A Simple Odd-Symmetric Filter for Digital Broadcasting

Michael Bank, Miriam Bank, Motti Haridim, and Jacob Gavan

Abstract—A new odd-symmetric filter for ISI suppression and VSB transmission is proposed. The frequency response of the proposed filter is similar to that of an ideal raised-cosine filter, but in contrast to raised-cosine the new filter is realizable, e.g., using simple active filters. Simulation results show that concerning ISI suppression the new filter is significantly more efficient than the classical Butterworth or Chebyshev filters. The proposed filter can be also used as a VSB filter for TV modulators and demodulators.

Index Terms—Intersymbol interference-ICI, odd-symmetric filter, raised-cosine filter, VSB filter.

I. INTRODUCTION

In most conventional digital communication systems, inter-symbol interference (ISI) is a principal cause of system performance being degraded. In particular, ISI constitutes a major factor in the reduction of channel capacity from theoretical limit. [1]–[4]. In modern wideband communication systems, the symbol time can be much smaller than the channel delay spread, and so ISI can be severe.

Theoretically, ISI can be completely eliminated using shaping filters so that the transmitted pulses pass through zero at all previous and future sampling moments. This condition is known as the Nyquist criterion for zero ISI [1]. The ideal Nyquist pulse has a rectangular frequency response band-limited to the minimum bandwidth required for the given symbol rate. The Nyquist criterion can also be satisfied by a raised-cosine filter (RCF). Although neither of these ideal filter responses is actually realizable, the RCF is usually used in actual practice, because it has no “brick-walls” and can be more closely approximated. Ref. [5] presented a novel filter that significantly reduces ISI, but also suffers from realization problems.

\[
H(f) = \left\{ \begin{array}{ll}
1 & \mbox{if } f_m - k f_m < f < f_m + k f_m \\
\frac{1}{2} \left[ 1 - \sin \left( \frac{\pi f_m}{2 f_m} (f - f_m) \right) \right] & \mbox{if } f_m - k f_m < f < f_m + k f_m \\
0 & \mbox{if } f < f_m - k f_m
\end{array} \right.
\]

(1)

The RCF has the following transfer function [2] where \( f_m \) is the symmetry point of \( H(f) \), and \( 0 \leq k \leq 1 \) is defined as the excess bandwidth factor or the roll-off factor. The special case of \( k = 0 \) corresponds to the minimum Nyquist bandwidth. Increasing \( k \), one can trade the cost of increased transmission bandwidth for the benefit of relaxed filtering and clock timing requirements [1]. The impulse response corresponding to the RCF of (1) is

\[
h(t) = A \frac{\sin(2\pi f_m t)}{2\pi f_m t} \cdot \cos(k2\pi f_m t) \cdot \frac{1 - \cos^2(\pi f_m t)}{\pi}
\]

(2)

The zeros of \( h(t) \) occur at \( t = n/2 f_s, n \neq 0 \), so that the ideal RCF is ISI-free.

Fig. 1 shows the frequency response of RCF given by (1), which is characterized by an odd-symmetrical roll-off about \( f_m \), i.e. in the frequency range \( f_m - k f_m < f < f_m + k f_m \), the function \( H(f) \) is odd-symmetric around \( f_m \).

Filters with an odd symmetry about a certain carrier frequency are also required in Vestigial Side Band (VSB) transmission, which is widely used in both broadcast and digital TV systems [6, 17].

The commonplace approach to implementation of analog RC filters is to use a conventional passive filter, e.g. Butterworth (BF), followed by a group delay equalization. This approximation approach suffers from lack of odd-symmetry, resulting in insufficient suppression of ISI.

In this section we present the basic properties of the proposed filter. We shall start with the case \( k = 1 \). In this case the transfer function of the proposed filter, \( H(s) \), is the inverse of a 4th order polynomial, satisfying the following conditions

\[
\begin{align*}
H(0) &= 1 \\
H'(0) &= 0 \\
H(s = j\omega_m) &= 0.5 \\
H(s) &\leq 0, \quad s \geq 0
\end{align*}
\]

(3)

The proposed filter is based on the following transfer function (written for normalized angular frequency)

\[
H(s) = \frac{1}{s^2 + \sqrt{2}s + 1} \cdot \frac{1}{s^2 + \sqrt{2}s + 1} \cdot \frac{1}{1 + \frac{1}{\omega^2}}
\]

(4)

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M. Bank, M. Haridim, and J. Gavan are with the Holon Academic Institute of Technology, Holon 58102, Israel (e-mail: michaelbank@bezeqint.net; mharidim@bait.ac.il; gavan@bait.ac.il).

M. Bank is with the Hebrew University of Jerusalem, Gerot-Ramat, Jerusalem 91904, Israel (e-mail: miriamb@math.huji.ac.il).

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which is highly asymmetric around the normalized angular frequency of 1 rad/sec, and satisfies conditions (3). This filter consists of the cascade connection of two identical 2nd order low pass filters, each with a damping factor of $\zeta = \sqrt{3}$ and a cut-off frequency $\omega_{oc} = 1$.

Next we consider the case $k < 1$, for which, the frequency response consists of a central constant part and a roll-off part as shown in Fig. 1.

By analogy with (4) we can write

$$H(s) = \frac{A}{(a^2 - k^2)^2 + B}$$

Strictly speaking, the roll-off portion of $H(s)$ starts from $f/f_m = (1 - k)$, and the constants $A$ and $B$ must be chosen so that the following conditions prevail

$$H(s = j\omega_m) = \frac{1}{2}$$

$$H(s = j\omega_m(1 - k)) = 1$$

Condition (6.1) provides the odd-symmetry feature of the filter that is necessary for IISI suppression. Therefore, it is essential to fulfill this condition as precisely as possible. Condition (6.2), on the other hand, can be approximated in practical design. For $k = 0.5$ we have

$$H(s) = \frac{0.936}{s^2 + 1.14s + 0.907} - \frac{0.936}{s^2 + 1.516s + 0.907}$$

$$= \frac{s^2 + 0.197 - 0.952s + 0.952^2}{0.936}$$

$$= \frac{s^2 + 1.592 - 0.952s + 0.952^2}{0.936}$$

The transfer function for the equivalent high pass filter appropriate for VSB TV with $k = 0.5$ will be

$$H(s) = \frac{0.936s^2}{s^2 + 1.197 - 0.952s + 0.952^2}$$

$$= \frac{0.936s^2}{s^2 + 1.592 - 0.952s + 0.952^2}$$

III. SIMULATION RESULTS AND COMPARISON WITH BUTTERWORTH AND CHEBYSHEV FILTERS

The circuit configuration used for simulations of the proposed filter is shown in Fig. 2. As Fig. 2 shows, the filter is implemented as an active R-C low pass filter [8]–[10].

The elements of a second order filter with transfer function:

$$H(s) = \frac{1}{s^2 + \omega_0^2 s + \omega_0^2}$$

can be calculated with the help of the following formulas [7]:

$$\omega_0^2 = \frac{1}{\omega_c^2}$$

$$\omega_0^2 = \frac{f_0}{\frac{1}{r_1 r_2 c_1 c_2}}$$

$$R_0 = \frac{r_1 r_2 c_1}{c_2}$$

$$R_c = \frac{f_c}{c_c}$$

$$C_c = \frac{c_c}{2\pi f_c R_0}$$

where $R_0$ is the resistance fiducial value, $i = 1, 2, \ldots, 6$ and $j = 1, 2$. In our simulations, the values of the filters elements are calculated for $f_0 = 240$ kHz and $R_0 = 10$ k\Omega in the manner described in [8].

Simulation results for the case of $k = 0.5$ are shown in Fig. 3(a) (frequency response) and Fig. 3(b) (time-domain response).

As Fig. 3(a) shows, the frequency response of the proposed filter extends to infinity and has an odd-symmetry around $f_m$. 

*Fig. 2.* Active filter circuit used for simulations.

*Fig. 3.* The characteristics of the proposed filter for $k = 0.5$: (a) the frequency response, (b) the time-domain response.
To evaluate the ISI level, we use the following formula for calculating the error caused by the residual effect of each pulse on adjacent sampling points [see Eq. (3)]

\[ K_{ISI} = \frac{\sqrt{V_1^2 + V_2^2 + V_3^2}}{V_0} \times 100 \]  

(11)

ISI can be minimized by adjusting the pulse duration \( \tau \). We have calculated values of \( K_{ISI} \) that minimize ISI for pulse durations \( \tau_1 = 1/f_s \) and \( \tau_2 = \tau_{opt} \) respectively.

To quantify the "amount" of odd-symmetry of various frequency responses, we define the parameter \( K_{os} \) that measures the difference between the values of \( H(s) \) at equal distances from \( \frac{5}{2} \pi \):

\[ K_{os} = \frac{\sum_{n=1}^{N} |V(1 - \Delta x) - V(1 + \Delta x)|^2}{N} \times 100\% \]  

(12)

where \( V(1 - \Delta x) \) and \( V(1 + \Delta x) \) represent the values of \( |H(s)| \) at the same distance from symmetry point.

We now compare the proposed filter with a fourth-order Butterworth filter [6]:

\[ H(s) = \frac{1}{s^2 + 0.7655s + 1} \cdot \frac{1}{s^2 + 1.848s + 1} \]  

(13)

and a fourth-order Chebyshev filter with \( \Delta H = 1 \text{ dB} \) [6]

\[ H(s) = \frac{\frac{1}{1.014s^2 + 0.283s + 1}}{\frac{1}{1.014s^2 + 0.283s + 1}} \cdot \frac{1}{3.579s^2 + 2.411s + 1} \]  

(14)

The values of \( \tau_2, K_{ISI} \), and \( K_{os} \) for the proposed filters, the BF, and the CF are shown in Table I.

As Table I shows, the ISI with the proposed filter both for \( k = 1 \) and for \( k = 0.5 \), is about 3.9% for \( \tau_1 = 1/f_s \). This is much smaller than it’s 13.4% ISI value with the BF or the 19.4% ISI value with the CF. The proposed filters exhibit minimal ISI for \( \tau_2 \) because of asymmetrical properties near \( f_m \).

IV. CONCLUSION

In conclusion, we have presented a new approximation for raised-cosine filters. The proposed filter exhibits a highly odd-symmetric transfer function as required for ISI suppression. It consists of two simple active filters arranged in cascade connection, which facilitates splitting the ISI-suppression filtering between the transmitter and each receiver. It is possible to use one of filter in transmitter and other in receiver. Simulation results show that the proposed filter can efficiently reduce the ISI. Comparison of the simulation results for the proposed filter with those of Butterworth and Chebyshev filters indicates that the proposed filter more closely approximates the ideal raised-cosine filter.

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