fully indicated in Table I.[20] Deleting them from the equations tends to increase residual variances, but it has very little effect on the parameter estimates in the tables.

Our empirical strategy is first to test the hypothesis $S_{mf} = S_{fm}$ using the results in Table I. Proceeding in this way we compute $\hat{S}_{mf} - \hat{S}_{fm} = .22$ and, using the estimated asymptotic covariance matrix of the estimates in Table I, a standard error of .226. This gives a ratio of estimated coefficient difference to estimated standard error of .98, which clearly would not allow a judgment that $S_{mf} \neq S_{fm}$ at conventional significance test levels. In view of this result we have computed the results in Table II by imposing the restriction $S_{mf} = S_{fm}$. As can be seen from comparison of Tables I and II, imposing this restriction onto the data results in a

THREE-STAGE LEAST SQUARES ESTIMATES OF (12) SUBJECT TO THE RESTRICTION $S_{mf} = S_{fm}$ FOR 100 STATISTICAL AREAS, 1960[a]

Labor supply of	Coefficients (and standard errors) of: ΔW_m	ΔW_f	ΔF
Husbands	.106	.127	−.102
	(.0430)	(.0673)	(.0671)
Wives	.127	1.233	−.886
	(.0673)	(.273)	(.177)

[a] See the note to Table I.

large reduction in estimated standard errors and sharpens our ability to test additional hypotheses rather dramatically. First, we have a substitution effect for males of $\hat{S}_{mm} = .106$, with a ratio of coefficient to standard error of 2.5 that is clearly significantly different from zero at conventional test levels. Although statistically significant, this implies a very small substitution elasticity of around .06 (evaluated at the means). Second, we have a substitution effect for females of $\hat{S}_{ff} = 1.233$, with a ratio of coefficient to standard error of 4.5 that is clearly significantly different from zero at conventional test levels. This implies a relatively large substitution elasticity of 1.154 (evaluated at the means), which is very close to Cain's [**9**, p. 52] estimate of 1.25 for 1950, although it is substantially higher than his estimate of .80 for 1960. Finally, we compute the value of the determinant (8) as $\hat{D} \equiv \hat{S}_{mm}\hat{S}_{ff} - (\hat{S}_{mf})^2 = .117$. Using the estimated asymptotic covariance matrix

[20] The two variables we did leave out were designed by Bowen and Finegan to capture so-called demand factors [**7**, pp. 159–163]. It seemed inappropriate to include these variables in equations that purport to estimate labor supply parameters.

of the estimated coefficients in (12) that is reported in Table III, we compute an estimate of the asymptotic standard error of $\hat{D}$ as .0538.[21] This gives a ratio of $\hat{D}$ to its estimated standard error of 2.17, which would clearly allow the judgment that $\hat{D}$ is significantly greater than zero at conventional test levels. We conclude, therefore, that each of the restrictions (6)–(8) based on the utility maximization hypothesis is consistent with this set of data.

TABLE III

ESTIMATED COVARIANCE MATRIX OF THE ESTIMATED COEFFICIENTS OF (12)[a]

	S_{mm}	S_{mf}	B_m	S_{ff}	S_{fm}	B_f
S_{mm}	.184	.217	−.265	.249	.217	−.304
S_{mf}		.454	−.419	.588	.454	−.672
B_m			.451	−.516	−.419	.609
S_{ff}				7.461	.588	−4.279
S_{fm}					.454	−.672
B_f						3.131

[a] All variances and covariances have been multiplied by 100 for ease of reading, and the lower triangle of the table has been omitted in view of its symmetry.

The estimated values of the income effects B_i are $-.102$ and $-.886$ for husbands and wives respectively, indicating that leisure is a normal good in accord with the presumption (9). Only the latter of these two estimates is large enough relative to its standard error to be judged significantly different from zero at conventional test levels, however. The uncompensated wage derivatives in this model, $\partial R_i/\partial W_j = S_{ij} + R_j B_i$, are not constants, of course, and depend systematically on the points of compensation R_j. Table IV contains the estimates of these "Cournot" wage derivatives evaluated at the mean values of R_m and R_f. These coefficients are the implied values of the slopes of the labor supply functions (4). First, the estimate of $\partial R_m/\partial W_m$ is essentially zero, indicating that the husband's labor supply function is wage inelastic. Second, the estimate of $\partial R_f/\partial W_f$ is large and positive, with an elasticity of .870. Third, we observe that although $\hat{S}_{mf} = \hat{S}_{fm}$ is very small and

TABLE IV

ESTIMATES (AND ESTIMATED STANDARD ERRORS) OF THE UNCOMPENSATED (COURNOT) WAGE EFFECTS AT THE MEAN VALUES OF R_m AND R_f

Dependent variable	Independent variable ΔW_m	ΔW_f
ΔR_m	.006	.092
	(.031)	(.047)
ΔR_f	−.740	.931
	(.146)	(.222)

[21] Arraying the S_{ij} in the vector S and denoting the covariance matrix of the estimator of these terms as Σ, we compute this estimated standard error by inserting the relevant estimates from Tables II and III into $(\partial D/\partial S)'\Sigma(\partial D/\partial S)$ and taking the square root.

positive, the estimated $\partial R_f/\partial W_m$ is very large and negative, and the estimated $\partial R_m/\partial W_f$ is essentially zero. Thus, the equality of the compensated cross-substitution parameters masks two very different uncompensated cross-substitution effects.[22] It is interesting to note that the signs of the wage effects in Table IV are very roughly consistent with the time-series behavior of the labor force rates that is depicted in Table V. For example, the large increases in real wage rates over this period for both males and females apparently had little effect on the labor

TABLE V

THE LABOR FORCE RATES OF HUSBANDS AND WIVES AGED 25–54 IN 1940, 1950, AND 1960[a]

	Husbands	Wives
1940	.951	.166
1950	.941	.244
1960	.964	.328

[a] The source for the data in column (1) is Bowen and Finegan [7, Table 4-3]. For the data in column (2) it is Mincer [**24**, Table 10], and Long [**23**, Table A-6].

force rate of husbands, which is consistent with the inconsequential estimate of $\partial R_m/\partial W_m + \partial R_m/\partial W_f$ in Table IV. Likewise, the large increases in real wage rates over this period apparently had a substantial effect on the labor force rates of wives, which is also very broadly consistent with the estimated positive value of $\partial R_f/\partial W_f + \partial R_f/\partial W_m$ in Table IV.[23] Finally, we compute an estimate of the sum of the wage-rate weighted income effects, $W_mB_m + W_fB_f$, evaluated at the mean wage rates of $-.341$ (with standard error .075). This estimate implies that the average family spends .34 of each additional dollar of unearned income on non-market time, and the other .66 of it on market goods and services, while a full 83 per cent of the additional non-market time is allocated to the wife's non-market activity.

In recent years, economists have paid more attention to the specification of the disturbances of estimated equations. In particular, Theil [**34**] has recently advanced several arguments for supposing that the covariance matrix of the vector $\varepsilon = (\varepsilon_m \varepsilon_f)'$ will be proportional to the leading sub-matrix of the inverse of the bordered Hessian that arises from the maximization of (1) subject to (2). In our case this amounts to the specification that the covariance matrix

$$\Omega = E(\varepsilon\varepsilon') = \sigma^2 \begin{bmatrix} S_{mm} & S_{mf} \\ S_{fm} & S_{ff} \end{bmatrix}. \tag{13}$$

[22] The small value of $\hat{S}_{fm}$ relative to $\hat{B}_f$ implies that a regression of R_f on the wage of the husband will give a reasonably close estimate of B_f. In fact, this is the procedure followed by Mincer [**24**], Cain [**9**], and others, and the results in Table IV provide some justification for this procedure. Interestingly enough, the coefficients reported by Cain [**9**, Table 13] for 1950 are quite similar to those in Table IV. By the same token, the large value of S_{mf} relative to B_m implies that a regression of R_m on the wage of the wife is not a viable procedure for estimating B_m.

[23] As other investigators have found, the estimated value of $\partial R_f/\partial W_f + \partial R_f/\partial W_m$ is *not* large enough to fully explain the increase in R_f over this period by wage changes alone.

TABLE VI
THE ESTIMATED RESIDUAL COVARIANCE MATRIX OF (12)[a]

	ε_m	ε_f
ε_m	.589	.883
ε_f	.883	9.703

[a] All elements of the table have been multiplied by 100 for ease of reading.

Table VI contains the estimated residual covariance matrix for the estimated version of (12). According to (13) this matrix may be used to obtain an independent estimate of the matrix of substitution terms up to the factor of proportionality σ^2. It may be of some interest, therefore, to see how close the estimates implied by Table VI are to those estimated directly in Table II. A simple informal procedure for doing this is to arbitrarily select the estimate of σ^2 so as to force the estimate of S_{mm} implied by Table VI to equal the direct estimate of S_{mm} in Table II. We may then compare the estimates of S_{mf} and S_{ff} implied by this procedure to the results in Table II.[24] Proceeding in this way, we have

$$\hat{\Omega} = 5.56 \begin{bmatrix} .106 & .159 \\ .159 & 1.745 \end{bmatrix}.$$

Comparing the two estimates of S_{mf} and S_{ff}, we have, .159 versus .127 and 1.745 versus 1.233, both of which seem comfortably close in view of the second order nature of Ω.

In view of the fact that the results in Table II are based on a single cross-section, it is of some interest to know whether the estimates of the parameters in (12) would differ in another body of data. Approximately comparable data for wives may be obtained for 1950, but data for husbands are not available. Using the information from Tables I and II that $S_{fm} \approx 0$, we have computed estimates of S_{ff} and B_f using the two-stage least squares estimator.[25] These estimates (and estimated standard errors) are $\hat{S}_{ff} = 1.267$ (.331) a d $\hat{B}_f = -.606$ (.229), which are not significantly different from the comparable estimates of 1.233 and $-.886$ in Table II.

Finally, we have computed estimates of the parameters of (12) that allow for the possibility that the demand for the labor of husbands and/or wives is not highly elastic *in the neighborhood of the observed ranges of the R_i and W_i*. The demand functions for the labor of each group are of course aggregations over differing occupational and industrial structures, so we will make no attempt to estimate them directly. If these demand functions are not highly elastic, however, treating the wage rates on the right-hand side of (12) as exogenous may lead to a serious

[24] An alternative approach is used by Phlips [**28**] for similar purposes.
[25] These data are discussed in Bowen and Finegan [**7**, Table B-101].

inconsistency in our estimator of the labor supply parameters. The elasticity of demand for each group will presumably depend on their substitution possibilities with each other and all other groups as well as the proportion that each group makes up of the total labor force.[26] Since wives make up only about one-half of the female labor supply and less than one-fifth of the total labor supply, it seems likely that the demand for the labor of this group would be highly elastic. On the other hand, husbands make up about three-fourths of the male labor supply and about one-half of the total labor supply, so that this may be less plausible in their case. In any event, we have estimated the parameters of (12) by adding a demand shift variable[27] to the list of exogenous variables noted in Table I and by treating both ΔW_m and ΔW_f as endogenous. The main results of this experiment are briefly summarized. First, the estimated parameters do not change substantially. For example, the estimates of S_{ff} and B_f produced in this way are 1.37 and $-.674$, to be compared with 1.23 and $-.886$ from Table II. Second, the estimated coefficient standard errors increase very substantially. For example, the estimated standard errors of $\hat{S}_{ff}$ and $\hat{B}_f$ produced in this way are 2.87 and 1.17, to be compared with .273 and .177 from Table II. We conclude, therefore, that treating ΔW_m and ΔW_f as endogenous has little effect on our parameter estimates, though it does result in a significant reduction in their estimated precision.

3. CONCLUSION

In this paper we have formulated the theoretical restrictions arising from the application of the classical theory of consumer behavior to the household demand for leisure in a way that makes them readily amenable to test. The results of applying these tests to one body of data do provide support for the classical restrictions, and the imposition of at least one of the restrictions onto the data resulted in a significant improvement in the precision of all parameter estimates. If it can be shown that the implications of the classical theory are fully consistent with other data, we may eventually be able to place substantial confidence in the economist's use of this tool in practical matters. Moreover, we may eventually be able to integrate the consumer's demand for non-market time with his demand for goods and services to produce estimates of a truly complete system of consumer demand functions.

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[26] Holding the wage rate of all other groups constant, an increase in the supply of the given group must result in a decrease in its wage rate unless enough employers exist for whom no significant wage premiums need to be paid to convince them of the acceptability of the additional employees. The extent to which this is likely to be true presumably depends on the heterogeneity of employer job requirements relative to worker qualifications in the group and the proportion that the group is of the labor force.

[27] This is an index of the industrial composition of each area's employment mix and is tabulated in Bowen and Finegan [7, 774–776].

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