Abstract—In Ad Hoc or sensor networks, communication may be achieved between mobile nodes without a central entity (base station), in a half-duplex manner. The impact of jamming signals on those kinds of networks is studied. The contributions of this paper can be divided into two parts. In the first part, the probability of success, throughput and the maximum throughput are derived for the half-duplex Slotted ALOHA in terms of the node transmitting probability, the node receiving probability and average jamming signals rate. Results show that the probability of success, throughput and maximum throughput decrease with the increase of average jamming signals rate. In the second part, the effect of jamming signals on the maximum throughput of retransmission cut-off half-duplex Slotted ALOHA is studied. A close form equation is obtained for the transmission probability from each mobile node that maximizes the channel throughput in the presence of jamming signals. This equation provides the relationship between the new packet transmission probability from each mobile node, jamming signals rate and the number of retransmission attempts. The number of retransmission attempts plays an important role for a lower value of jamming signals rate. The results of this study may be used for system design of half-duplex contention-based multiple access schemes with and without jamming signals and retransmission cut-off.

Index Terms—Ad Hoc, half-duplex, jamming signals, node transmitting probability, number of retransmission attempts, sensor, Slotted ALOHA.

I. INTRODUCTION

SELECTING an efficient multiple access scheme for sharing the limited radio resources among the mobile nodes is one of the most challenging issues in the wireless networking field. Among the radio resource sharing multiple access schemes Code Division Multiple Access (CDMA) system has a special resistance against the interference jamming signals. Consider a simplified Direct Sequence CDMA (DS-CDMA) transmission system in which the spread signals experience jamming interference signals when it is transmitted over radio channels. With the knowledge of spreading code, the receiver is able to spread further the power density of the jamming interference and enhance the user signals power density, if the jamming interference is narrowband. In case of broadband jamming interference signals, it is possible to transmit the user signals with lower spectral power density than that of the jamming interference power and to hide the user signals into the jamming signals. The user signals are only detectable from the jamming signals with the knowledge of the spreading code. Thus CDMA may be the first choice as a multiple access scheme in the presence of jamming interference signals.

In Ad Hoc and sensor networks, mobile terminals not only behave as transmitters and receivers but also as network elements, i.e. switches or routers, without any established network infrastructure. Almost all mobile terminals are powered by battery, and replacement or recharging a battery during the time of operation is very difficult and most of the time impossible. Especially sensor networks that require lot of retransmissions which requires lot of battery power consumption. Therefore low power consumption systems are becoming important. The CDMA scheme was compared with ALOHA scheme in terms of power consumption in [1]. A simple linear transformation of conventional ALOHA access, called spread ALOHA was made to compare it with CDMA. It is not possible to find a multiple access method at the same average power and the same bandwidth which is more efficient than spread ALOHA [1]. In addition, a Slotted ALOHA system is more efficient than that of the pure ALOHA system [2].

The Slotted ALOHA multiple access scheme is a widely used random access protocol independently and as a part of different multiple access protocols, for its adequate working capability for distributed wireless nodes having bursty traffic. One inherent problem of the Slotted ALOHA is its lower throughput. Thus, in the design of some mobile communication systems, maximizing the throughput of Slotted ALOHA is an important issue. Stability and maximum throughput of retransmission cut-off Slotted ALOHA is studied in [3]. The retransmission cut-off Slotted ALOHA attains its maximum throughput when the optimum packet generation rate is applied [3]. Slotted ALOHA with retransmission cut-off is used in some modern mobile communication systems. For instance, it is used in the random access channels of Global System for Mobile (GSM) communications [4] and its extension General Packet Radio Services (GPRS) [5], [6], Wideband Code Division Multiple Access (WCDMA) system [2], cdma2000 [7] [8] etc.

The maximum throughput of Slotted ALOHA with retransmission cut-off is investigated in [9]. It is shown that the Slotted ALOHA never reaches its maximum throughput, if the new packet generation rate is below a critical limit irrespective of the number of retransmission attempts [9]. The value of this critical limit is the same as the maximum throughput. The number of retransmission attempts has to
decrease abruptly, if the new packet generation rate exceeds this critical limit [9]. Few similar results were also found in [10]. Note that, although [9] and [10] followed two different approaches for derivation, few results obtained were similar.

In all the above mentioned ALOHA systems [1-10] mobile nodes transmit their packets to a central entity (base station). The maximum throughput of Slotted ALOHA for infrastructure-less wireless Ad Hoc, sensor and mesh networks is studied in [11] and [12]. Maximizing the throughput of half-duplex Slotted ALOHA with multiple power levels is examined in [11]. The throughput is less than maximum throughput, if the new packet generation rate from each mobile node is less than a critical limit [12]. The jamming signals is not considered in all the above mentioned ALOHA systems [1-12].

The effect of jamming signals on the Slotted ALOHA scheme without retransmission cut-off is investigated [13-19]. However, the maximum throughput of half-duplex Slotted ALOHA by optimization of three parameters, the new packet transmission probability from each mobile node, average jamming signals rate and the number of retransmission attempts, has not been previously investigated. In this paper a generalized relationship between the three parameters, the new packet transmission probability from each mobile node, the average jamming signals rate and the number of retransmission attempts that also achieves the maximum throughput, is presented.

In the infrastructure-less wireless networks, mobile nodes may communicate with each other directly in a half-duplex manner, to reduce the components of the mobile nodes [26, 27]. The optimum use of the fewer components in each mobile node reduces the overall cost [28]. A half-duplex CDMA system is considered in [26, 27, 28]. An ALOHA system is more efficient than that of the CDMA system, in terms of power and bandwidth [1], [2]. In general ALOHA system is an attractive protocol due to its simplicity and easy to implement. Because of that, we have considered the improved version of Slotted ALOHA with and without retransmission cut-off and jamming, although the simple ALOHA system is invented in early 1970’s.

The paper is organized as follows. Section II describes the authorized and jamming packet arrivals on multiple receiving nodes of the half-duplex Slotted ALOHA system. The effect on probability of success, throughput and maximum throughput is derived in Section III. Section IV provides the optimum new packet generation probability from each mobile node that maximizes the throughput in the presence of jamming signals. The conclusion is provided in Section V.

II. AUTHORIZED AND JAMMING PACKET ARRIVAL MODEL ON THE MULTIPLE RECEIVING NODES

Let us consider an area where mobile nodes are uniformly distributed in that area. Assume that mobile nodes’ distribution is homogenous Poisson Point Process. This assumption has been widely used in analyzing multi-hop wireless Ad Hoc networks [11, 12, 20, 21, 22]. We will consider a circular area of a as our operating range for calculation. If the density of mobile nodes is $\rho$ per unit area then the average number of nodes in the area $a$ is $\gamma = \rho a$.

Under these assumptions, the probability that there are $i$ nodes in the area $a$ is given by [11, 12, 20, 21, 22]

$$E(d = i) = \frac{\mu^i}{i!} e^{-\mu}$$

(1)

Where $\gamma$ is the average number of nodes in the area $a$, as described earlier.

The mobile nodes can be divided into three parts: transmitting nodes, receiving nodes, idle nodes (neither transmitting nor receiving). Now we have to evaluate the average number of transmitting nodes, average number of receiving nodes and average number of idle nodes in the area of $a$.

Assuming that the transmission radio channels in multi-hop Ad Hoc network is divided into time frames with equal duration called time Slotted. Each mobile node transmits its data packets by fitting those time slots. These data slots are used when few nodes are transmitting packets and few other nodes receive packets. Because of that in Ad Hoc networks, mobile nodes can not be in a transmission mode and in a receiving mode at the same time. Multiple Slotted ALOHA channels are present in the system. The number of Slotted ALOHA channels is same as the number of receiving mode nodes. Each mobile node occupies one Slotted ALOHA channel during its receiving mode. These occupations are known to all other mobile nodes.

Assume that each node become in any of these three modes independently. Any of the nodes can become in transmission mode with probability $b$.

A. Packets generation from jammers and transmitting nodes

It is already assumed that each node’s transmits mode, reception mode and idle mode in a given time slot is independent in nature. One node can cover the area of $a$ and there are $i$ nodes in that area and the distribution of the number of those nodes is given in Eq. (1). Consider that out these $i$ nodes, in a given time slot duration, $j$ nodes are in transmitting mode. If each node becomes in transmission mode with probability $b$, the probability that $j$ nodes will be in transmission mode in the area $a$ is [11]

$$E(j) = \frac{(\mu j)^j}{j!} e^{-\mu j}$$

(2)

Eq. (2) shows that the distribution of nodes in the transmission mode are also Poisson Point Process within the area $a$. The average number of transmitting nodes covered by a node (within the area of $a$) is $Z = \gamma j$ [11]. Although, Eq. (1) and Eq. (2) have exactly similar behavior, their average values are different.

A random packet destruction denial of Services (DoS) attack signals [23] is considered as jamming signals. The jammers are producing short period of noise/jamming signals. The jammer does not need to pretend as a legal user. Thus, current anti-jamming measures such as encryption, authentication and authorization cannot prevent these types of attacks.

A constant noise jamming signal needs more power comparing with the random packet destruction Denial of Services (DoS) attack signals [23]. The power consumption is also critical for jammers. Because of that we have considered this type of jammers in this paper.
Assuming that the jamming packet generation can also be modeled as Poisson Point Process with an average rate of $\gamma_j$. It is well known that the addition of two Poisson Point Processes is another Poisson Point Process, which average rate is sum of two averages. Thus the probability that $k$ packets are generated by authorized mobile nodes and jammer in the area $a$ is

$$E(k) = \frac{(\gamma_j + \gamma)^k}{k!} e^{-(\gamma_j + \gamma)}$$

Now we have to calculate the average number of receiving nodes within the area $a$. These receiving nodes will receive not only the authorized users’ packets but also the jamming signals (noise packets).

$\text{B. Average number of receiving nodes and idle nodes}$

It is already assumed that each node’s transmits mode, reception mode and idle mode in a given time slot is independent in nature. If each node can be in a receiving mode independently with probability $c$, then the probability that $l$ nodes are in receiving mode within the region of $a$ is given by [11]

$$E(l) = \binom{\gamma}{l} c^l (1-c)^{\gamma-l}$$

Eq. (4) indicates the probability of receiving nodes covered by area $a$, which is also Poisson Point Process. The average number of receiving nodes in the area $a$ is $Y = \gamma$ [11].

It is already shown that the distribution of transmitting nodes and receiving nodes in the area of $a$ are Poisson Point Processes. Using the same procedure, it can be shown that the distribution of the ideal nodes in the same area is also Poisson Point Process.

It has also been shown in the beginning of the Section that the average number of nodes in the area $a$ is $\gamma = \rho a$. The average number of transmitting nodes $Z = \gamma_j$. The average number of receiving nodes $Y = \gamma c$. Therefore, the average number of idle nodes in the region $a$ is $X = \gamma - \gamma_j - \gamma c$.

Where, $(\gamma_j + \gamma c) \leq 1$. If, $(\gamma_j + \gamma c) = 1$, then there is no ideal nodes in the area $a$. If there are also idle nodes in the area $a$, then $(\gamma_j + \gamma c) < 1$.

We have already shown in Eq. (2) that $j$ nodes are ready to transmit authorized packets within the area $a$. Eq. (3) shows that $k$ packets are transmitted from authorized users and jammer. Few packets are transmitting outside the border and few packets are coming from the outside of the area $a$. An additional assumption in this matter is that the number of outgoing (authorized and jamming) packets from the area $a$ are equals to the number of incoming packets from the outside to inside of the area. Under this assumption, we can say that on average there are $(\gamma_j + \gamma) \gamma_j$ packets those should be received by $Y$ receiving nodes.

Based on that, we have to find the packet arrival model of a half-duplex system without any central entity.

$\text{C. Authorized and jamming packets arrival model to a receiving node}$

According to Eq. (3), the number of packets or nodes ready to transmit is $k$ which can be any number: from zero to very larger. It is already shown in Eq. (4) that the number of receiving nodes is $l$ and the average value of the receiving node is $Y$ in the area $a$.

Now according to system model presented above, (in the area of $a$) $k$ packets are ready to transmit and $l$ receiving nodes will accept those packets. For simplicity, let us assume that $k$ packets will be directed to $Y$ receiving nodes, since the average number of receiving node is $Y$. If all $k$ packets are uniformly directed to $Y$ receiving nodes, then $m$ packets out of $k$ packets directed to a given receiving node is

$$W = \frac{\binom{k}{m} \gamma_m Y^m (1-Y)^{k-m}}{m!}$$

The necessary condition for Eq. (6) is $Y \geq 1$ or $\gamma \geq 1$. In other words, each transmitting node and jamming signals should be surrounded by at least one receiving node. Now form total probability theory, the probability that $m$ (authorized and jamming) packets are ready to transmit to the direction of a given receiving node is

$$Tr = \sum_{l=m}^{\infty} \binom{k+j}{l} e^{-(j+c)} \binom{l}{m} \gamma_m Y^m (1-Y)^{k-m}$$

Eq. (7) shows the probability that $m$ packets are ready to transmit to any receiving node. It is important to note that the probability equation derived in (7) is independent of average number of transmitting nodes or receiving nodes within the range of operation (area $a$). The probability equation (7) depends only on the ratio of node transmitting probability, $b$, node receiving probability, $c$, and average rate of jamming signals. We also modeled the jammer signals in such a manner that it will be independent of the area $a$. And we obtain this equation under the assumption that each transmitting node has at least one receiving node in its range of operation.

In addition, Eq. (7) shows that the aggregate of authorized and jamming packet arrival in a half-duplex system can be modeled as Poisson and the arrival rate is the ratio of packet transmission probability from each mobile node plus average jamming packet arrival rate and the packet receiving probability of each mobile node.

The packet arrival to any receiving node without jamming signals can be obtained by setting $J=0$ in Eq. (7) as

$$Tr = \frac{b}{c} e^{-\frac{b}{c}}$$

Eq. (8) is same as the packet arrival model without jamming signals as described in [11].

$\text{III. Effect of Jamming Signals on Half-Duplex Slotted ALOHA}$

The aggregate of authorized and jamming signals packet arrival in each mobile node is another Poisson process having rate of $(b+c)/c$. One mobile node can receive only one packet per time slot, if it is in receiving mode. A packet is successfully received in a time slot is the probability of transmitting an authorized packet in a given slot and no other interference packet transmitted to the same slot.
The probability of success in the presence of jamming signals can be written as
\[ P(Su) = P(\text{a packet will be message packet}) \times P(\text{no other interference packet}) \]
\[ = \frac{(b/c)}{(b+c)} \exp\left(-\frac{(b+J)}{c}\right) \tag{9} \]

Figure 1 depicts the probability of success with and without jamming signals. The curve indicated by \( J=0 \) is without jamming signals. Figure 1 shows that the original signals level \((b/c)\) should be sufficiently high to obtain higher probability of success, \( P(Su) \), in the presence of jamming signals.

![Figure 1. Probability of success with and without jamming signals.](image)

The probability of success in the presence of jamming signals has two components: first is the probability that a packet will be an authorized packet and the second component is when there is no other interfering packet. The transmission probability from each mobile node is zero, then the probability that a packet will be an authorized packet is also zero. The probability of success starts to increase with the increase of transmission probability from each mobile node, by increasing the probability that a packet will be an authorized packet. Note that the probability that a packet will be an authorized packet is one without any jamming signals, and this is considered in [1-12].

Throughput of the half-duplex Slotted ALOHA in the presence of jamming signals is [13-12].

\[ S = (b/c)P(Su) = \left(\frac{b}{b+c}\right) \exp\left(-\frac{(b+J)}{c}\right) \tag{10} \]

Assume that each node is either in transmission mode or in receiving mode. Thus, \( b+c=1 \). Using this relationship in Eq. (10), the throughput can also be written as
\[ S = \left\{ b \left(1-b\right) \right\} \exp\left\{ \left(\frac{b+J}{1-b}\right) \right\} = \left\{ \frac{1-c}{c} \right\} \exp\left\{ \left(\frac{1-c+J}{c}\right) \right\} \tag{11} \]

Throughput of half-duplex Slotted ALOHA in the presence of jamming signals is depicted in Figure 2. Obviously, the throughput decreases with the increase of jamming signals. The jammers know the behavior of original signals. Those jammers produce jamming signals having exactly same behavior of the original signal. Because of that, the throughput decreases with the increase of jamming signal at all traffic load condition.

![Figure 2. Throughput with and without jamming signals.](image)

To obtain the maximum throughput in the presence of jamming signals, differentiating Eq. (11) with respect to \( c \) and setting it equals to zero, we obtain a quadratic equation as
\[ 2q^2 - q(1+J)3+J) + (1+J)^2 = 0 \tag{12} \]

In Eq. (12), \( q \) has two solutions
\[ q = \frac{(1+J)(3+J)}{4} \pm \frac{(1+J)}{4} \sqrt{1+6J+J^2} \tag{13} \]

The one solution is in the operating range and defines the optimum receiving probability from each mobile node as
\[ q_{opt} = \frac{(1+J)(3+J)}{4} - \frac{(1+J)}{4} \sqrt{1+6J+J^2} \tag{14} \]

The optimum transmitting probability of each mobile node is
\[ p_{opt} = 1-q_{opt} = 1 - \frac{4J-J^2}{4} + \frac{(1+J)}{4} \sqrt{1+6J+J^2} \tag{15} \]

The maximum throughput in the presence of jamming signals can be obtained using the optimum receiving probability of each mobile node from Eq. (14) or optimum transmitting probability of each mobile node from Eq. (15) to Eq. (11). The maximum throughput in the presence of jamming signals \( J \) is
\[ S_{\text{max}} = \frac{1-4J-J^2 + (1+J)(3+J) - (1+J)\sqrt{1+6J+J^2}}{3+4J+J^2 - (1+J)\sqrt{1+6J+J^2}} \exp\left\{-\left(\frac{1-J+\sqrt{1+6J+J^2}}{3+J-\sqrt{1+6J+J^2}}\right) \right\} \tag{16} \]

The numerical results of Eq. (16) are depicted in Figure 3. Obviously, the maximum throughput decreases with the increase of jamming signals.

![Figure 3. Maximum throughput.](image)

IV. RETRANSMISSIONS CUT-OFF HALF-DUPLEX SLOTTED ALOHA WITH JAMMING SIGNALS

In the case of a normal data transmission system, every packet must be transmitted successfully. Thus, the new packet
generation rate should be enough low to keep the system throughput lower than that of the maximum throughput. On the other hand, in the case of access to obtain dedicated channel or real-time traffic transmission, we can cut the retransmission number to keep the system throughput up to its maximum limit. The main intension of a random access protocol is to keep the system operation stable by adjusting its throughput up to maximum limit.

Consider the new packet generation (transmission) probability from each mobile node is \( x \) packet per time slot. The new packet receiving probability of each receiving node is \( y \) packet per time slot. Thus the new packet arrival rate to the direction of a receiving node is \((x/y)\) packet per time slot. Arrival of newly generated and retransmitted packets can be assumed as Poisson [24]. Let the total number of retransmission attempts of a packet be \( r \). The total mean offered traffic to a receiving node is then given by [25]

\[
S = \left[ \frac{b}{c} \right] P(Su) = (x/y) \left[ 1 - \left( 1 - P(Su) \right)^{r-1} \right] \quad (17)
\]

Let us consider a special case, where each mobile node is either in transmitting mode or in receiving mode, i.e., \( y = 1-x \). In that situation, and combining the Eq. (17) we can rewrite as

\[
x = \frac{S}{S + \left[ 1 - \left( 1 - P(Su) \right)^{r-1} \right]} \quad (18)
\]

Maximum throughput is the one of the most important issues specially for designing a random access protocol. Maximum throughput of half-duplex Slotted ALOHA in the presence of jamming signals is investigated in previous section. The channel throughput shows its maximum value when the receiving probability from each mobile node

\[
q_{\text{opt}} = \frac{(1+J)(3+J)}{4} \sqrt{1+6J + J^2}, \quad \text{as given in Eq. (14)}
\]

Combining Eqs. (9), (14), (15), (16) and (18), the optimum new packet transmission probability that also provides the maximum throughput with and without jamming signals and retransmissions cut-off can be written as

\[
x_{\text{opt}} = \frac{S_{\text{opt}}}{S_{\text{opt}} + \left[ 1 - \left( 1 - P(Su) \right)^{r-1} \right]}^{\left( \frac{1}{3} \right)}
\]

The value of \( S_{\text{opt}} \) shown in Eq. (19) can be evaluated from Eq. (16). Eq. (19) is the close form equation for the maximum throughput of retransmission cut-off half-duplex Slotted ALOHA in the presence of jamming signals.

Figure 4 shows the optimum new packet transmission probability from each transmitting node, \( x_{\text{opt}} \), with the variation of the retransmission attempts, \( r \), using Eq. (19). Thus, the optimum transmission probability from each mobile node, \( x_{\text{opt}} \), has to decrease with the increase of the retransmission attempts, \( r \).

Figure 5 also represents the optimum new packet transmission probability from each mobile node of half-duplex Slotted ALOHA with the variation of jamming signals rate that provides maximum throughput.

Let us consider a special case of Eq. (19), where the system operates without retransmission cut-off. In that situation, the number of retransmission attempts, \( r \), is very large. Under that condition, we can define a new parameter as the ‘critical limit’ of the new packet transmission probability, below which the retransmission cut-off is not needed to achieve the maximum throughput of half-duplex Slotted ALOHA with jamming signals. Thus, the ‘critical limit’ of the new packet transmission probability below which the retransmission cut-off is not needed, can be written using Eq. (19) (as for a large number of retransmission attempts, \( r \))

\[
CL1 = \frac{S_{\text{opt}}}{1 + S_{\text{opt}}}
\]

Where the value of \( S_{\text{opt}} \) is given in Eq. (16).

From Figure 4 and 5 and/or from Eq. (20), it can be said that the system can achieve maximum throughput without retransmission cut-off (for a large value of \( r \)), if new packet transmission probability from each transmitting node is less than a ‘critical limit’ and this value of critical limit can be evaluated using Eq. (20). On the other extreme is no retransmission attempt is allowed. Under that condition, to achieve the maximum throughput of the system, the new packet transmission probability from each transmitting node is

\[
C_{\text{CL2}} = \frac{S_{\text{opt}}}{\frac{1-4J-J^2}{3+4J-J^2} + \frac{(1+J)(1+6J+J^2)}{3+4J-J^2}} \quad \text{exp} \left[ \frac{1-J+\sqrt{1+6J+J^2}}{3+J-\sqrt{1+6J+J^2}} \right]
\]

Additionally, Figures 4 and 5 and/or Eq. (19) show that the number of retransmission trials, \( r \), plays an important role, if
the jamming signals rate $J$ is small. If the jamming signals rate $J$ is medium or high, the maximum throughput is very low. Nothing can help during that situation.

V. CONCLUSIONS

In this paper, an analytical approach for the maximum throughput of half-duplex Slotted ALOHA with and without jamming signals and retransmission cut-off scheme has been investigated.

The probability of success of the half-duplex Slotted ALOHA with jamming signals shows interesting results. Without any jamming, the probability of success decreases linearly with the increase of transmission probability from each mobile node, $b$ (Figure 1 a). In the presence of jamming signals, the probability increases initially with the increase of transmission probability from each mobile node. It reaches its maximum value in a certain value of transmission probability from each mobile node. The probability of success starts to decrease when the transmission probability from each mobile node increases further.

Maximum throughput decreases with the increase of jamming signals rate, $J$ (Figure 3 and Eq. (16)). If the jamming signals rate, $J$ is relatively low, the maximum throughput is high enough to optimize it by reducing the retransmission attempts, $r$. Because of that, the optimum new packet transmission probability from each mobile node, $x_{opt}$, can be increased by decreasing the retransmission attempts, $r$.

Each mobile node acts as a receiver during its receiving mode. Therefore, it can measure the original packet arrival rate and the jamming packet arrival rate. Consequently, each mobile node can adjust the optimum transmission probability, $x_{opt}$. In this way, these results can be used for multiple receiving nodes half-duplex Slotted ALOHA with and without jamming signals and retransmission cut-off.

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