Performance Calculation for a High Speed Solid-Rotor Induction Motor

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Abstract-The paper deals with analytical method of design and prediction of performance characteristics for induction motors with solid steel rotor coated with copper layer. On the basis of the distribution of the 2D electromagnetic field, the equivalent impedance of the rotor have been derived. The edge effect and nonlinear magnetic permeability of solid steel have been included. The presented analytical method has been verified with laboratory test results. A 300 kW, 60 000-rpm, three-phase solid rotor induction motor for air compressor has been investigated. The accuracy of analytical approach is acceptable and can be recommended for rapid design of solid rotor induction motors.

I. INTRODUCTION

Although the principle of operation of solid rotor induction machines is similar to that of other induction machines, the analysis of physical effects in solid rotors on the basis of classical electrodynamics of nonlinear bodies is intricate. Problems arise both due to nonlinearity of solid ferromagnetic bodies and complex structures of certain types of these machines [1,4,5,8,9,12,14]. The electromagnetic field in the rotor is strictly three-dimensional (3D), even if the rotating magnetic field excited by the stator system can be assumed as two dimensional (2D) [1,5,12]. The performance of the machine depends on the intensity and distribution of vectors of the electromagnetic field, in particular, of the vectors of current density and magnetic flux density [5].

Recent interest in electric machines with alternating electromagnetic field in solid ferromagnetic parts is motivated by new applications of electrical machines as, for example, motors for high speed direct drive compressors, motors for pumps, motors for drills, high speed generators, electric starters for large turbogenerators, eddy current couplings and brakes, etc. Before the vector control era, there were attempts to use solid rotors covered with thin copper layer for very small-diameter rotors of two-phase servo motors, in which the cage winding and back iron (yoke) were very difficult to accommodate. Research is also stimulated by trends in improvements of other types of electrical machines, e.g., machines with rotors made of soft magnetic powders (magnetodielectrics and dielectromagnetics), shields for end windings of large turbogenerators, shields for superconducting machines, retaining sleeves for high speed permanent magnet (PM) machines and losses in PMs.

In comparison with cage-rotor induction motors of the same dimensions, solid rotor induction motors have lower output power, lower power factor, lower efficiency, higher no-load slip and higher mechanical time constant. Worse performance characteristics are due to high rotor impedance, eddy current in solid ferromagnetic rotor body, higher reluctance of solid steel than laminated steel and greater influence of higher harmonic of the magnetic field than in other types of induction machines [1,4,5,8,9,11-14]. There are wide possibilities of reduction of the rotor impedance that improves the performance characteristics through [5]:

- selecting the rotor solid material with small relative magnetic permeability – to – electric conductivity ratio $\mu/\sigma$ and adequate mechanical integrity;
- using a layered (sandwiched) rotor with both high magnetic permeability and high conductivity materials [11];
- using a solid rotor with additional cage winding.

Sensible application of the above recommendations that leads to optimization of the design is only possible on the basis of the analysis of the electromagnetic field distribution in the machine.

![Layered halfspace for analysis of 2D electromagnetic field.](image)

Fig. 1. Layered halfspace for analysis of 2D electromagnetic field.

II. ANALYSIS

A. Distribution of electromagnetic field

If the distribution of the magnetic flux density normal component on the surface of a double-layer structure Cu-Fe (Fig. 1) is known, i.e.,

$$b(x, t) = \sum_{\nu=1}^{\infty} \left[ B_\nu^+ e^{j(\alpha x + \beta_\nu z)} + B_\nu^- e^{j(\alpha x - \beta_\nu z)} \right]$$  (1)

where $B_\nu^+$, $B_\nu^−$ are magnitudes of the $\nu$th harmonics of the forward and backward-rotating magnetic flux density waves, the 2D electromagnetic field equations have the following form [5,6]:

- for $0 \leq z \leq d_2$

$$H^{(2)}_{\nu z} = \frac{1}{\mu_2} B_\nu e^{j\beta_\nu z} \left[ \frac{\mu_1}{\mu_2} \cos[k_\nu(z-d_2)] - \frac{\mu_2}{\mu_1} \sin[k_\nu(z-d_2)] \right]$$  (2)

$$H^{(1)}_{\nu z} = \frac{1}{\mu_1 M_{\nu z}} B_\nu e^{j\beta_\nu z} \left[ \frac{K_{\nu 1}}{M_{\nu z}} \cos[k_\nu(z-d_2)] - \frac{K_{\nu z}}{M_{\nu z}} \sin[k_\nu(z-d_2)] \right]$$  (3)
The attenuation coefficient for a conductive medium with its tangential magnetic component at \( z = d_2 \) is calculated as a parallel connection of solid rotor impedance.

The magnetic permeability of free space is \( \mu_0 = 4\pi \times 10^{-7} \text{H/m} \). For paramagnetic or diamagnetic bodies \( a_K = a_\sigma = 1 \) and for ferromagnetic bodies \( a_K = 1.45, a_\sigma \approx 0.85 \). \( K_{d} \approx \frac{k \omega \mu_0 \mu_r \sigma}{2} \). The following relationship exists between coefficients \( a_v, \kappa_v, \) and \( \beta_v \):

\[
\kappa_v = (\alpha_v + \beta_v) = (a_{\sigma} + j a_{K_v}) k_v \\
\alpha_v = \frac{1}{\sqrt{2}} \left[ 4a_\sigma^2 a_v + \left( a_v - a_\sigma + \frac{\mu_r \kappa_v}{\kappa_1} \right) \right]^{1/2} \\
\beta_v = \frac{\pi}{\tau} \\
\]

### Surface Impedance

The surface impedance of the infinitely thick inner layer 1 is equal to the ratio of the tangential electrical component to the tangential magnetic component at \( z = d_2 \):

\[
z_{vi1} = \frac{E^{(1)}_{vi}}{H^{(1)}_{vi}} = -\frac{j \omega \mu_0 \mu_r}{\kappa_{d1}} + \frac{j \omega \mu_0 \mu_0 \sigma_1}{\kappa_{d1}} = \frac{\sigma_{d1}}{\kappa_{d1}} - \frac{\sigma_{d1}}{\kappa_{d1}} (15)
\]

The surface impedance of layers 1 and 2 is equal to the ratio of the electrical component to tangential magnetic component at \( z = 0 \), i.e.,

\[
z_{v2} = \frac{E^{(2)}_{v2}}{H^{(2)}_{v2}} = +\frac{j \omega \mu_0 \mu_r}{\kappa_{d2}} + \frac{j \omega \mu_0 \mu_0 \sigma_2}{\kappa_{d2}} = \frac{\sigma_{d2}}{\kappa_{d2}} - \frac{\sigma_{d2}}{\kappa_{d2}} (16)
\]

After multiplying the numerator and denominator by \( \frac{1}{j \omega \mu_0 \mu_0 \sigma} \), the impedance (16) takes the form:

\[
z_{v2} = \frac{E^{(2)}_{v2}}{H^{(2)}_{v2}} = +\frac{j \omega \mu_0 \mu_r}{\kappa_{d2}} + \frac{j \omega \mu_0 \mu_0 \sigma_2}{\kappa_{d2}} + \frac{1}{\kappa_{d2} + \tan \left( \kappa_{d2} \right)} (17)
\]

where \( z_{vi} \) is the impedance of the infinitely thick inner layer 1 (half-space). Thus, the impedance of the outer layer 2 is

\[
z_{v2} = \frac{E^{(2)}_{v2}}{H^{(2)}_{v2}} = +\frac{j \omega \mu_0 \mu_r}{\kappa_{d2}} + \frac{1}{\kappa_{d2} + \tan \left( \kappa_{d2} \right)} (18)
\]

If the thickness \( d_1 \) of the inner layer 1 smaller than the equivalent depth of penetration, its impedance is

\[
z_{v1} = \frac{j \omega \mu_0 \mu_r}{1} (19)
\]

If \( d_1 \to \infty \), the hyperbolic function in the denominator \( \tan \left( \kappa_{d1} \right) \to 1 \) and \( z_{v1} = j \omega \mu_0 / \kappa_{d1} \).

### Rotor Impedance

If the outer layer 1 is a copper layer and the inner layer 2 is a solid steel layer, then for the fundamental harmonic \( V = 1 \), \( d_2 = d_{Cu} \), \( d_1 = d_{Fe} + d_{Cu} \), \( \mu_1 = \mu_0 \mu_0 \sigma_1 \) [6], \( \mu_2 = \mu_0 \mu_0 \sigma_2 \), \( \kappa_1 = \kappa_{Cu} \), \( \kappa_2 = \kappa_{Fe} \), \( \omega_0 = \omega \). The impedance is

- for solid steel

\[
Z_{Fe} = \frac{j \omega \mu_0 \mu_r}{1} \frac{1}{\kappa_{Fe} + \tan \left( \kappa_{Fe} d_{Fe} \right)} \tau (20)
\]

- for copper layer

\[
Z_{Cu} = \frac{j \omega \mu_0 \mu_r}{1} \frac{1}{\kappa_{Cu} + \tan \left( \kappa_{Cu} d_{Cu} \right)} \tau (21)
\]

where \( L \) is the axial length of the solid ferromagnetic core and \( \tau \) is the pole pitch. The resultant impedance of a double-layer solid rotor is calculated as a parallel connection of \( Z_{Fe} \) and \( Z_{Cu} \), i.e.,

\[
Z_{2} = \frac{Z_{Fe} + Z_{Cu}}{Z_{Fe} + Z_{Cu}} = \frac{j \omega \mu_0 \mu_r}{1} \frac{1}{\kappa_{Fe} + \tan \left( \kappa_{Fe} d_{Fe} \right)} \frac{1}{\kappa_{Cu} + \tan \left( \kappa_{Cu} d_{Cu} \right)} L (22)
\]

After multiplying the numerator and denominator of the impedance \( Z_2 \) according to eqn (22) by

\[
\frac{\kappa_{Fe} + \tan \left( \kappa_{Fe} d_{Fe} \right)}{j \omega \mu_0 \mu_r} \kappa_{Cu} + \tan \left( \kappa_{Cu} d_{Cu} \right)
\]

the rotor impedance becomes
If $d_{fe} \to \infty$, the hyperbolic function in the denominator \( \tanh(k_{Fe}d_{Fe}) = 1 \) and

$$Z_2 = \frac{j \omega L_{cu}}{\kappa_{Fe}} \frac{1}{\frac{L_{cu}}{\kappa_{Fe}} + \tanh(k_{Fe}d_{Fe})} \tag{24}$$

The rotor impedance can also be found straightforward from eqn (17), i.e.,

$$Z_2 = \frac{j \omega L_{cu}}{\kappa_{Fe}} \frac{1}{\frac{L_{cu}}{\kappa_{Fe}} + \tanh(k_{Fe}d_{Fe})} \tag{25}$$

Since

$$\frac{1}{\kappa_{Fe}} \frac{j \omega L_{cu}}{\kappa_{Fe}} \tanh(k_{Fe}d_{Fe}) << 1 \tag{26}$$
eqns (24) and (25) are equivalent. Introducing the coefficient \( k_{ov} \) of transformation of the rotor impedance to the stator winding

$$k_{ov} = \frac{m_z(N_{k_{ov}})^2}{m_z(N_{k_{ov}})^2} = \frac{2m_z(N_{k_{ov}})^2}{p}$$

the rotor impedance referred to the stator system is

$$Z_2 = \frac{j \omega L_{cu}}{\kappa_{Fe}} \frac{1}{\frac{L_{cu}}{\kappa_{Fe}} + \tanh(k_{Fe}d_{Fe})} \frac{1}{\kappa_{Fe}} \frac{L_{cu}}{\kappa_{Fe}} \frac{k_{ov}}{\tau} \tag{28}$$

For a solid rotor induction machine the number of rotor phases \( m_z = 2p \), number of rotor turns per phase \( N_z = 0.5 \) and rotor winding factor for fundamental harmonic \( k_{ov} = 1 \).

To include the edge effect [5,10] the conductivity of solid steel \( \sigma_{Fe} \) in the parameter \( \kappa_{Fe} \) should be multiplied by the reciprocal of the edge effect factor \( k_{e} \), i.e.,

$$\sigma_{Fe} = \frac{1}{k_{e}} \sigma_{Fe} \tag{29}$$

where

$$k_{e} = 1 + \frac{2}{\frac{\tau}{L_{Fe}}+\frac{\tau}{L_{Cu}}}$$

and the conductivity of copper layer \( \sigma_{Cu} \) in \( \kappa_{Cu} \) should be multiplied by Russell-Northworhy’s coefficient \( k_{RN} \) [10], i.e.,

$$\sigma_{Cu} = k_{RN} \sigma_{Cu} \tag{30}$$

where [10]

$$k_{RN} = 1 - \frac{\tanh(0.5\beta_{Fe}L)}{0.5\beta_{Fe}L[1 + k_{ov}\tanh(0.5\beta_{Fe}L\tanh(\beta_{Fe}w_{Cu}))]} \tag{31}$$

with the correction factor for \( t_{ov} \) [6]

$$k_{ov} = 1 + \frac{1.2(t_{ov} - d_{Cu})}{d_{Cu}} \tag{32}$$

Dimensions \( t_{ov} \) and \( w_{Cu} \) of the solid rotor coated with copper layer are given in Fig. 2.

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**Fig. 2.** Solid steel rotor coated with copper layer. To minimize the rotor impedance, the thickness of the copper layer behind the stator stack is thicker than under the stack (US54732111 [2]).

The equivalent circuit of the solid rotor induction motor is discussed in Appendices I and II. In Appendix I an equivalent circuit corresponding to a classical cage-rotor induction motor has been derived, while in Appendix II the equivalent circuit for solid rotor is compared with that for deep-bar rotor.

### III. Computations

On the basis of the analysis presented in Section II, a computer program for the design and calculation of performance characteristics has been created. A 300-kW, 60,000-rpm induction motor with solid steel rotor coated with copper layer (Fig. 2) has been investigated. Specifications are listed in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>kW</td>
<td>300</td>
</tr>
<tr>
<td>Voltage (line-to-line)</td>
<td>V</td>
<td>400</td>
</tr>
<tr>
<td>Synchronous speed</td>
<td>rpm</td>
<td>60,000</td>
</tr>
<tr>
<td>Number of poles</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Class of insulation</td>
<td></td>
<td>H (180ºC)</td>
</tr>
<tr>
<td>Stator outer diameter</td>
<td>mm</td>
<td>250</td>
</tr>
<tr>
<td>Stator inner diameter</td>
<td>mm</td>
<td>115</td>
</tr>
<tr>
<td>Stator stack length</td>
<td>mm</td>
<td>173</td>
</tr>
<tr>
<td>Air gap (mechanical clearance)</td>
<td>mm</td>
<td>3</td>
</tr>
<tr>
<td>Total length of rotor</td>
<td>mm</td>
<td>315.5</td>
</tr>
</tbody>
</table>

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**Fig. 3.** Solid steel rotor coated with copper layer for a 300-kW, 60,000-rpm induction motor. Courtesy of Sundyne Corporation, Espoo, Finland.

This motor has been developed by Sundyne Corporation, Espoo, Finland. Sundyne manufactures high-speed solid-rotor induction motors for air compressors. The solid-rotor runs up to 400 m/s, i.e., at twice the tip speed of a cage-rotor. The speed limit for this technology is 550 m/s [3].
shows a solid steel rotor integrated with two impellers. The copper layer is thicker in the rotor end zones than under the stator stack. The air compressor driven by the 300-kW solid-rotor induction motor is shown in Fig. 4.

Fig. 4. Air compressor driven by a 300 kW, 60,000-rpm induction motor with solid rotor coated with Cu layer. The rotor is shown in Fig. 3. Courtesy of Sundyne Corporation, Espoo, Finland.

Fig 5 shows the calculated magnetic flux density distribution in the air gap. Fig. 6 shows the resistances and reactances of the copper layer and solid steel body referred to the stator system. The impedance given by eqn (28) has been separated into impedance of the copper layer and solid steel core, similar to eqns (20) and (21).

![Fig. 5](image)

Fig. 5 Distribution of the normal component of the air gap magnetic flux density $B_g$ along the pole pitch. $B_{av}$ = average flux density.

The electromagnetic power $P_{elm}$ and electromagnetic torque $T_{elm}$ have been calculated in the same way as those for a cage rotor induction motor, i.e., using the T-type equivalent circuit, in which the rotor nonlinear impedance (dependent on the magnetic permeability of the rotor core) referred to the stator winding is expressed by eqn (28). The shaft power has been calculated by subtracting from the electromagnetic power $P_{elm}$ the rotor electrical losses, windage losses and bearing friction losses.

Figs 7, 8 and 9 shows the steady-state performance characteristics, i.e., input and output power versus speed (Fig. 7), torques versus speed (Fig. 8), and efficiency and power factor versus speed (Fig. 9).

![Fig. 7](image)

Fig. 7. Output power $P_{out}$ and input power $P_{in}$ versus speed at 400 V and 1000 Hz.

![Fig. 8](image)

Fig. 8. Shaft torque $T_{sh}$ and electromagnetic torque $T_{elm}$ versus speed at 400 V and 1000 Hz.

![Fig. 9](image)

Fig. 9. Efficiency $\eta$ and power factor $pf = \cos \phi$ versus speed at 400 V and 1000 Hz.

Fig. 6. Resistances and reactances of the Cu layer and solid rotor body referred to the stator winding versus slip at 1000 Hz input frequency.
IV. LABORATORY TESTS

A 300-kW, 60,000-rpm high-speed solid-rotor induction motor built by Sundyne Corporation was tested at Lappeenranta University of Technology, Finland [7]. Specifications including main dimensions of the motor are shown in Table I. The 300-kW induction motor was loaded with a two-stage turbocompressor connected directly to the motor shaft. The high-speed motor compressor under testing is shown in Fig 10.

The test loop arrangement is presented in Fig 11. The air flow is first sucked by the stage 1 and then cooled down before it goes to the stage 2. After discharging from the stage 2, the flow is cooled back to the ambient temperature. The rotor of the motor is integrated with the compressor impellers as shown in Fig. 3.

Fig 12 shows the power flow chart for the tested motor. In a high speed machine the sum of windage and friction losses is of the same order as electrical losses. The motor input power is $P_{in}$, the motor mechanical power is $P_{mech}$ and the output power to the compressor stages is $P_{out}$. The total electrical motor losses (stator winding, stator core, rotor and stray losses) are $P_{el}$ and the windage and friction losses are $P_{wf}$.

\[
P_{out} = \frac{P_1}{\eta_1} + \frac{P_2}{\eta_2}
\]

in which $P_1$ and $P_2$ are the gas compressor powers for stage 1 and stage 2, respectively and $\eta_1$ and $\eta_2$ are the corresponding isentropic efficiencies. All these values can be calculated by measuring the inlet and outlet flow parameters (pressure, temperature, humidity) as well as the mass flow rate for both stages separately.

In order to obtain the motor mechanical power, the windage and friction losses $P_{wf}$ must be added to the calculated output power. The windage and friction losses of the machine have been measured by deceleration tests without the compressor impellers. Based on the deceleration time, the windage and friction losses can be calculated from equation

\[
P_{wf} = J \frac{d\Omega}{dt}
\]

where $J$ is the rotor moment of inertia, $\Omega = 2\pi n$ is the shaft angular speed and $t$ is the time.

The tested motor has been cooled with an air flow forced through the stator winding and the air gap. The required power of the blower is 3.0 kW, which is relatively small number for a 300 kW motor. A low blower power has been achieved by a careful design of the air cooling paths. In addition, water circulation in the motor housing reduces the needed air cooling. As all the heat produced by the motor losses have been taken out by the cooling air and water, it has been possible to measure the total losses of the motor by a calorimetric method. The obtained results have been in good agreement with the losses calculated on the basis of the electrical input power and output power.

V. COMPARISON OF CALCULATIONS WITH TEST RESULTS

Analytical calculations performed according to the presented theory in Section II have been compared with
laboratory measurements performed on the 300-kW, 60,000-rpm solid rotor induction motor (Table I).

The input power, output power, efficiency and power factor curves obtained from calculations and laboratory tests are plotted versus speed in Figs 13 and 14.

<table>
<thead>
<tr>
<th>Rotor speed, rpm</th>
<th>Synchronous speed, rpm</th>
<th>Frequency, Hz</th>
<th>Slip</th>
<th>Stator winding temperature, °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>56064</td>
<td>56520</td>
<td>942</td>
<td>0.00807</td>
<td>119</td>
</tr>
<tr>
<td>57126</td>
<td>57660</td>
<td>961</td>
<td>0.00931</td>
<td>129</td>
</tr>
<tr>
<td>58062</td>
<td>58680</td>
<td>978</td>
<td>0.011</td>
<td>140</td>
</tr>
<tr>
<td>58836</td>
<td>59550</td>
<td>992.5</td>
<td>0.012</td>
<td>152</td>
</tr>
<tr>
<td>59300</td>
<td>60000</td>
<td>1000</td>
<td>0.012</td>
<td>152</td>
</tr>
</tbody>
</table>

Although, the performance calculation for a solid rotor induction motor is similar to that of a cage induction motor, the electromagnetic effects are more complex. The rotor copper layer and solid steel body conduct both the electric current and magnetic flux, the impedance of the rotor solid body depends on the nonlinear magnetic permeability and the magnetizing current is high due to presence of the nonferromagnetic copper layer [5]. The rotor branch of the equivalent circuit contain the rotor nonlinear resistance and reactance, both dependent on the magnetic permeability of the solid steel and slip.

The authors have decided to use analytical approach based on the 2D analysis of the electromagnetic field to the design and performance calculation of solid rotor induction motors. The created computer program allows for rapid design of such induction machines.

The obtained accuracy is acceptable. Figs 13 and 14 shows that the analytically calculated and measured curves for a 300-kW, 60,000-rpm solid rotor induction motor are in good agreement.

APPENDIX I. EQUIVALENT CIRCUIT

For simplicity, only solid rotor without external copper layer, fundamental harmonic \( v = 1 \) and balanced three-phase system will be considered. For \( f_{sp} \rightarrow \infty \), the hyperbolic function \( \tanh(\kappa_f d_{fr}) = 1 \) and eqn (20) takes the form

\[
Z_{rs} = \frac{j\omega \mu_r L_s}{\kappa_f} = \frac{j\omega \mu_r L_s}{\kappa_f \sigma_r} = = (a_k + jA_k) \frac{k_{fr}}{\sigma_r} L (34)
\]

where \( \alpha_r \) is according to eqns (10) and (13) and the attenuation coefficient \( k_{fr} \) for solid steel is according to eqn (12). Thus, including the edge effect [5,10] the impedance of the solid steel rotor is

\[
Z_{rs} = (a_k + jA_k) \frac{k_{fr}}{\sigma_r} L (35)
\]

where \( \mu_r \) is the magnetic relative magnetic permeability at the surface of the rotor. Typical values of damping coefficients in solid steel rotors with \( \tau = 5 \text{mm}, \mu_r = 100 \), \( \sigma_r = 5 \times 10^6 \text{ S/m}, a_k = 1.45 \) and \( A_k = 0.85 \) are: for \( f = 50 \text{ Hz} \), \( k_{fr} = 314.16 \text{ l/m} \), for \( f = 60 \text{ Hz} \), \( k_{fr} = 344.14 \text{ l/m} \), for \( f = 400 \text{ Hz} \), \( k_{fr} = 888.58 \text{ l/m} \) and for \( f = 1000 \text{ Hz} \), \( k_{fr} = 1404.96 \text{ l/m} \).

The rotor impedance referred to the stator system is obtained by multiplying the impedance coefficient of transformation \( k_{ir} \), i.e.,

\[
Z_{2s} = (a_k + jA_k) k_{ir} L \frac{\pi f \mu_r \mu_s}{\sigma_r} = R_{2s} + jX_{2s} (36)
\]

\[
R_{2s} = a_k k_{ir} L \frac{\pi f \mu_r \mu_s}{\sigma_r} = a_k Z_s \sqrt{\mu_r} \sqrt{s} (37)
\]

\[
X_{2s} = a_k k_{ir} L \frac{\pi f \mu_r \mu_s}{\sigma_r} = a_k Z_s \sqrt{\mu_r} \sqrt{s} (38)
\]
where
\[ Z_c = k_n k_r \frac{L}{\tau} \frac{\pi f \mu_s}{\sigma_f} \] (39)

is the constant value of the rotor impedance independent of the slip \( s \) and surface value of the relative magnetic permeability \( \mu_r \). The coefficient of transformation \( k_n \) of the rotor impedance to the stator system is expressed by eqn (27).

\[ \sigma \approx + E_220 (41) \]

where
\[ \mu \approx \frac{\pi f \mu_s}{\sigma_f} \] (42)

are the resistance and reactance in the rotor branch of the equivalent circuit referred to the stator system. Fig. 15 shows the development of the equivalent circuit (transformer analogy) of a solid rotor induction motor.

**APPENDIX I. ANALOGY TO DEEP-BAR ROTOR**

In a deep bar rotor induction machine with the rotor bar height \( h_{2b} \) the rotor branch contains the following resistance and reactance
\[ R_1' = \frac{R_{2b}'}{s} = a_n Z_c \sqrt{\frac{\mu_r}{s}} \] (43)
\[ X_1' = \frac{X_{2b}'}{s} = a_n Z_c \sqrt{\frac{\mu_r}{s}} \] (44)

Assuming \( (\mu_r)_{1n} = \mu_r \), the reduced height of the current conductive layer is
\[ \xi = \frac{h_{2b}}{\sqrt{\pi}} \frac{1}{\mu_n} \sqrt{\frac{s f \sigma}{\mu_r}} \] (47)

Similar to a deep bar rotor, the reduced height of the layer on the rotor surface that conducts the current is
\[ \xi = \frac{h_{2b}}{\sqrt{\pi}} \frac{1}{\mu_n} \sqrt{\frac{s f \sigma}{\mu_r}} \] (48)

where the equivalent depth of penetration of the electromagnetic field into rotor body at \( n = 0 \) (s = 1) is
\[ \delta = \frac{1}{\sqrt{\pi \mu_n (\mu_r)_{1n}}} \] (49)

Assuming \( (\mu_r)_{1n} = \mu_r \), the reduced height of the current conductive layer is
\[ \xi = \frac{h_{2b}}{\sqrt{\pi}} \frac{1}{\mu_n} \sqrt{\frac{s f \sigma}{\mu_r}} \] (50)

Thus,
\[ R_2' = \xi a_{n} k_{n} k_{r} \frac{L}{\delta_{f e} \tau \sigma_{Fe}} \] (51)
\[ X_2' = \frac{1}{\xi s a_{n} k_{n} k_{r} \delta_{f e} \tau \sigma_{Fe}} \] (52)

Denoting
\[ Z_c = k_n k_r \frac{L}{\tau} \frac{\pi f \mu_s}{\sigma_{Fe}} \] (53)

The resistance in the rotor branch is inversely proportional to the slip
\[ R_1' = \frac{R_{2b}'}{s} = a_n Z_c \sqrt{\frac{\mu_r}{s}} \] (54)

and reactance in the rotor branch has the same form as that in standard cage induction motors, i.e.,
\[ X_1' = \frac{X_{2b}'}{s} = a_n Z_c \sqrt{\frac{\mu_r}{s}} \] (55)
So that the rotor current referred to the stator system is equal to the EMF $E_I$ divided by the rotor branch impedance, i.e.,

$$I_z' = \frac{E_I}{\left(\xi a_R Z_a \sqrt{\mu_n} \frac{1}{s}\right)^2 + \left(\frac{1}{\xi} a_X Z_a \sqrt{\mu_n}\right)^2}$$  \(56\)

The alternative equivalent circuit of a solid rotor induction motor, similar to that of a deep bar induction motor, is shown in Fig. 16.

If $a_R \approx 1.45$ and $a_R \approx 0.85$ (typical values) the coefficient of the solid ferromagnetic rotor for resistance $\xi a_R \approx 1.45 \xi$ and for reactance $a_X / \xi \approx 0.85 / \xi$. For a deep bar rotor the same coefficients are $\xi$ and $1.5 / \xi$. Note, that the equivalent depth of penetration $\delta_{Fe}$ of electromagnetic field into solid steel rotor body is small in comparison with the height of a nonferromagnetic copper or aluminum bar, the electric conductivity of steel is about 10 times lower than the conductivity of copper and about 7 times lower than the conductivity of aluminum. In addition, the surface magnetic permeability of solid rotor body $\mu_{ns} >> 1$ versus

![Fig. 16. Equivalent circuit of a solid rotor induction motor with rotor branch similar to a deep bar rotor.](image)

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**REFERENCES**


