Resource-Allocation for OFDMA Multi-hop Relaying Downlink Systems

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Abstract—This paper proposes resource-allocation algorithms to maximize the sum-rate in orthogonal frequency-division multiple-access multi-hop relaying downlink systems. With advance subchannel allocation to users and relay stations, Lagrange dual-decomposition solves the power-allocation problem. This paper shows the optimal solution to be a modified water-filling algorithm, where an inner-outer bisection method determines the optimal water-levels. Further, proposed heuristic resource-allocation algorithms rely on the Karush-Kuhn-Tucker conditions. Simulation results show that the multi-hop OFDMA relaying systems outperform the conventional OFDMA systems by more than 30% with use of the proposed resource-allocation algorithms.

I. INTRODUCTION

Figure 1 shows a multi-user wireless relaying system that provides high data rates and enlarges the coverage area by relaying the data from a base station (BS) to users [1], [2], [3]. The presence of relay stations (RS) complicates the resource optimization in system-throughput maximization.

Researchers have proposed resource-allocation algorithms for a variety of situations with many assumptions: The route-selection algorithm and the smart channel-selection algorithm for a fixed RS model have been investigated in [4], [5]. Wu and Qiao [6] proposed a load-balancing algorithm in integrated cellular and ad-hoc relay systems. The authors in [7] formulated the subchannel-allocation problem in an orthogonal frequency-division multiple-access (OFDMA) multi-hop relaying system as a linear programming (LP) problem. However, [7] only focused on subchannel reuse without power optimization. Under time-division multiple-access systems, timeslot-allocation algorithms have been investigated in [8] and [9]. Although the optimal timeslot-allocation problem to maximize the common throughput for all the users in a cell can be formulated as an LP, the model used in [8] is too simple for most practical situations. For such simplified models, power-allocation algorithms are also developed in [10], [11] and [12].

To obtain a practical solution, this paper focuses on the resource-allocation algorithms that maximize the sum-rate in OFDMA multi-hop relaying downlink system. The difficulty of resource-allocation is the nonconvexity of the problem, which is caused by the interrelationship of all resources such as power and subchannels. This paper formulates the optimization problem as a convex problem by using preassigned subchannels because the subchannel-allocation might not be a significant factor for system performance [4]. Then, the use of the Lagrange dual-decomposition method leads to the resource-allocation algorithms. This paper shows that the optimal power-allocation algorithm is a modified water-filling algorithm, which is efficiently solved by using an inner-outer bisection method. Also, this paper proposes a heuristic power-allocation algorithm that is simple in complexity and achieves near-optimal performance. Further, this result motivates a heuristic subchannel-allocation algorithm in which the user with the lowest modified inverse subchannel signal-to-noise ratio (MISSNR) occupies each subchannel.

The organization of this paper is as follows: Section II presents the system model and the problem formulation. Section III develops efficient resource-allocation algorithms by using the Lagrange dual decomposition method and Karush-Kuhn-Tucker conditions (KKT). Section IV discusses numerical results. Section V concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

This paper considers an OFDMA multi-hop relaying downlink system of \(K\) users and \(N_r\) RSs, where each transmitter and receiver have a single antenna. The number of downlink subchannels is \(N\), and the OFDMA receiver completely removes the inter-symbol interference. Figure 2 shows the overall downlink frame structure. Transmission between the BS and the users occurs on subchannel set \(B_0\) of subframe 1, and subchannel set \(B_j\) (\(1 \leq j \leq N_r\)) in subframe 1 is for the transmission between the BS and RS \(j\). Similarly,
subchannel set $S_j$ ($1 \leq j \leq N_r$) in subframe 2 is for the transmission between RS $j$ and the users associated with RS $j$. $N_j'$ denotes the number of subchannels in a set $B_j$, i.e., $|B_j| = N_j'$. In both subframes, at most one user or one RS occupies each subchannel to avoid interference which dramatically increases the complexity of the problem, hence, $B_j \cap B_j' = \emptyset$ for $j \neq j'$, $S_j \cap S_j' = \emptyset$ for $j \neq j'$, $\cup_{j=1}^{N_j} B_j \subseteq \{1, \ldots, N\}$, and $\cup_{j=1}^{N_r} S_j \subseteq \{1, \ldots, N\}$. For subchannel $i$ in subframe $k$, $h_k(i)$ denotes the subchannel gain, and a zero-mean i.i.d. Gaussian noise with variance $\sigma_k^2(i)$ adds at the receiver. $c_k(i) = |h_k(i)|^2/\sigma_k^2(i)$ defines the signal-to-noise ratio for subchannel $i$ in subframe $k$. $P_k(i)$ and $r_k(i)$ denote the power and the rate for subchannel $i$ in subframe $k$ such that $r_k(i) = 0.5 \log_2(1 + P_k(i)c_k(i))$ bits per dimension. $i_1$ and $i_2$ denote subchannel $i$ in subframe 1 and 2 respectively.

In general, (1) is not a convex optimization problem since to find optimal subchannel sets $\{B_j\}$ and $\{S_j\}$ is a combinatorial problem, whose complexity exponentially increases with $N$. However, for fixed subchannel sets, (1) becomes a convex power-allocation optimization problem. Although a subgradient method [13] can solve the convex optimization problem, its convergence requires a huge number of iterations and provides no insight into the subchannel-allocation algorithm, which is essential to the convexity and the performance of (1). Thus a new method is required.

The total power constraint couples the objective function over RSs, but the Lagrange dual-decomposition method decouples the overall problem into RS-by-RS problems. However, the dual method does not easily apply to the current form of (1). Hence, the following equivalent problem is considered:

\[
\text{maximize} \quad \sum_{i_1 \in B_0} r_1(i_1) + \sum_{i_2 \in S_1} r_2(i_2) \\
\text{subject to} \quad t_j \leq \sum_{i_1 \in B_j} r_1(i_1) \quad \forall j = 1, \ldots, N_r, \\
\sum_{i_1 \in B_j} P_1(i_1) + \sum_{i_2 \in S_j} P_2(i_2) \leq P_{tot}, \\
P_k(i_k) \geq 0 \quad \forall k \quad \text{and} \quad \forall i_k. \quad (2)
\]

The Lagrangian of the sum-rate maximization problem (2) over domain $\mathcal{D}$ is

\[
\mathcal{L}(\{P_1(i_1)\}, \{P_2(i_2)\}, \{r_1(i_1)\}, \{r_2(i_2)\}, t_1, \ldots, t_N, \lambda_1, \lambda_2, \ldots, \lambda_{N_r}, \lambda'n) = \sum_{i_1 \in B_0} r_1(i_1) + t_1 + \cdots + t_N' \\
+ \lambda_1(\sum_{i_2 \in S_1} r_2(i_2) - t_1) + \lambda'_1(\sum_{i_1 \in B_1} r_1(i_1) - t_1) + \cdots \\
+ \lambda_{N_r}(\sum_{i_2 \in S_{N_r}} r_2(i_2) - t_{N_r}) + \lambda'_N(\sum_{i_1 \in B_{N_r}} r_1(i_1) - t_{N_r}) \\
+ \lambda'n(P_{tot} - \sum_{i_1} P_1(i_1) - \sum_{i_2} P_2(i_2)), \quad (3)
\]

where the domain $\mathcal{D}$ is the set of all non-negative $\{P_1(i_1)\}$ and $\{P_2(i_2)\}$. Then, the Lagrange dual function is
Because (4) is a convex function in $P_1(i_1)$ and $P_2(i_2)$ for all $i_1$ and $i_2$, the optimal solution satisfies the following KKT conditions:

\[
\begin{align*}
P_1(i_1) &= [k_1 - \frac{1}{c_1(i_1)}]^+ \quad i_1 \in B_0, \\
P_1(i_1) &= [k_{j_2} - \frac{1}{c_1(i_1)}]^+ \quad i_1 \in B_j \quad \forall j, \\
P_2(i_2) &= [k_{j_3} - \frac{1}{c_2(i_2)}]^+ \quad i_2 \in S_j \quad \forall j, \\
k_1 &= k_{j_2} + k_{j_3} \quad i_1 \in B_j \quad i_2 \in S_j \quad \forall j, \\
\sum_{i_1 \in B_j} r_1(i_1) &= \sum_{i_2 \in S_j} r_2(i_2) \quad \forall j, \\
\sum_{i_1} P_1(i_1) + \sum_{i_2} P_2(i_2) &= P_{tot}, \\
P_k(i_k) &\geq 0 \quad \forall k \quad \text{and} \quad \forall i_k,
\end{align*}
\]

where $k_1$, $\{k_{j_2}\}$, and $\{k_{j_3}\}$ are constant water-levels. This type of solution is called a water-filling solution. The solution is unique because the minimized function is convex. In case of a conventional OFDMA downlink system, the resource-allocation algorithm for the sum-rate maximization problem assigns each subchannel to the user with the highest subchannel SNR, and the optimal power-allocation algorithm is a conventional water-filling algorithm [14]. However, the conventional water-filling algorithm does not solve this problem because the water-levels for all subchannel sets $\{B_j\}$ and $\{S_j\}$ are different, i.e. $k_{jk} \neq k_{j'k}$ for $j \neq j'$ and $\forall k$, and all water-levels are interrelated through the water-level $k_1$. Further, Fig. 3 illustrates that the system requires only the portion of the allocated power for relaying.

The modified water-filling algorithm solves (2). This algorithm employs an inner-outer bisection method. The outer bisection method initially determines the upper bound of $k_1$ by using the conventional water-filling algorithm on subchannels in $B_0$ with $P_{tot}$. Using the $k_1$, the inner bisection method adjusts $\{k_{j_2}\}$ and $\{k_{j_3}\}$ to satisfy (6)-(9) and determines $P_1(i_1)$ for $i_1 \in B_j$ and $P_2(i_2)$ for $i_2 \in S_j$. If this power distribution violates (10), the inner-outer bisection method readjusts the $k_1$, $\{k_{j_2}\}$, and $\{k_{j_3}\}$. This process is performed until the power distribution also satisfies (10) and the width of each water-level is small enough.

**Algorithm 1 Inner-outer bisection method**

Inner bisection method for each $j$

1. $t_{k2} := (u_{k2}(j) + l_{k2}(j))/2$.
2. $k_{j3} := t_{k1} - t_{k2}$.
3-1. $P_1(i_1) := (t_{k2} - \frac{1}{c_1(i_1)})^+$ \quad $\forall i_1 \in B_j$.  
3-2. $P_2(i_2) := (k_{j3} - \frac{1}{c_2(i_2)})^+$ \quad $\forall i_2 \in S_j$.
4-1. $r_2 := \sum_{i_1 \in B_j} 0.5 \log_2(1 + P_1(i_1)c_1(i_1))$.
4-2. $r_3 := \sum_{i_2 \in S_j} 0.5 \log_2(1 + P_2(i_2)c_2(i_2))$.
5. $\text{error} := r_2 - r_3$.
6. if $\text{error} > 0$, $u_{k2}(j) := t_{k2}$;
else $l_{k2}(j) := t_{k2}$.

until $u_{k2}(j) - l_{k2}(j) < \epsilon_2$.

Outer bisection method

given $k_1$ and tolerance $\epsilon_1$, let $u_{k1} := k_1$ and $l_{k1} = 0$.

repeat
1. $t_{k1} := (u_{k1} + l_{k1})/2$.
2. Do inner bisection method for all $j = \{1, ..., N_r\}$.
3. $P_1(i_1) := (t_{k1} - \frac{1}{c_1(i_1)})^+$ \quad $\forall i_1 \in B_0$.
4. $\text{error} := \sum_{j} P_1(i_1) + \sum_{j} P_2(i_2) - P_{tot}$.
5. if $\text{error} > 0$, $u_{k1} := t_{k1}$;
else $l_{k1} := t_{k1}$.

until $u_{k1} - l_{k1} < \epsilon_1$.

The convergence proof of the inner-outer bisection method simply uses the fact that the error functions required to adjust the water-levels in the bisection method are the increasing function of $k_{j_2}$ and $k_1$ respectively and intersect 0. Details are not presented here due to page limits.

**B. Heuristic Solutions with Total Power Constraints**

This subsection suggests the heuristic power-allocation and sub-channel-allocation algorithms. Figure 4 shows the optimal power distribution obtained from the KKT conditions. $P_1$, $P_2$, and $P_3$ in Fig. 4 denote the total power of subchannels $B_0$, $B_j$, and $S_j$ respectively and $P_a$ in Fig. 4 is,

\[
P_a = \sum_{i_2 \in S_j} (P_1(i_1) + P_2(i_2)), \quad i_1 \in B_j.
\]
In general, \( P_1 + P_4 \) is not equal to \( P_{\text{tot}} \) because,

\[
P_1 + P_4 - P_{\text{tot}} = \sum_{i=1}^{N_r} \left( \sum_{i \in S_j} P_1(i_1) - \sum_{i \in B_j} P_1(i_1) \right).
\] (12)

However, (12) is negligible in most practical situations such as the channel between the BS and the RS is line-of-sight (LOS). In this case, (12) reduces to

\[
\sum_{j=1}^{N_r} |S_j|^* - N_j^* P_1(i_1),
\]

where \(|S_j|^*\) denotes the number of power allocated subchannels in \( S_j \) and is negligible because \( P_2 \) are small. This observation leads to the heuristic power-allocation algorithm. The algorithm assumes that \( P_1 + P_4 = P_{\text{tot}} \) instead of (8) and distributes \( P_{4j} \) to all subchannels \( B_j \) and \( S_j \) for all \( j \) where

\[
P_{4j} = \sum_{i \in S_j} (P_1(i_1) + P_2(i_2)).
\]

The algorithm is as follows: Step 1 generates MISSNRs for all \( \{ S_j \} \), i.e. \( 1/c_1(i_1) + 1/c_2(i_2) \). Step 2 performs the conventional water-filling algorithm on subchannels in \( B_0 \) and modified subchannels in \( S_j \) with \( P_{\text{tot}} \) to determine \( k_1 \) and \( P_1(i_1) \) for \( i_1 \in B_0 \). Step 3 calculates \( P_{4j} \) from the water-filling result in step 2 and distributes it to subchannels in \( B_j \) and \( S_j \) with constraint (9) by using a bisection method. It also converges because the error function is the increasing function and intersects 0. The algorithm repeats the same procedure for all \( j \).

The single water level in (5) and (8) suggests the heuristic subchannel-allocation algorithm. The user with the highest subchannel SNR occupies subchannels in \( B_0 \) and the user with the lowest MISSNR occupies subchannels in \( S_j \). For choosing subchannels \( \{ B_j \} \) where \( j \neq 0 \), it is the optimal solution to assign the worst \( \sum_{j=1}^{N_j} N_j \) subchannels of the BS to \( \{ B_j \} \) where \( j \neq 0 \) because these are the least efficient subchannels in subframe 1.

### C. Separate Power Constraints

This subsection investigates the power-allocation problem when the BS and RSs have their own power constraints. For preassigned subchannel sets, the power constraints in (1) become

\[
\sum_{i} P_1(i_1) \leq P_{\text{Base}},
\]

\[
\sum_{i \in S_j} P_2(i_2) \leq P_{RS_j}, \quad \forall j.
\] (13)

where \( P_{\text{Base}} \) and \( P_{RS_j} \) are the power constraints of the BS and RS \( j \). Since all \( \{ S_j \} \) are orthogonal (no interference), each RS independently manages its own power. In case of an optimal solution, the elements of the minimum function in (1) should be equal. Then,

\[
\min \left( \sum_{i \in B_j} r_1(i_1), \sum_{i \in S_j} r_2(i_2) \right) = \min \left( \sum_{i \in B_j} r_1(i_1), c_j^* \right)
\] (14)

where \( c_j^* \) is the solution of the following problem,

maximize \( \sum_{i \in S_j} r_2(i_2) \)
subject to \( \sum_{i \in S_j} P_2(i_2) \leq P_{RS_j}, \quad P_2(i_2) \geq 0 \quad \forall i \in S_j. \)

Independent conventional water-filling algorithms can determine each \( c_j^* \). From the above arguments and with fixed \( \{ B_j \} \) and \( \{ S_j \} \), the optimization problem becomes

maximize \( \sum_{i \in B_0} r_1(i_1) + f_1 + \cdots + f_{N_r} \)
subject to \( \sum_{i \in B_0} P_1(i_1) \leq P_{\text{base}}, \quad P_1(i_1) \geq 0 \quad \forall i_1, \)

where \( f_j \) denotes the right side of (14). This is an easy convex problem because only \( \{ P_1(i_1) \} \) are variables in (16). After solving (16), a water-filling algorithm determines \( \{ P_2(i_2) \} \) for each RS through margin maximization.

### IV. Simulation Results

This section provides simulation results for the proposed resource-allocation algorithms in OFDMA multi-hop relaying downlink systems. The radius of a cell where 1000 users are uniformly distributed is 20 km and there are 4 RSs in the cell. For each given location of users, shadowing is assumed log-normal and spatially uncorrelated with a standard deviation of 10 dB. The model for propagation loss used in the simulations is the large city and metropolitan area COST 231-Hata model [15]. The channel between the BS and RSs is line-of-sight. The height of the BS is set to 50 m, the height of the RS...
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