Performance Analysis of an Atmospheric Optical M-PPM CDMA System with Optical Orthogonal Code and Partial Modified Prime Code

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Abstract- In this research a theoretical analysis is provided to evaluate the performance of an optical M-ary Pulse Position Modulation (M-PPM) taking into account the effects of Scintillation, avalanche photodiode noise, thermal noise and multi-user interference. The expression for the signal current at the output of an optical direct detection receiver with sequence inverse keying operation considering the effect of scintillations is derived. The effect of atmospheric scintillation on the Bit error rate (BER) performance is determined in the forms of power penalty and the results are compared for OOC’s and PMP codes and optimum system parameters are determined.

I. INTRODUCTION

Optical wireless Communication is attractive more and more attention because of its ability to provide increased capacity, lower cost and flexibility. It becomes very popular in terrestrial application as well as in inter-satellite to ground communication [7, 8] and also the places where fiber can’t be installed very easily but large volume of data needs to be transferred. It can be used as information bridges between buildings containing cables or wireless subnet. When the atmosphere forms part of the propagation channel, scintillation, multi-scattering phenomena are recognized as being the major source of communication performance degradation, leading to link failure and system downtime.

As a multiple access technique in optical communication, optical code-division multiple-access (CDMA) schemes have attracted much attention, particularly in the field of fiber-optic networks, because they allow multiple users to access the network asynchronously and/or simultaneously [1]. Higher order PPM offers better performance than on-off keying (OOK) at a given BER[2-5]. Optical CDMA is advantageous in that it makes channel assignment easier than in time-division multiple-access (TDMA) or frequency-division multiple-access (FDMA) systems. Thus, optical CDMA is an attractive option for future optical access networks.

The performance of atmospheric optical systems is strongly influenced by atmospheric molecular absorption, aerosol scattering and turbulence. These influences can be specified by the attenuation and the fluctuation of the received optical power. In the atmospheric optical communication system using intensity –modulation and direct-detection (IM/DD) the primary factors affecting the performance of the systems is intensity fluctuation that is known as the log-normal intensity scintillation. In optical CDMA systems, multi-user interference is one of the most serious problems. It is important to analyze the performance of an optical CDMA system over a free space optical link considering the above system limitation. However the effect of atmospheric scintillation on the BER performance in the forms of power penalty and the results compare for different types of codes are yet to reported (known to authors).

2. Construction of Partial Modified Prime Code

A major problem associated with prime codes is that their code weight \( w \) is always fixed to the number of code words (i.e. code size) and must be a prime number \( p \) [9]. To accommodate more users in an OCDMA system, a larger \( P \) is required, so is the code weight \( w \). Since all-optical CDMA encoders and decoders for prime codes use a 'parallel' configuration, the resulting optical power losses and complexity of an encoder or decoder would be high if \( w \) becomes large. For example, the power loss of an all-parallel encoder (or decoder) is as high as 35.4 dB if \( p = 59 \), and the required number of optical delay lines per encoder (or decoder) is equal to \( w = p = 59 \). In this case, the encoders and decoders are also bulky, which may prevent their implementation by using integrated optics. As reported in [9-12], the bit error rate (BER) of incoherent OCDMA systems using a prime code is decreased with increasing \( w \). When \( P \) is large enough (e.g., \( P \geq 41 \) for OCDMA systems without optical hard-limiting), the BER becomes very low even if all the users simultaneously transmit data in the system. Thus, it is expected that we can choose a lower \( w \) than that of the large-weight prime code to ensure a prescribed BER (e.g. \( 10^{-9} \)) by removing some 'redundant' pulses from the original prime code of \( P \) pulses. This results in the construction of the modified prime (PMP) codes for OCDMA applications. Subsequently, we will prove that PMP codes still preserve the same cross-correlation constraint as original prime codes.

Then we will show how to reasonably choose the weight and length of PMP codes for OCDMA systems to optimize the system performance.

The prime codes are a set of code sequences with code length \( L = p^w \) derived from prime sequences, where \( p \) is a prime number [9]. Elements of a prime sequence can be obtained by multiplying each element in the Galois field GF \((p) = \)
(0, 1, ..., (p-1)) by a preset number chosen from GF(p). Hence, there are p prime sequences. We then map these prime sequences to binary code sequences to form the prime codes. For example, the prime sequence \( S_{x,0} = (S_{x,0,0}, S_{x,1}, \ldots, S_{x,(p-1)}) \) is mapped to the code sequence \( C_x = (C_{x,0}, C_{x,1}, \ldots, C_{x,(p-1)}) \) according to
\[
C_{x,i} = \begin{cases} 1 & \text{for } i = S_{x,i} + j \cdot p, \quad j = 0, 1, \ldots, p-1, \\ 0 & \text{otherwise} \end{cases} 
\]

On the other hand, the modified prime codes, which are used in synchronous systems, are time-shift versions of the prime codes [12]. To construct the modified prime codes, we take a prime sequence \( S_x \) and rotate it by \((p-1)\) times to create new prime sequences \( S_x = (S_{x,0,0}, S_{x,0,1}, \ldots, S_{x,(p-1)}) \), where \( x \) refers to the number of left-rotations. Likewise, a mapped code sequence \( C_x = (C_{x,0,0}, C_{x,0,1}, \ldots, C_{x,(p-1)}) \) can be obtained according to
\[
C_{x,i} = \begin{cases} 1 & \text{for } i = S_{x,i} + j \cdot p, \quad j = 0, 1, \ldots, p-1, \\ 0 & \text{otherwise} \end{cases} 
\]

Each prime code sequence can generate \( p-1 \) new code sequences to form a code group. Hence, the modified prime codes can be divided into \( p \) groups, and the total number of codes is \( p^2 \). An example of the modified prime codes derived from is GF (5) is shown in Table I. Under synchronized condition, the cross-correlation between the \( x \)th and \( y \)th modified prime codes is

\[
\Gamma_{x,y} = \begin{cases} p, & x = y, \\ 0, & x \text{ and } y \text{ are in the same group}, \\ 1, & x \text{ and } y \text{ are in different groups} \end{cases} 
\]

According to (3), the cross-correlation between the \( x \)th and \( y \)th codes is zero when they belong to the same group or one otherwise. Since the cross-correlation of the modified prime codes is never larger than one, the modified prime codes are superior to the modified quadratic congruence (MQC) codes [11], whose cross-correlation is equal to one, for suppressing PIIN. However, the cross-correlation of the modified prime codes is also equal to one in most situations, i.e., the situation that the codes are in different groups. Hence, the improvement is insignificant.

In order to further suppress the PIIN, we relax the constraint of the cross-correlation of the modified prime codes and propose the PMP codes in this paper to reduce the beating rate between the code sequences. The PMP codes are divided versions of the modified prime codes. Each of the modified prime sequences can be used to generate several new sequences. This means that the modified prime sequence \( S_{x,r} \) constructed from GF(p) can be divided to form \( M \) new prime sequences \( S_{x,r,m} = (S_{x,r,m,0}, S_{x,r,m,1}, \ldots, S_{x,r,m,p-1}) \), where \( M \) is a factor of \((p-1)\) and \( m \in \{0, \ldots, M - 1\} \).

For instance, if, \( M=2 \), the modified prime sequence can be separated as \( S_{x,r} = (S_{x,r,0,0}, S_{x,r,0,1}, \ldots, S_{x,r,0,(p-2)}, S_{x,r,0,(p-1)} = X) \), \( S_{x,r,0,0} = X \), \( S_{x,r,0,1} = X \), \( S_{x,r,0,2} = X \), \( S_{x,r,0,3} = X \), \( S_{x,r,0,4} = X \), and \( S_{x,r,0,5} = X \).

Example of the Modified Prime Codes for GF (5)

<table>
<thead>
<tr>
<th>Group</th>
<th>Sequence</th>
<th>Code Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>S0000</td>
<td>C0000</td>
</tr>
<tr>
<td>1</td>
<td>S0001</td>
<td>C0001</td>
</tr>
<tr>
<td>2</td>
<td>S0002</td>
<td>C0002</td>
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<tr>
<td>3</td>
<td>S0003</td>
<td>C0003</td>
</tr>
<tr>
<td>4</td>
<td>S0004</td>
<td>C0004</td>
</tr>
</tbody>
</table>

Example of the PMP Codes for GF (5)

TABLE I

\[
C_{x,r,m,i} = \begin{cases} 1 & \text{for } i = S_{x,r,m,i} + j \cdot p, \quad j = 0, 1, \ldots, p-1, \\ 0 & \text{otherwise} \end{cases} 
\]

Where X=null. The mapped code sequences \( C_{x,r,m} = (C_{x,r,m,0,0}, C_{x,r,m,0,1}, \ldots, C_{x,r,m,0,(p-1)}) \) can be obtained according to (4)

\[
C_{x,r,m,i} = \begin{cases} 1 & \text{for } i = S_{x,r,m,i} + j \cdot p, \quad j = 0, 1, \ldots, p-1, \\ 0 & \text{otherwise} \end{cases} 
\]

By using this scheme, the size of each group of the modified prime codes is expanded \( M \) times. The code size of the PMP codes is \( Mp^2 \) and the code weight is reduced to \( (p-1)/M \). Therefore, the beating rate of any two PMP code sequences can be reduced as the value of the dividing factor \( M \) is increased. The PIIN can be further suppressed by optimizing the value of \( M \). An example of the PMP codes for GF (5) and \( M=2 \) is shown in Table I. Fig.1 illustrates an exemplary
procedure for generating (0, 1) sequence based on the partial modified prime sequence $S_{t,m,n}$.

Under synchronized condition, the cross-correlation between the $x$th and $y$th PMP codes is

$$\Gamma_{x,y} = \begin{cases} 
(p-1)/M , & x = y \\
0, & x \text{ and } y \text{ are in the same group} \\
\leq 1, & x \text{ and } y \text{ are in different groups}
\end{cases} \quad (5)$$

Hence, if $C_m(g)$ denotes the $g$th element of the $n$th PMP code sequence, the relation between the code sequences can be written as:

$$\sum_{g=0}^{L-1} C_m(g) C_n(g) = \begin{cases} 
\frac{p-1}{M}, & m = n \\
\leq 1, & m = n
\end{cases} \quad (6)$$

Where $L$ is the code length, i.e. $L=p^2$

### TABLE II

Example of the PMP Codes for GF (5) and $M=2$

<table>
<thead>
<tr>
<th>Group</th>
<th>Sequence</th>
<th>Code Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1011</td>
<td>10000000000000000000</td>
</tr>
<tr>
<td></td>
<td>1012</td>
<td>10000000000000000000</td>
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<td></td>
<td>1013</td>
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<td>10000000000000000000</td>
</tr>
<tr>
<td>1</td>
<td>1002</td>
<td>10000000000000000000</td>
</tr>
<tr>
<td></td>
<td>1003</td>
<td>10000000000000000000</td>
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<td></td>
<td>1103</td>
<td>10000000000000000000</td>
</tr>
<tr>
<td></td>
<td>1104</td>
<td>10000000000000000000</td>
</tr>
<tr>
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<td>1105</td>
<td>10000000000000000000</td>
</tr>
<tr>
<td></td>
<td>1106</td>
<td>10000000000000000000</td>
</tr>
<tr>
<td></td>
<td>1107</td>
<td>10000000000000000000</td>
</tr>
<tr>
<td></td>
<td>1108</td>
<td>10000000000000000000</td>
</tr>
</tbody>
</table>

III. Mathematical Analysis

The probability that a specified number of photons are absorbed from an incident optical field by an APD detector over a chip interval with is given by a Poisson distribution [64-66]. The average number of absorbed photons over $T_c$ is

$$\lambda_s = \frac{P_w \eta}{hf} \quad (7)$$

Where $\lambda_s$ is the photon absorption rate, $P_w$ is the received laser power, $\eta$ is the APD efficiency, $h$ is Planck’s constant, and $f$ is the optical frequency. Through an avalanche multiplication process, the APD outputs some electrons in response to the absorption of $\lambda_s T_c$ primary photons on the average: $\lambda$ represents the total photon absorption rate due to signal, background, and APD bulk leakage current

$$\lambda = \begin{cases} 
k\lambda_s + k\lambda_s + \frac{I_b}{e} & \text{for a mark} \\
k\lambda_s + k\lambda_s + \frac{I_b}{e} & \text{for a space}
\end{cases} \quad (8)$$

Where $\lambda_s$ is the photon absorption rate due to the actual background light, $e$ is an electron charge, $I_b/e$ represents the contribution of the APD bulk leakage current to the APD
output, and \( M_e \) is the modulation extinction ratio of the laser output power.

The probability that a user causes interference to the \( i \)th slot of the desired user is given by [6]

\[
2 \frac{P_e}{MF} \tag{9}
\]

We assume the 0th user to be the desired user. We define the interference state vector at the \( i \)th slot as

\[
K_i \equiv (l_i(1), l_i(2), \ldots, l_i(N-1))
\]

Where

\[
l_i(j) = \begin{cases} 
1, & \text{if the } j \text{th user hits on the } i \text{th slot} \\
0, & \text{otherwise}
\end{cases}
\]

And \( N \) is the number of users (transmitters). We also denote the output of the optical correlator in the \( i \)th slot \( i \in \{0, 1, \ldots, M-1\} \) by \( Y_i \).

Assuming equally likely data, the word error probability is expressed as

\[
P_e = \sum_{i=0}^{M-1} P[e|i] \Pr[i] \tag{10}
\]

where \( \Pr[i] = 1/M \) represents the occurrence probability of the \( i \)th word. Using a union bound, the conditional probability is expressed as

\[
P[e|i] = \Pr[Y_j \geq y_i, \text{some } j \neq i]
\]

\[
\leq \sum_{j=0, j \neq i}^{M-1} \Pr[Y_j \geq y_i]
\]

Please note that we analyze the performance in the chip synchronous and slot asynchronous case. In this case, the conditional probability \( P[e|i] \) differs for each \( i \), because \( \Pr[Y_j \geq y_i] \) depends on the absolute value of \( |i - j| \) [11].

Denoting the union bound of the word error probability \( P_e \) by \( P_e^U \) and using that \( \Pr[Y_j \geq y_i] \) depends only on the absolute value of the difference \( |i - j| \), \( P_e^U \) is expressed as [11]

\[
P_e^U = \sum_{i=0}^{M-1} \sum_{j=0, j \neq i}^{M-1} \Pr[Y_j \geq y_i] \Pr[e]
\]

\[
= \sum_{d=0}^{M-1} \frac{2}{M} \left( M - d \right) \Pr[Y_j \geq y_i, |i - j| = d]
\]

\[
= \sum_{d=0}^{M-1} \frac{2}{M} \left( M - d \right) \Pr[Y_d \geq y_0, |d|]
\] \tag{11}

Where the probability \( \Pr[Y_d \geq y_0] \) in (11) is evaluated as follows:

\[
\Pr[Y_d \geq y_0] = \sum_{k_0} \sum_{k_d} \Pr[Y_d \geq y_0, k_0 = k_d] \Pr[k_0, k_d]
\]

\[
= \sum_{k_0} \sum_{k_d} \Pr[Y_d \geq y_0, k_0 = k_d] \Pr[k_0, k_d]
\] \tag{12}

Where \( L \) is the set of all the interference vectors.

The upper bound on the conditional probability in (12) is given by

\[
\Pr[Y_d \geq y_0, k_0 = k_d] \leq \Pr[Y_d \geq y_0, k_0 = 0, k_d]
\]

Substituting into (12), we get the following conditional equation [6]:

\[
\Pr[Y_d \geq y_0, k_0 = 0] \leq \sum_{k_d} \Pr[Y_d \geq y_0, k_0 = 0, k_d] \Pr[k_0, k_d]
\]

\[
= \sum_{k_d} \Pr[Y_d \geq y_0, k_0 = 0, k_d] \Pr[k_0, k_d]
\] \tag{13}

Please note that \( \Pr[k_d] \) is expressed as

\[
\Pr[k_d] = \left( \frac{K^2}{MF} \right)^{|k_d|} \left( 1 - \frac{K^2}{MF} \right)^{N-1-|k_d|}
\]

Substituting (13) into (11), we get the upper bound on \( P_e^U \) as follows:

\[
P_e^U \leq \sum_{k_d} \left( \frac{K^2}{MF} \right)^{|k_d|} \left( 1 - \frac{K^2}{MF} \right)^{N-1-|k_d|}
\]

\[
\Pr[Y_d \geq y_0, k_0 = 0, k_d]
\] \tag{14}
Where $|k_d|$ is the number of nonzero elements in $k_d$. At an optical CDMA receiver, the optical intensity received at the mark positions of the code sequence for the desired channel is summed up at the last chip by the optical correlator consisting of a set of optical delay lines inversely matched to the pulse spacings [67]. The output of the optical correlator is converted into the electrical signal by the APD. The receiver integrates the APD output over the last chip interval $T_c$. Using a Gaussian approximation of the APD output, the probability $P_{Y_d \geq Y_0|k_d=0,k_0=0}$ in (14) is derived as

$$P_{Y_d \geq Y_0|k_d=0,k_0=0} = \frac{1}{\sqrt{2\pi}\sigma_d^2} \int_{-\infty}^{\infty} e^{-\frac{(x-\mu_d)^2}{2\sigma_d^2}} \, dx,$$

where $\mu_d(X) = G_T e_{\lambda_d(X)} = \sum_{i=1}^{N-1} I_j(i) X_i \lambda_{d,i} + \frac{K X_0 \lambda_{d,0}}{M_e} + \sum_{i=1}^{N-1} (K - I_i(i)) \frac{X_i \lambda_{d,i}}{M_e} + K \lambda_b + \frac{I_b}{e}$

and $\sigma_d^2 = G^2 F_e T_c$

Here, $\lambda_{d,i}$ is the photon-absorption rate of the $i$th user, $G$ is the average APD gain, $I_b$ is the APD surface leakage current, and $F_e$ is the excess noise factor given by

$$F_e = k_{no} G + \left(2 - \frac{1}{G}\right) \left(1 - k_{eff}\right)$$

where $k_{no}$ is the APD effective ionization ratio, and $\sigma_{th}^2$ is the variance of thermal noise written as

$$\sigma_{th}^2 = \frac{2k_B T_r R_L}{(e^2 R_L)}$$

where $k_B$ is Boltzmann’s constant, $T_r$ is the receiver-noise temperature, and $R_L$ is the receiver-load resistor. Therefore, the BER of the atmospheric optical M-ary PPM CDMA systems is derived as

$$P_b \leq \frac{M}{2(M-1)} \rho_e^U$$

Please note that our performance involves two levels of bounds and a Gaussian approximation and that (4.16) is the average over log normal power of Gauss exceeding another.

### IV. Result

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background Noise, $P_b$</td>
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</tr>
<tr>
<td>Wavelength, $\lambda$</td>
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</tr>
<tr>
<td>APD Gain, $G$</td>
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<tr>
<td>Receiver Noise Temperature, $T_r$</td>
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</tr>
<tr>
<td>Surface Leakage current, $I_s$</td>
<td>10 nA</td>
</tr>
<tr>
<td>Bulk Leakage Current, $I_b$</td>
<td>0.1 nA</td>
</tr>
<tr>
<td>Bit Rate, $B_r$</td>
<td>1 Gbps</td>
</tr>
<tr>
<td>Modulation Extinction Ratio, $M_e$</td>
<td>100</td>
</tr>
<tr>
<td>Receiver Load Resistor, $R_L$</td>
<td>1030</td>
</tr>
<tr>
<td>APD Quantum efficiency, $\eta$</td>
<td>0.6</td>
</tr>
<tr>
<td>APD effective Ionization Ratio, $k_{no}$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table III. Nominal Parameters of Optical Wireless Communication Link
Fig. 4 BER Versus the received laser power without scintillation $P_w$ for four-ary PPM-CDMA ($M=4, \sigma_s^2=0, \sigma_i^2=0.005$)

Fig. 5 BER Versus the received laser power without scintillation $P_w$ for $M$-ary PPM CDMA ($\sigma_s^2=0$)

Fig. 6 BER Versus the received laser power without scintillation $P_w$ for four-ary PPM CDMA in the presence of different values of $\sigma_i^2$

Fig. 7 Power penalty as a function of variance of scintillation for $M$-ary PPM

Fig. 8 BER Versus Number of Users for $M$-ary PPM-CDMA ($\sigma_s^2=0.01$)

Fig. 9 BER Versus the received laser power without scintillation $P_w$ considering $M$-PPM CDMA system with OOC's and PMP codes in the presence of different values of $\sigma_i^2$ ($M=2, N=3$)
V. Conclusion
It is found that the performance of optical direct detection M-PPM systems is very sensitive to atmospheric scintillation. There is a significant degradation in BER performance due to atmospheric scintillation and the penalty is higher for higher value of scintillation variation. It is also observed that higher order PPM offers better performance. The numerical results confirm the superior performance of the PMP-coded OCDMA system in comparison to OOC-OCDMA system even under the scintillation effect. The result is useful in determining the receive power level of the system to guarantee a given receiver symbol error performance.

Reference