Parameter Extraction of HEMT Models from Multibias S-Parameters

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Abstract—The aim of the paper is to show how to identify the parameters of various nonlinear circuit models of a high electron mobility transistor from the multibias s-parameter dataset measured on a pHEMT device. A three-step identification procedure is described that uses tested optimization methods. Two original model modifications are suggested based on empirical relations for the transconductance dependence on gate-source voltage. Various models including the modified ones are compared in terms of the root-mean-square errors of multibias s-parameters.

I. INTRODUCTION

It is generally known that the nonlinear models of MESFETs implemented in contemporary CAD tools can well serve to perform both time- and frequency-domain simulations of microwave HEMTs. Scattering (s-) parameters of the transistors measured at different bias voltages provide valuable information needed to extract the parameters of such nonlinear models. Model accuracy is dependent on a formulation of nonlinear equations characterizing internal physical processes. However, a model identification procedure which extracts the values of parameters from the measured data is important too. In this paper, a suitable extraction procedure is described and tested using the pHEMT measured dataset of I-V characteristics and related multibias s-parameters. The parameters of several popular nonlinear models are identified by the proposed method and the results are compared.

The nonlinear dependence of the drain (channel) current on the gate-source voltage affects the overall behavior of HEMT. The first derivative (transconductance) and higher-order derivatives influence a nonlinear distortion that is examined by designers when the large-signal characteristics of the HEMT circuits are studied. Two new formulae of the drain current vs gate voltage dependence are proposed in the paper which provide smooth higher derivatives. The higher-order derivatives are also compared with the measured data to assess the quality of the nonlinear formulation of various models.

II. EXTRACTION METHODS

Optimization methods are often used for the extraction of model parameters of transistors and other devices from the measured characteristics. They minimize an objective function that describes the difference between measured and simulated characteristics. Different optimization methods may exhibit different behavior in terms of efficiency and robustness. In this section, four well-known methods are tested and compared. The parameter identification of the linear model in Fig. 1 from s-parameters is an important step of the whole nonlinear model identification. Twelve circuit parameters are determined by means of Levenberg-Marquardt (LM), genetic (G), simulated annealing (SA), and Nelder-Mead (NM) simplex algorithms. A starting 12-dimensional vector of parameter values is chosen randomly within an interval in a parameter space that is bounded by the lower and upper boundary vectors $x \_L$ and $x \_H$. Parameter space is widened by a variable factor: $x \_H = x \_\ast \times \text{factor}$, $x \_L = x \_\ast / \text{factor}$, where $x \_\ast$ is a near-optimum vector. Thirty optimizations for every value of factor are executed to obtain the statistics of the resulted root-mean-square errors $\text{rms}$ of the simulated s-parameters. The minimal $\text{rms}$ values are plotted in Fig. 2.

For this particular circuit, SA is the most robust method reaching the minimum regularly even with higher factor. On the other hand, LM is very efficient method when the starting vector is sufficiently near the optimum.

III. NONLINEAR HEMT MODELS

The following nonlinear equivalent circuit models of HEMT are compared: Curtice, Statz, TOM, TOM2 (all described in [1]), Materka [2], Ahmed [3], [4], and Dobeš [5] models. The models differ primarily in the definition of the nonlinear elements. Low- (DC) and high- (RF) frequency model parameters have to be differentiated in order to take the frequency dispersion effects into account. A circuit diagram of the model is shown in Fig. 3. The drain-current nonlinear dependence on the gate- and drain-source voltages forms the basis of HEMT
IV. MODEL IDENTIFICATION

In order to demonstrate the behavior of various nonlinear models and to present the performance of the identification procedure, measured multibias $s$-parameter data of the transistor GaAs pHEMT with a gate length 0.25 µm is used [6], [7]. The dataset consists of frequency characteristics of two-port $s$-parameters (common source) measured in a great number of bias points.

A. Identification Procedure

The identification procedure which uses the optimization methods described above proceeds as follows:

1) The least-squares LM method is applied to minimize the difference of measured and simulated input and output V-I characteristics. The DC parameters of the nonlinear model are obtained in this way.

2) The SA and then LM methods are applied to minimize the difference of measured and simulated $s$-parameters at one operating point. The parameters of the linearized HEMT model are obtained.

3) The LM method with starting vector set given by the previous results is performed to identify all the parameters of the dispersive nonlinear model. An objective function evaluates the differences between measured and simulated multibias $s$-parameters as well as the differences between the static characteristics.

B. Static Characteristics

The static characteristics of the identified model TANH are shown in Fig. 4. A detail for lesser values of the channel current is shown in Fig. 5. The absolute error achieved with the TANH model is depicted in Fig. 6. One can see that the largest differences occur for greater channel currents and lesser drain voltages. Table I summarizes the root-mean-square errors of the channel current and its derivatives achieved with various models. The error is defined by the formula

$$\text{rms}_{\text{abs}} = \left( \frac{1}{n} \sum_{i=1}^{n} (x_i^{\text{meas}} - x_i^{\text{simul}})^2 \right)^{\frac{1}{2}},$$

where $n$ is number of bias points and $x$ stands for the channel current and its derivatives, respectively. The Ahmed and XEXP models attained the best results.

C. Multibias S-Parameters

Table II summarizes the results of the identification procedure applied to various models. The Curtice and Statz models were excluded because of the significant error of their static characteristics.
**TABLE I**

**ROOT-MEAN-SQUARE ERROR OF THE DRAIN CURRENT AND ITS DERIVATIVES**

<table>
<thead>
<tr>
<th>Model</th>
<th>$i_d$</th>
<th>$\frac{\partial i_d}{\partial V_{ds}}$</th>
<th>$\frac{\partial^2 i_d}{\partial V_{ds}^2}$</th>
<th>$\frac{\partial i_d}{\partial V_{gs}}$</th>
<th>$\frac{\partial^2 i_d}{\partial V_{gs}^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahmed</td>
<td>0.0017</td>
<td>0.0042</td>
<td>0.0241</td>
<td>0.1615</td>
<td>0.0104</td>
</tr>
<tr>
<td>Dobes</td>
<td>0.0031</td>
<td>0.0046</td>
<td>0.0243</td>
<td>0.1748</td>
<td>0.0206</td>
</tr>
<tr>
<td>Curtice</td>
<td>0.0110</td>
<td>0.0114</td>
<td>0.0302</td>
<td>0.1812</td>
<td>0.0558</td>
</tr>
<tr>
<td>Statz</td>
<td>0.0077</td>
<td>0.0105</td>
<td>0.0371</td>
<td>0.1892</td>
<td>0.0371</td>
</tr>
<tr>
<td>TOM</td>
<td>0.0035</td>
<td>0.0066</td>
<td>0.0357</td>
<td>0.2821</td>
<td>0.0200</td>
</tr>
<tr>
<td>TOM2</td>
<td>0.0018</td>
<td>0.0046</td>
<td>0.0267</td>
<td>0.2262</td>
<td>0.0122</td>
</tr>
<tr>
<td>Materka</td>
<td>0.0020</td>
<td>0.0054</td>
<td>0.0288</td>
<td>0.1872</td>
<td>0.0130</td>
</tr>
<tr>
<td>TANH</td>
<td>0.0016</td>
<td>0.0045</td>
<td>0.0251</td>
<td>0.1634</td>
<td>0.0110</td>
</tr>
<tr>
<td>XEXP</td>
<td>0.0015</td>
<td>0.0046</td>
<td>0.0256</td>
<td>0.1647</td>
<td>0.0094</td>
</tr>
</tbody>
</table>

and the error of the $s$-parameters at all bias points together is defined analogically by

$$\text{rms}_{RF} = \sqrt{\frac{1}{4mn} \sum_{l=1}^{m} \sum_{k=1}^{n} \sum_{i,j=1}^{2} \left( \frac{|s_{ij,l}^{\text{meas}}(\omega_k) - s_{ij,l}^{\text{simul}}(\omega_k)|^2}{|s_{ij,l}^{\text{meas}}(\omega_k)| + \text{NUL}_{RF}} \right)}$$

(7)

where $m$ is number of frequencies. Metrics of the $s$-parameters $s_{11}$, $s_{12}$, $s_{21}$, and $s_{22}$

$$M_{s_{ij}} = \frac{\sum_{l=1}^{m} \sum_{k=1}^{n} |s_{ij,l}^{\text{meas}}(\omega_k) - s_{ij,l}^{\text{simul}}(\omega_k)|^2}{\sum_{l=1}^{m} \sum_{k=1}^{n} |s_{ij,l}^{\text{meas}}(\omega_k)|^2}$$

(8)

are also computed at the bias points defined by the voltages $V_{GS} = (-1.1 : 0.2 : 0.3)$ V and $V_{DS} = (0.5 : 0.5 : 10)$ V for the frequencies $f = (4 : 0.5 : 80)$ GHz. These intervals are the same as those in [6]. The results obtained here for various models can be compared with the results of the EEHEMT1 model taken from [6]. $N_{RF}$ in the last column of Table II denotes the number of model parameters involved.

The measured and calculated $s$-parameters of the TANH model at the operating points where the best and worst fits were detected are shown in Figs. 7 and 8, respectively. The larger points mark the lowest frequency, i.e., 4 GHz.

Moreover, large-signal properties of the suggested models have been recently tested by series of the steady-state analyses [8] of a tunable distributed microwave oscillator.

**Fig. 6.** Absolute error of the drain current computed by model TANH.

**Fig. 4.** Output characteristics computed by model TANH in comparison with measured data for various values of $V_{GS}$.

**Fig. 5.** Output characteristics computed by model TANH in comparison with measured data for various values of $V_{GS}$ – a detail for lesser values of $I_D$.

I-V characteristics. The root-mean-square error of the output I-V characteristics is defined by the formula

$$\text{rms}_{DC} = \sqrt{\frac{1}{n} \sum_{j=1}^{n} \left( \frac{|I_{d,j}^{\text{meas}} - I_{d,j}^{\text{simul}}|}{\sqrt{I_{d,j}^{\text{meas}} + \text{NUL}_{DC}}} \right)^2},$$

(6)
### TABLE II

Comparing the Identification Results of Various Nonlinear Dispersive HEMT Models

<table>
<thead>
<tr>
<th>Model</th>
<th>rms&lt;sub&gt;DC&lt;/sub&gt;</th>
<th>rms&lt;sub&gt;RF&lt;/sub&gt;</th>
<th>M&lt;sub&gt;s11&lt;/sub&gt;</th>
<th>M&lt;sub&gt;s12&lt;/sub&gt;</th>
<th>M&lt;sub&gt;s21&lt;/sub&gt;</th>
<th>M&lt;sub&gt;s22&lt;/sub&gt;</th>
<th>N&lt;sub&gt;P&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahmed</td>
<td>0.0193</td>
<td>0.1354</td>
<td>0.077</td>
<td>0.326</td>
<td>0.168</td>
<td>0.110</td>
<td>31</td>
</tr>
<tr>
<td>Dobes</td>
<td>0.0267</td>
<td>0.1847</td>
<td>0.176</td>
<td>0.559</td>
<td>0.185</td>
<td>0.127</td>
<td>31</td>
</tr>
<tr>
<td>TOM</td>
<td>0.0224</td>
<td>0.2305</td>
<td>0.243</td>
<td>0.475</td>
<td>0.239</td>
<td>0.129</td>
<td>31</td>
</tr>
<tr>
<td>TOM2</td>
<td>0.0204</td>
<td>0.1270</td>
<td>0.072</td>
<td>0.308</td>
<td>0.145</td>
<td>0.095</td>
<td>35</td>
</tr>
<tr>
<td>Materka</td>
<td>0.0213</td>
<td>0.1649</td>
<td>0.096</td>
<td>0.350</td>
<td>0.180</td>
<td>0.096</td>
<td>38</td>
</tr>
<tr>
<td>TANH</td>
<td>0.0193</td>
<td>0.1218</td>
<td>0.077</td>
<td>0.318</td>
<td>0.136</td>
<td>0.087</td>
<td>37</td>
</tr>
<tr>
<td>XEXP</td>
<td>0.0150</td>
<td>0.1358</td>
<td>0.097</td>
<td>0.359</td>
<td>0.148</td>
<td>0.079</td>
<td>37</td>
</tr>
<tr>
<td>EEHEMT1</td>
<td>-</td>
<td>-</td>
<td>0.169</td>
<td>0.547</td>
<td>0.220</td>
<td>0.156</td>
<td>51</td>
</tr>
</tbody>
</table>

**Fig. 7.** Measured and simulated frequency dependence of s-parameters in complex plane, frequencies from 4 to 90 GHz, s<sub>21</sub>/5, s<sub>12</sub> × 5, V<sub>GS</sub> = −0.5 V, V<sub>DS</sub> = 4 V, rms<sub>RF</sub> = 0.066 (the best fit).

**Fig. 8.** Measured and simulated frequency dependence of s-parameters in complex plane, frequencies from 4 to 90 GHz, s<sub>21</sub>/5, s<sub>12</sub> × 5, V<sub>GS</sub> = −0.5 V, V<sub>DS</sub> = 1.5 V, rms<sub>RF</sub> = 0.214 (the worst fit).

### V. CONCLUSION

The parameters of several popular nonlinear HEMT models have been extracted from multibias s-parameters using a three-step identification procedure. Results reported in Table II show the effectiveness of the procedure and the performance of various models including the two new ones – modifications TANH and XEXP. As observed by comparing with the results of the EEHEMT1 model published before, the errors of modeled multibias s-parameters described by the metrics M<sub>s1j</sub> are well reduced despite the lower number of model parameters.

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### REFERENCES


