Scheduling for Time-Varying Broadcast Channels

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Abstract—To maximize spectral efficiency for multiple users with time-varying broadcast channel, the scheduling order of the users needs to be optimized. The scheduling problem naturally is a combinatorial optimization problem with high complexity, which exponentially increases with the number of users. In this paper, several low-complexity scheduling algorithms are proposed. Computer simulation results show that the proposed algorithms achieve performance close to the optimal one.

I. INTRODUCTION

For conventional wireless communications systems that use Orthogonal Frequency Division Multiplexing (OFDM) or Orthogonal Frequency Division Multiple Access (OFDMA), a base station (BS) supports multiple mobile stations (MS) in an orthogonal way such that each time slot and subcarrier is dedicated to only one user. For example, in IEEE 802.16e standard, within each downlink frame, multiple bursts are allocated to MSs across a time-frequency domain according to the channel state information (CSI) feedback from the previous uplink frame [1],[2],[3]. Therefore, how to assign MSs to available slots and how to allocate power to the slots have been an extensively active area of research recently. For instance, a scheduling algorithms based on utility functions is proposed in [4], [5]. The resource-allocation problems for multiuser OFDM systems with frequency-selective channels are solved in [6],[7],[8].

As the need of mobility support increases, it is also necessary to consider time-varying nature of channels for the resource-allocation problems in order to further enhance spectral efficiency. Most of the previously developed algorithms need instantaneous CSI of MSs, such as achievable data rates and channel gain values, rather than statistical CSI of MSs, such as the distribution of the channel state values and mobile speed of MS’s, which tend to remain constant for a relatively long time. However, it is often impractical and inefficient to update CSI every time when the BS tries to send data to one of the MSs. With its limited availability, assigning too much resource to get CSI would deplete the resources needed to send actual data, resulting in reduced overall throughput [9],[10]. Furthermore, inherent delay is usually inevitable between the time when MS’s channel state is measured and the time when the MS is supported by the BS always exists.

Therefore, for practical wireless systems, it is reasonable to assume that the CSI is updated infrequently, e.g., only at the beginning of each frame. Since the CSI becomes outdated as time passes, the effect of outdated CSI should be considered to design a scheduling algorithm. For a single user case, a dynamic CSI framework that incorporates channel gain feedbacks and channel statistics, mean, covariance, and time-correlation, is constructed and the optimal transmit strategy to maximize an ergodic capacity is found in [11]. This paper, by noting that the accuracy of CSI of MSs decreases as time passes, proposes several low-complexity algorithms for a multi-user case to maximize spectral efficiency, which is defined as the sum of outage capacities.

This paper is organized as follows: Section II describes a system model, while section III defines the spectral efficiency in terms of outage capacities. Section IV proposes several scheduling algorithms to maximize spectral efficiency with reduced complexity. The performance of the proposed scheduling algorithms is evaluated by computer simulations in Section V. Finally, Section VI provides conclusion.

II. SYSTEM MODEL

The system model has one BS and K MS’s. The MSs have its CSI perfectly all the time and the feedback channel is error-free. For expositional simplicity, the BS and the MSs have only one antenna as the extension to the multiple-antenna case is straightforward. If the kth MS is assigned to the ith slot and the signal \( x_k^i \) is sent from the BS, then the received signal \( y_k^i \) is given by,

\[
y_k^i = h_k^i x_k^i + n_k^i, \quad 1 \leq i, k \leq K,
\]

where \( h_k^i \) and \( n_k^i \) are the channel gain value and the zero mean additive white complex gaussian noise for the kth MS at the ith slot, respectively. Both are assumed to be i.i.d. across k and i, and the noise variances are equal to \( \sigma^2 \). When a slot is composed of multiple symbols, \( y_k^i, x_k^i, \) and \( n_k^i \) change to vectors.

It is assumed that the channel is flat and \( h_k^i \) follows a complex Gaussian distribution, \( h_k^i \sim CN(0,1) \), for all k and i after normalization without loss of generality. The BS is assumed to transmit with fixed power, i.e., \( E[|x_k^i|^2] = P \). As a result, the expected signal-to-noise ratio (SNR) is,

\[
SNR = \frac{E[|h_k^i|^2|x_k^i|^2]}{\sigma^2} = \frac{P}{\sigma^2}.
\]

With flat fading channel assumption, each frame consists of K slots and only one MS is supported in each slot in Time Division Multiple Access (TDMA) manner. The framework of this paper can be easily extended to the case when MSs are supported in OFDMA fashion for frequency-selective channels. At the beginning of each frame, MSs feedback their CSI. Doppler frequencies of each MS, \( f_{D,k} \), can be either feedback by the MS or estimated by the BS. The BS
schedules MSs according to the received CSI and the Doppler frequencies to maximize spectral efficiency.

| ... | Uplink 1,2,3 | Downlink 1 | Downlink 2 | Uplink 1,2,3 | Downlink 1 | Downlink 2 | ... |

Fig. 1. Timing Diagram

As an example, Fig. 1 shows a timing diagram of the system when the number of MSs is 3. The scheduling order of MSs for downlink can change frame by frame. Note that the later MS is scheduled, the less accurate is the CSI of the MS, because the feedback CSI becomes more outdated. \( f_{D,k} \) is assumed to be constant within each frame and \( h_i^k \) is assumed to be time-varying during a frame while constant within each slot. By assuming Jakes model, the time correlation between user \( k \)'s channel at the \( 0^{th} \) slot and the \( i^{th} \) slot is

\[
\rho_k^i = E[h_k^0 h_k^i] = \rho_k^0 f_{D,k} T i, \tag{3}
\]

where \( T \) is a slot duration.

### III. PROBLEM STATEMENT

The BS tries to maximize spectral efficiency by appropriately scheduling MSs. However, the BS has uncertainty regarding the CSI of MSs, i.e., \( h_k^0 \) given \( h_k^0 \) and \( f_{D,k} \) follows a conditional distribution as will be shown below. Therefore, an ergodic capacity may be inappropriate as a measure for spectral efficiency, because it applies to the case when each codeword is long enough to experience a sufficient number of different channel states. Thus, outage capacity is considered and the BS’s objective is to maximize the sum of the outage capacities.

From a standard MMSE estimation theory [13], given the CSI at the \( 0^{th} \) slot, the channel gain estimate for the \( i^{th} \) slot and its mean squared error are respectively,

\[
\hat{h}_k^i = \rho_k^0 h_k^i, \tag{4}
\]

\[
MSE_{\hat{h}_k^i} = 1 - |\rho_k^0|^2. \tag{5}
\]

It can be written in a following simple form,

\[
h_k^i h_k^0 \sim CN(\rho_k^0 h_k^0, 1 - |\rho_k^0|^2). \tag{6}
\]

| \( |h_k^i|^2 \) | \( |h_k^0| \) | \( \text{denoted as} \ Y \), is a noncentral chi-square random variable with degrees of freedom, \( n \), equal to 2. The cumulative distribution function of \( Y \) is given as

\[
F_Y(y) = \int_0^y \frac{1}{2\sigma^2} e^{-\frac{u}{2\sigma^2}} I_{\frac{n - 2}{2}}(\frac{\sqrt{us}}{\sigma}) du = 1 - Q_1\left(\frac{\sqrt{\gamma}}{\sigma}\right), \tag{7}
\]

where \( s^2 = |\rho_k^0 h_k^0|^2, \sigma^2 = 1 - |\rho_k^0|^2 \) and \( Q_1(\cdot) \) is Marcum’s Q function.

\[
Q_1(a, b) = e^{-\frac{a^2 + b^2}{2}} \sum_{k=0}^{\infty} \left(\frac{a}{b}\right)^k I_k(ab), \quad b > a > 0, \tag{8}
\]

and \( I_a(\cdot) \) is the \( a^{th} \) order modified Bessel function of the first kind.

Assuming that each MS faces a common outage probability constraint \( p_{out} \),

\[
p_{out} = F_Y(y), \tag{9}
\]

the outage capacity for the \( k^{th} \) MS assigned to the \( i^{th} \) slot is given as

\[
C_k^i\{h_k^0, f_{D,k}\} = \log\left(1 + \frac{F_Y^{-1}(p_{out})P}{N_0}\right). \tag{10}
\]

By using a bisection method [14], the value of \( F_Y^{-1}(p_{out}) \) can be easily obtained. The outage capacity decreases as time passes and decreases faster if the channel varies faster. Therefore, \( C_k^i\{h_k^0, f_{D,k}\} \) decreases as \( i \) or \( f_{D,k} \) increases.

Finally, the sum rate of each frame, which consists of \( K \) slots, given \( h_k^0 \) and \( f_{D,k} \) for \( k = 1, \ldots, K \) is

\[
C_{sum}^\pi = \sum_{i=1}^K C_{\pi(i)}^i\{h_1^0, \ldots, h_1^0, f_{D,1}, \ldots, f_{D,K}\}, \tag{11}
\]

where \( \pi \) denotes a downlink scheduling order of MS’s. The objective of the scheduler algorithm used at the BS is to find \( \pi \) that maximizes \( C_{sum}^\pi \) given \( \{h_1^0, \ldots, h_1^0, f_{D,1}, \ldots, f_{D,K}\} \).

### IV. SCHEDULING ALGORITHMS

A straightforward scheduling algorithm that achieves the optimal performance would try every possible scheduling order and select the best. Though optimal, this algorithm suffers from combinatorial complexity that increases exponentially as the number of MS’s increases. Therefore, this paper proposes several scheduling algorithms with only moderate complexity but achieves close to the optimal performance.

Consider the following optimization problem over \( \pi: a \) permutation of \( \{1, 2, \ldots, K\} \),

\[
\max_{\pi} \sum_{i=1}^K C_{\pi(i)}^i. \tag{12}
\]

The following theorem holds for the above optimization problem.

**Theorem 1:** If there exists a permutation \( \pi \) of \( \{1, 2, \ldots, K\} \) such that

\[
C_{\pi(k_1)}^i - C_{\pi(k_1)}^{i+1} > C_{\pi(k_2)}^i - C_{\pi(k_2)}^{i+1}, \tag{13}
\]

for all \( i, k_1, k_2 \) such that \( 1 \leq i \leq K - 1, 1 \leq k_1 < k_2 \leq K \), then \( \pi \) is the optimal solution for (12).

**Proof.** Without loss of generality, it can be assumed that \( \pi(i) = i \) for \( 1 \leq i \leq K \). Then \( c_i^i - c_i^{i+1} > c_m^{i+1} - c_m^i \) for \( l < m \). Suppose \( \pi \) is not the optimal solution, then there exists an optimal solution \( \pi' \) such that \( C_{\pi'(k_1)}^i > C_{\pi'(k_2)}^i \) for some \( 1 \leq k_1 < k_2 \leq K \). Let \( \pi'' \) denote another permutation...
with \( \pi''(i) = \pi'(i) \) for \( i \neq k_1, k_2 \), \( \pi''(k_1) = \pi'(k_2) \) and \( \pi''(k_2) = \pi'(k_1) \). Thus,
\[
\sum_{i=1}^{K} C_{\pi'''(i)} - \sum_{i=1}^{K} C_{\pi''(i)}
\]
\[
= C_{\pi''(k_1)} - C_{\pi''(k_2)} + C_{\pi''(k_2)} - C_{\pi''(k_2)}
\]
\[
(a) = C_{\pi'(k_1)} - C_{\pi'(k_1)} + C_{\pi'(k_1)} - C_{\pi'(k_2)}
\]
\[
= C_{\pi'(k_1)} - \sum_{j=k_1}^{k_2-1} (C_{\pi'(k_1)} - C_{\pi'(k_2)}) - C_{\pi'(k_2)}
\]
\[
(b) > C_{\pi'(k_1)} - \sum_{j=k_1}^{k_2-1} (C_{\pi'(k_2)} - C_{\pi'(k_1)}) - C_{\pi'(k_2)}
\]
\[
= 0.
\]
where (a) is from \( \pi''(k_1) = \pi'(k_2) \) and \( \pi''(k_2) = \pi'(k_1) \), and (b) is from \( C_{i} - C_{i+1} > C_{m} - C_{m+1} \) for \( l < m \) and \( \pi'(k_1) < \pi'(k_2) \). Thus \( \pi' \) is not an optimal solution. This contradicts the assumption.

Figure 2 illustrates the example with \( K = 3 \), where the assumption of (13) of Theorem 1 is satisfied when \( \pi = (\pi(1) = 1, \pi(2) = 2, \pi(3) = 3) \). Thus, this \( \pi \) is the optimal scheduling. In the figure, MS 1 experiences the steepest decrease in the outage capacity throughout slots, while the outage capacity of MS 3 decreases relatively slowly. Hence, it is desirable that the first slot should be allocated to MS 1 and the last to MS 3, which agrees to the result suggested by Theorem 1.

A. The Proposed Scheduling Algorithms

Theorem 1 gives the exact solution for the scheduling problem (12) under the condition that there exists a permutation that satisfies (13). For example, when the difference of the outage capacity from slot \( i \) to \( i + 1 \), \( C_{k} - C_{k+1} \), is constant over each MS, e.g. the \( k^{th} \) MS, then (13) is satisfied. In this case, the optimal scheduling is trivial: allocate the first slot to the MS with the largest decrement, the second slot to the MS with the second largest decrement, and so on. When a permutation that satisfies (13) does exist, the optimal scheduling can be obtained by the same method used for the above example. Even though the assumption of Theorem 1 does not hold, Theorem 1 hints that the decrement of the outage capacity plays a more important role than the magnitude of the outage capacity itself. From this observation, the following four suboptimal scheduling algorithms are proposed:

1) Steepest Difference for Every instance (SDE): In this algorithm, each slot is allocated to the MS with the largest difference of outage capacity among remaining MS’ s. For slot 1, the MS with the largest \( C_{1} - C_{2} \) is selected. Likewise, the MS with the largest \( C_{k} - C_{k+1} \) is selected from the rest of MS’s for slot \( i \). When the condition (13) holds, SDE is optimal. Even if the condition does not hold, it finds the locally optimal user selection for each slot.

2) Steepest First Difference (SFD): Unlike SDE, SFD only uses the first difference for scheduling: slot 1 is allocated to the MS with the largest outage capacity among remaining MS’s. For slot 1, the MS with the largest \( C_{1} - C_{2} \) is selected. Likewise, the MS with the largest \( C_{k} - C_{k+1} \) is selected from the rest of MS’s for each slot. When the condition (13) holds, SDE is optimal. Even if the condition does not hold, it finds the locally optimal user selection for each slot.

3) Largest Outage capacity for Every instance (LOE): This greedy algorithm selects the MS with the largest outage capacity for each slot: for slot 1, the MS with the largest \( C_{1} \) is selected; for slot 2, the MS with the largest \( C_{k} \) is selected among the rest of MS’s, and so on. Although LOE looks irrelevant to what Theorem 1 suggests, it shows only slightly inferior performance to SDE. This finding is caused by the correlation of the decrement of the outage capacity and the outage capacity itself for each instance.

4) Largest First Outage capacity (LFO): In this algorithm, only the outage capacities of slot 1 are used for scheduling: the slot 1 is allocated to the MS with the largest \( C_{1} \); the slot 2 is allocated to the MS with the second largest \( C_{2} \); and so on.

B. Computational Complexity Analysis

The computational load for the scheduler consists of two parts: calculating the outage capacities, i.e. \( C_{k} \), and scheduling MS’s with given these outage capacities. Table 1 shows the numbers of calculations of the outage capacities and the asymptotic complexities of scheduling algorithms with given outage capacities, denoted by \( i_{\text{outage}} \) and \( i_{\text{scheduling}} \), respectively. Moreover, the computational complexity of the exhaustive search, which is the optimal scheduling, is also shown for reference.

As shown in the table, SDE and LOE are more complex schemes than SFD and LFO in the sense of computational complexity. However, the complexities of SDE and LOE are still significantly less than that of the exhaustive search.
TABLE I
COMPUTATIONAL COMPLEXITIES OF THE FOUR ALGORITHMS

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>$t_{\text{outage}}$</th>
<th>$t_{\text{scheduling}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDE</td>
<td>$K^2$</td>
<td>$O(K^2)$</td>
</tr>
<tr>
<td>SFD</td>
<td>$2K$</td>
<td>$O(K^2)$</td>
</tr>
<tr>
<td>LOE</td>
<td>$K^2 - K$</td>
<td>$O(K^2)$</td>
</tr>
<tr>
<td>LFO</td>
<td>$K$</td>
<td>$O(K)$</td>
</tr>
<tr>
<td>Exhaustive</td>
<td>$K^2$</td>
<td>$O(K^K)$</td>
</tr>
</tbody>
</table>

V. SIMULATION RESULTS

This section provides simulation results to compare the sum rate of the four proposed algorithms: SDE, SFD, LOE, and LFO. For simulations, the number of MS's is set to 4 unless described otherwise, 10% and 1% outage capacities are used, and the sum of outage capacities normalized with respect to $K$, $C_{\text{sum}}/K$, will be averaged over $h_{k}^0$’s and presented in the $y$-axes of the figures.

For the first part of the simulations, the users are assumed to have the same Doppler frequency. Under this assumption, the effects of Doppler frequency, SNR, and the number of MS’s are shown in Fig. 3, Fig. 4, and Fig. 5, respectively. For the second part of the simulations, it is assumed that each user has different Doppler frequency and the effect of SNR is presented in Fig. 6.

The outage capacities for the exhaustive search, which has the optimal performance, and for the random scheduling, which randomly allocates slots to MS’s and is equivalent to fixed-order scheduling, are also shown as references. The outage capacities of these two scheduling algorithms form an upper bound and an lower bound for the performance of scheduling, respectively. The difference between exhaustive search and random scheduling is that exhaustive search optimizes the order of MS’s to further maximize spectral efficiency while random scheduling does not optimize at all. Yet, both scheduling algorithms consider that the data rate changes as time passes and select the highest possible rate when a user is assigned to a slot. If time-variability of the data rate is neglected at the BS, scheduling should assume that every user is in its worst state, i.e. scheduled at the last slot. In this case, scheduling order have no impact on spectral efficiency. This scheme is denoted as conventional scheme because the time-varying nature of broadcast channels has been neglected conventionally when designing a scheduler. The performance of this scheme serves as a lower bound for a scheduler that does not consider that CSI becomes outdated as time passes.

A. Effect of Doppler frequency

Figure 3 shows the average sum capacities, $C_{\text{sum}}^\pi/K$, of the proposed algorithms with $f_DT$ ranging from 0.01 to 1. For example, when $f_c = 2$GHz, $v = 30$mph, and $T = 1$ms, corresponding $f_D T$ is 0.1 because $f_D = \frac{v}{c} f_c$, where $v$, $c$, and $f_c$ are the speed of the MS, the speed of light, and the carrier frequency, respectively. The average sum capacities of the algorithms decreases as $f_D T$ increases because estimation of the channel becomes less accurate as $f_D T$ increases, hence resulting in decreased outage capacities. The proposed four algorithms give almost the same average sum capacities as the optimal case. When $f_D T = 0.1$, the performance gain of the proposed scheduling algorithms over random scheduling are approximately 25% and 33% for 10% and 1% outage probabilities, respectively. When compared to the conventional scheme, the performance gains of the proposed algorithms are approximately 50% and 90% for 10% and 1% outage probabilities, respectively, at $f_D T = 0.1$. Among the four algorithms, only SFD results in a slightly inferior performance to the optimal case. Interestingly, the gap between the upper bound and the lower bound is small either at low or high Doppler frequency compared to that at around $f_D T = 0.06$. With a high Doppler frequency, the channel varies so rapidly that the estimation at the beginning of the frame is already outdated and the sum rate becomes saturated. On the other hand, a low Doppler frequency means the channel is almost constant. Hence, scheduling has an insignificant effect on the average sum capacity with high or low Doppler frequency. In other words, there is an appropriate range of $f_D T$ where the scheduling plays a significant role on the performance of the system.

B. Effect of SNR

In Fig. 4, the normalized sum capacities of the four algorithms with SNR ranging from 0dB to 30dB, ($f_D T = 0.06$, and $K = 4$) are shown. As the SNR increases, the absolute differences of the normalized sum capacities between the four algorithms increase while the relative differences decrease. According to the figure, the order of the performance of the four algorithms is determined clearly as follows: SDE for every instance is the best; LOE and LFO then follow; SFD shows the smallest throughput, with still better than that of the random scheduling. In addition, three algorithms except SFD have the total throughput close to the upper bound. When $SNR = 15$dB, the performance gain of the proposed scheduling algorithms over those of the random scheduling
are approximately 13% and 22% for 10% and 1% outage probabilities, respectively. When compared to the conventional scheme, the performance gains are approximately 25% and 75% for 10% and 1% outage probabilities, respectively, at $SNR = 15$dB.

C. Effect of the number of mobile stations

In Fig. 5, the normalized sum capacities of the four algorithms with the number of MS’s varying, $f_D T = 0.06$, and $SNR = 5$dB. As the number of MS’s increases, the average sum capacities of all the four algorithms decrease. Since large $K$ means a longer frame, i.e. larger delay from the channel estimation to the actual data transmission, smaller outage capacities are achieved. Moreover, the gaps between the normalized sum capacities of the algorithms increase as $K$ increases. With large $K$, the channel values for MS’s vary more than those with small $K$, hence the scheduling becomes more important as the number of MS’s increases. When the number of MS’s is 8, the performance gain of the proposed scheduling algorithms over those of the random scheduling and the conventional scheme are approximately 40% and 60%, respectively.

D. Effect of SNR with different Doppler frequencies for mobile stations

Figure 6 shows the normalized sum capacities when MSs have different Doppler frequencies. When there are four MS’s, each with Doppler frequencies 0.02, 0.05, 0.01, and 0.2, respectively and under $SNR = 15$dB, the performance gain of the proposed scheduling algorithms over that of the random scheduling are approximately 13% and 30% for 10% and 1% outage probabilities, respectively. When compared to the performance of the conventional scheme, the proposed algorithms show 24% and 50% for 10% and 1% outage probabilities, respectively, at $SNR = 15$dB.

The order of the performance of the four algorithms are different from the previous results: SDE is the best; SFD then follows; LOE and LFO show small sum rates, which are above the lower bound by only a small amount. It can be interpreted as that the decrement of the outage capacity is more useful information than the outage capacity itself.

VI. CONCLUSION AND REMARKS

The spectral efficiency of time-varying broadcast channel can be enhanced by properly scheduling users. This scheduling problem naturally is a combinatorial optimization problem with high complexity. Several scheduling algorithms that have smaller complexity are proposed in this paper: SDE, DFD, LOE, and LFO. It is shown by simulations that compared to the case without scheduling, the proposed algorithms achieve performance gain from 13% up to 40%. Among the proposed algorithms, SDE always provides the best performance, which is very close to the optimal one.

It is expected that the proposed scheduling algorithms can also be applied to the case when a BS and MSs have multiple
antennas. With the knowledge of the distribution of mutual information, the outage capacities of each MS can be obtained for each slot and the proposed algorithms can work with those numbers to maximize the total throughput. Besides, for MIMO systems, an option of beamforming is another factor to be considered for scheduling algorithms. However, the distribution of mutual information for MIMO systems with and without beamforming is still an open problem in general and further research is needed on characterizing the distribution.

REFERENCES