Side Lobe Suppression Based on Optimized Phase Rotation Sequence

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SUMMARY This paper proposes an optimized phase rotation sequence method for side lobe suppression by complementing and regulating the side lobe suppression sequence set. The sequence set is efficiently enhanced through the quadratically constrained least square model. The theoretical suppression performance of our method is discussed. Furthermore, our scheme is also suitable for cognitive radio, which is analyzed in the simulation part. The simulation results confirm the effectiveness of our schemes.

key words: OFDM, out-of-band radiation, side lobe suppression, phase rotation

1. Introduction

With the increasing needs of consumers and the fast evolution of modern technology, the conflict between urgent demand for precious spectrum resources and the limitation of the spectrum itself has become more and more manifest. These glorious merits, such as high spectrum efficiency, excellent anti multi-path ability, etc., have led the OFDM communication scheme to a prominent position in the field of modern wireless communication. Lots of wireless standards have chosen the OFDM scheme. However, the high side lobe of the OFDM spectrum has rather impeded the OFDM system’s application. What’s more, the promising cognitive radio which seeks to use the spectrum gap, is handicapped by this seemingly low but actually high side lobe interfering with the existing legacy system. Therefore, side lobe suppression is a hot research spot all along, especially for the frequency co-existing system [1].

There are several prevailing solutions to suppress the side lobe. The most commonly used ones are to apply some kind of window in the time domain [2]; others are trying to null a few subcarriers as a guard band [3]. The obvious drawbacks of these methods are that the signal is expanded in the frequency domain by windowing and that the efficiency is deteriorated by the extra guard band. By inserting Cancellation Carriers, [4], [5] give a relatively deep suppression, but the computation load can’t be neglected and overlooked. Similar to the water filling, [6] assigns different weights to each individual subcarrier, so as to get smaller side lobes. However, when water filling is really needed, this “fake” water filling can do nothing.

Recently, [7] has proposed a multiple-choice sequence suppressing method that works by rearranging the phase or the order of the data after serial to parallel transforming at the transmitter side, which easily decreases the side lobe without impacting BER performance. However, this suppressing approach needs to be improved due to some inherent flaws: first, there is no randomness guarantee to secure the suppressing depth, which induces the performance of the proffered method to behave unstably. Further, and conspicuously, the efficiency of [7] would deteriorate dramatically when the size of the candidate sequence set increases, because [7]’s strategy can only decide whether a candidate sequence is the one by trial.

Thus a new side lobe suppression method has been proposed, aiming to solve the above problems. We adopt a quadratically constrained least square model [8] to regulate the phase rotation sequence set and speed up the selection-making procedure. Our method could cooperate with other methods, such as windowing, and achieve a joint performance. In order to analyze our method effectively, we do not discuss joint performance here.

This paper is organized as follows: in Sect. 2, the system model is described. Section 3 comparatively analyzes the critical ingredients of the existing methods, and then, from a different light, the optimized phase rotation sequence is explained. Section 4 presents the simulation results. Finally, Sect. 5 concludes the paper.

2. System Model

In this section, we will introduce the system model briefly based on [7]. In Fig. 1(a), s is the data after data mapping; after serial to parallel transforming, the data are sent to the side lobe suppression block. This block performs the side lobe suppression function by operations such as amplitude re-assignment, subcarrier interleaving, or phase rotation. Index Q stands for the selected sequence number [7].

At the receiver side, as shown in Fig. 1(b), in order to get a proper output, [7] does the reverse by the information through signaling channel; thus, the side lobe inverter operation could correctly and exactly proceed.

In order to guarantee the BER performance, we only discuss phase related methods. The phase rotation methods have no relationship with the signal’s amplitude, thus they would not deteriorate BER due to SNR degradation. [9] tries to adopt a kind of encoding method to eliminate signaling channel, but if this seemingly applicable scheme is employed here, there would be BER deterioration.

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As [7]'s symbol constellation method is very similar to its phase method, we will not spare extra words on it.

3. Side Lobe Suppression Method

[7] offers three approaches to construct the sequence sets: the symbol constellation approach, the interleaving approach and the phase approach. These schemes could be interpreted as kinds of scrambling, while how to measure this scrambling according to a yardstick is not provided, except for the pool size, which is just a parameter that reflects the general statistics of desirable sequences. Thus, aiming to cope with the above problems, we propose a new phase rotation sequence in this section.

3.1 Optimized Phase Rotation Sequence

According to [7], the phase rotation sequences are chosen in an entirely random manner and they supposedly possess the same probability of side lobe suppression depth. Here we intend to adopt a quadratically constrained least square model [8] to regulate the phase rotation sequence set. The sequences used for side lobe suppressing could be calculated in advance and preset in the side lobe suppressing block, however with a smaller group size and nearly same side lobe suppressing performance.

Assume that $X$ is a data vector of $1 \times N$ after data mapping with zeros padding before IFFT block at the transmitter side. The transform kernel $A$ is constructed according to [11]. The OFDM signal from transmitter to receiver with oversampling is given by:

$$
x(n) = \sum_{k=0}^{N-1} X(k) \exp(j2\pi nk/N) \tag{1}
$$

$$
Y(l) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \exp(-2j\pi n/N \cdot \frac{l}{L}) \tag{2}
$$

where the index of $Y(l)$ is from 0 to $L \cdot N - 1$ and $L$ is the oversampling times. Matrix $A$ is the subset of this $\frac{1}{N} \cdot T_{FFT}$ in (3) and contains only the columns corresponding to the zero padding band (assume $\zeta$ is the zero padding index on either side, the responsible elements of $T_{FFT}$ are column 1 to $L \cdot \zeta$ and column $(N - \zeta) + 1$ to $(L \cdot N)$).

We summarize the side lobe suppressing problem in following Eq. (4).

$$
\begin{cases}
\|Ax - b\|_2 = \min \\
\|x\|_2 = \beta
\end{cases}
$$

where $x$ is a phase vector of one row and $N$ columns, which is reasonable for side lobe strength, while $b$ is the side lobe suppressing level. Here we confine its norm to a value $\beta$, intending to limit the variation of $x$ to phase only, but there are other cases included; nevertheless, the proper phases for a reasonable side lobe are definitely involved. $\|\cdot\|_2$ means Euclidean Norm.

These two equations could be solved by finding a proper $A$ of (5) instead of referring to Lagrange Multipliers according to [8].

$$
\left[ \begin{array}{c} A \\ \sqrt{\beta} \end{array} \right] x = \left[ \begin{array}{c} b \\ 0 \end{array} \right] \min \|x\|_2 \tag{5}
$$

The solution of (5) can be calculated through LSQR [12]. The optimized sequence set $S$ is constituted of several pre-computed $x$ with same or different suppressing level $b$. Sometimes, $x$ is not phase related only; then, $x$ is normalized by its modulus element-wisely.

3.2 Analysis of Optimized Phase Rotation Sequence

It has been determined that an individual sequence for phase rotation possesses the probability that the side lobe will exceed an threshold $\alpha$ before phase rotation is $p_\alpha$, then the Multiple-Choice Sequences reduces this probability to (6) in [7].

$$
\bar{p}_\alpha = p_\alpha \cdot p_\alpha \cdot \cdots \cdot p_\alpha = p_{\alpha}^{(M)} \tag{6}
$$

Well, as the optimization has excluded some phase candidates or their combinations with contact to a higher side lobe, this probability $p_\alpha$ could be reduced to $p_\alpha'$ which is the probability of a certain side lobe exceeding $\alpha$ after rotation by a single optimized candidate phase vector. Although not all of the unpromising candidates are eliminated, concerning the side lobe suppressing ability of our sequence group we are sure that:
\[ p_0 > p'_0, \text{ thus } \]
\[ p_0^{(M)} > p'_0 = p'_0 \cdot p'_0 \cdot \ldots \cdot p'_0 = p'^{(M)}_0 \quad \ldots \quad (7) \]

4. Simulation Results

The performance of the proposed method is compared with [7] in this part. Similar to [7], \( P \) indicates candidate sequence group size, \( N \) is the FFT size in Fig. 1 and \( K \) is the number of subcarriers carrying data. The data mapping type is selected as BPSK; of course, the other types of mapping are applicable.

In the following simulations, \( b \) is set to the value corresponding to suppressing depth \(-30 \text{ dB} \) with a random tiny perturbation with each element, for example \( \pm 5\% \). As sometimes phase vector of (5) is not phase rotated only, it is being normalized element-wisely. Thus the suppressing depth is not as deep as desired, but it is indeed deeper and performs better than that of [7], which is shown by the following figures.

4.1 Side Lobe Suppression Level

Figure 2 shows the relationship of suppression depth with different candidate sequence group size \( P \). FFT size \( N \) is selected as 64 with \( K=48 \) carrying data, while the others on either side are padded with zeros.

According to Fig. 2, the proposed optimized phase rotation sequence approach could achieve an identical suppressing depth with a smaller \( P \) compared with others approaches. This characteristic is helpful for computation load alleviation.

4.2 Side Lobe Suppression Fault Rate

[7] has discussed a threshold \( \alpha \) and the probability that indicates how frequently the side lobe power will exceed this threshold \( \alpha \) after phase rotation, which we call Side Lobe Suppressing Fault Rate here.

According to analysis results of Sect. 3.2, this \( p_0 \) is reduced to \( p'_0 \). Figure 3 illustrates the distribution of \( p^{(M)}_0 \) and \( p'^{(M)}_0 \) with \( P=4 \); other settings are the same as those in Sect. 4.1.

In Fig. 3, the “proposed one” is our optimized phase rotation sequence method, which behaves best compared to the current ones. About 5 dB is gained over the unsuppressed case.

4.3 Application in Cognitive Radio

An interesting application of side lobe suppression is spectrum notching in a Cognitive Radio system. In spectrum co-existence scenarios in a Cognitive Radio system, to guarantee the soundness of the licensed primary user (PU), the subordinate system needs to adjust its frequency to null the overlapped spectrum. Consider a case: a licensed system operates at a frequency band; in the meantime there is another subordinate system that operates in its frequency vicinity, which intends to take advantage of the frequency space. Therefore, the side lobe overlapping area is favored to clear most. This case is similar to spectrum nulling and more general in a sense.

In order to make a clear sense of the application of our proposed method, Fig. 4 shows the Cognitive Radio co-existence scenarios. We set the parameters similar to those of the previous parts. Figures 4(a) and (b) are composed by two “parallel” OFDM spectrum. Each of these spectrum has FFT size \( N=64; K=48 \).

In Fig. 4, the proposed one could get about 5 dB depth, while [7]’s behaves inadequately; therefore, our method could benefit the co-existence of different systems.

5. Conclusion

This paper proposes a new side suppression method. By mathematic modeling, the phase rotation sequence could be optimized, which enhances the efficiency (achieves comparable performance with a relatively smaller \( P \)) without...
Fig. 4 Cognitive Radio co-existence scenarios.

any impact on BER performance. Among the sequences discussed here, the optimized phase rotation sequence outperforms the others at the cost of requiring an extra pre-computation step.

Acknowledgments

This work was supported by the Korea Science and Engineering Foundation (KOSEF) through the National Research Lab. Program funded by the Ministry of Education, Science and Technology (MEST) (No. M10600000194-06J0000-19410). This research was also supported by the MKE (Ministry of Knowledge Economy), Korea, under the ITRC (Information Technology Research Center) support program supervised by the IITA (Institute of Information Technology Assessment) (IITA-2008-C1090-0801-0019).

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