An optically-interrogated microwave-Poynting-vector sensor using cadmium manganese telluride

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Abstract: A single <110> cadmium-manganese-telluride crystal that exhibits both the Pockels and Faraday effects is used to produce a Poynting-vector sensor for signals in the microwave regime. This multi-birefringent crystal can independently measure either electric or magnetic fields through control of the polarization of the optical probe beam. After obtaining all the relevant electric and magnetic field components, a map of the Poynting vector along a 50-Ω microstrip was experimentally determined without the need for any further transformational calculations. The results demonstrate that this sensor can be used for near-field mapping of the Poynting vector. Utilizing both amplitude and phase information from the fields in the microwave signal, it was confirmed for the case of an open-terminated microstrip that no energy flowed to the load, while for a microstrip with a matched termination, the energy flowed consistently along the transmission line.

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References

1. Introduction

In recent years, the behavior of near-field electromagnetic (EM) signals, often involving the evanescently decaying waves common to plasmonic devices, has attracted considerable attention as efforts to understand and improve metamaterial structures has increased [1, 2]. Moreover, given appropriate knowledge of the EM-wave behavior, some applications, like invisibility cloaking and superlensing, have been realized through controlling propagation of the Poynting vector [3, 4]. Currently, the most commonly used numerical methods to study Poynting-vector propagation are the finite-difference time-domain (FDTD) technique and the finite-element method (FEM) [5–7]. These methods, however, often reveal discrepancies between the simulated and actual behavior of real devices. Thus, experimental measurement of the Poynting vector remains an active research topic for any device that radiates energy [8,9].

Even without considering their intrusiveness, one serious limitation of existing field-measurement techniques is that they only detect either the electric or the magnetic-field component, and hence further transformational calculation is necessary to estimate the real Poynting vector of a signal [10–12]. In particular, when one is in the near field where the electric and magnetic fields are not orthogonal, it is not sufficient to determine the Poynting vector solely through knowledge of either the electric or magnetic field, but rather it is necessary to know the characteristics of both. In this paper, we report on the development of a noninvasive Poynting-vector probe that does not require knowledge of the physical properties of the device-under-test, whereas typical measurement techniques require the permittivity or dimensions of the device-under-test to execute the necessary transformational calculations.

Here, cadmium manganese telluride (CMT) has been used to extract Poynting-vector maps at microwave frequencies in proximity to a microstrip transmission line without the need for transformational calculations. CMT is a diluted magnetic semiconductor with a strong exchange interaction between the d spins of its manganese ions and its electrons. Given these properties, a circular birefringence can be induced on an optical beam due to a relatively strong Zeeman splitting caused by a small applied magnetic field [13, 14]. As a noncentrosymmetric crystal, CMT also exhibits the Pockels effect, allowing a linear birefringence to be induced on an appropriately polarized input optical beam. Therefore, CMT may be used to independently measure either electric or magnetic fields if the linear polarization angle of an incident optical beam can be adjusted precisely [15]. Considering the definition of the Poynting vector, if both electric and magnetic field can be measured by a...
single sensor, it is possible to experimentally obtain the energy flow on a local scale and in
the near field.

2. Poynting vector

From the Poynting theorem, the density of power carried by electromagnetic waves through a
surface, also called the Poynting vector (S), is defined as

\[ S = \frac{1}{2} \mathbf{E} \times \mathbf{H}^* \]  \hspace{1cm} (1)

where \( \mathbf{E} \) represents the electric field and \( \mathbf{H} \) the magnetic field, and the asterisk denotes the
complex conjugate. Equation (1) shows that only mutually perpendicular components of \( \mathbf{E} \)
and \( \mathbf{H} \) contribute to power flow; the direction of the flow is normal to the plane containing \( \mathbf{E} \)
and \( \mathbf{H} \). Thus, in rectangular coordinates, the complex Poynting vector per unit area normal to
the y-z plane can be written as

\[ S_\perp = \frac{1}{2} (E_x H_y^* - E_y H_x^*) \]  \hspace{1cm} (2)

According to Eq. (2), if the CMT can be used to measure the four separate components of
the electric fields and magnetic fields indicated, it would allow the Poynting vector of the
device-under-test to be extracted at a particular point in space.

Measuring the electric and magnetic fields separately by applying one electro-optic (EO)
crystal and one magneto-optic (MO) crystal, such as, for example, zinc telluride and terbium
gallium garnet, may not provide an accurate Poynting vector value [16, 17]. This error can
arise due to different field perturbations that occur when the refractive indices, \( n \), of the
crystals differ [18]. This is because the field inside a dielectric is weighted by the index of the
medium through

\[ E_{\text{crystal}} = \frac{E_{\text{air}}}{\sqrt{n}} \]  \hspace{1cm} (3)

where Eq. (3) assumes that there are no reflection and refraction losses when the field enters
the crystal. Therefore, from Eq. (3), it is obvious that the measured electric field is less than
the actual radiating electric field in the air by a factor of the square root of \( n \). Since the
magnetic field is also perturbed by the crystal following the same trend as the electric field,
the measured Poynting vector is less than the actual Poynting vector by a factor of \( n \). In other
words, the actual Poynting vector value can be found by utilizing the same crystal, CMT, if
the measured electric and magnetic field strengths are multiplied by a constant, the refractive
index of CMT at the operating frequency of the device-under-test.

3. Experimental setup

A single, <110>-oriented CMT crystal that was doped with 25% manganese (Cd\textsubscript{0.75}Mn\textsubscript{0.25}Te)
was used as the sensing medium in this study. The linear electro-optic coefficient, \( r_{41} \) for
CMT at radio frequency (RF) is 3.5 ± 0.2 pm/V. At 800 nm optical wavelength, the Verdet
constant of CMT is 2.2 min/Gs-cm and the refractive index is 2.82. The CMT crystal of 1 mm
thickness was coated with a high-reflection dielectric layered stack on one side. This coating
was employed so that when the sensor crystal was oriented to measure the electric field out of
the plane of the microstrip surface and also the magnetic field within this plane, the optical
pulses could be easily reflected back to the probe-beam photodetector (Fig. 1). A mixing
process within the nonlinear-optical crystal converts the continuous-wave RF signal to be
measured to a lower frequency using a harmonic of the 80-MHz repetition frequency of an
ultrafast, pulsed laser as a local oscillator. Eventually only the down-mixed version of the
high-frequency signal, with signal amplitude and phase matching that of the input RF, is
output from the photodetector into a lock-in amplifier. The experimental implementation is shown in Fig. 1, where the CMT with the high reflection coating can be seen to reflect the probe beam, and the pellicle has a 50/50 transmission/reflection ratio. (A detailed description of the sideband-harmonic-mixing method has appeared elsewhere [19].)

![Fig. 1. Experimental configuration for the Poynting-vector measurement using a CMT sensor coated with a high reflection dielectric stack.](image)

The dashed-box in Fig. 1 includes the CMT and the device-under-test (depicted as the source of the electromagnetic (EM) fields), which was a 50-Ω microstrip transmission line. In order to measure all necessary field components, the CMT is mounted in two different ways as shown in Fig. 2. The vertically mounted CMT in Fig. 2(a) can measure the E\(_z\) or H\(_y\) field components since the <110> face is along the x-z plane. On the other hand, the horizontally mounted CMT in Fig. 2(b) can measure the E\(_y\) or H\(_z\) field. Combining the results from Fig. 2(a) and (b) using Eq. (2) will yield the complete Poynting-vector amplitude and direction. Since the Poynting vector should be different for the 50-Ω microstrip transmission line with an open termination and one with a matched load, both conditions are measured in Fig. 2(a) and (b) to help substantiate the measurement capability.

![Fig. 2. Experimental Setup for measuring (a) E\(_z\) or H\(_y\) field component and (b) E\(_y\) or H\(_z\) field components of a 50-Ω microstrip transmission line by using the same CMT crystal.](image)
The 50-Ω microstrip was 14 cm long with a 4-mm-wide top strip, and it was constructed from copper electrodes on an epoxy resin substrate. The fields just above the microstrip were mapped by raster-scanning the transmission line using a computer-controlled translation stage, while the CMT, mounted independently of the microstrip, remained stationary with its c-axis oriented vertically or horizontally above the microstrip. The free-space probe beam of 80-fs pulses from a mode-locked Ti:sapphire oscillator was focused through the crystal with a 1 mm beam diameter in both Fig. 2 (a) and (b). The CMT was slightly suspended above the transmission line for both probe orientations so that the optical beam intersected the CMT at a height of 300 µm above the plane of the microstrip. Since the optical beam illuminated closely to the bottom of the CMT crystal in Fig. 2 (a), both Fig. 2 (a) and (b) measured the average field strength inside the CMT crystal with the same 1 mm³ volume. In this study, the 1-mm-diameter spot size of the probe beam limits the spatial resolution, although a spatial resolution of 8 µm has been achieved in a purely electro-optic field-mapping application [19]. The measurements from the fringing EM fields were periodically obtained as the microstrip was stepped in 100 µm intervals, while the optical probe beam also remained in a fixed position relative to the stationary CMT probe. Each scan is 5 cm by 2 cm with the center at the middle of the transmission line, and 4.403 GHz was used as the RF input.

For EO sensing, a quarter waveplate with its slow axis oriented 45° relative to a crossed input polarizer and output analyzer was used, and the input optical polarization was set 45° relative to the in-plane crystal principal axes. Thus, CMT measures only the $E_z$ and $E_y$ field of the transmission line as shown in Fig. 2(a) and (b), respectively. As for MO sensing, the linear polarization angle of the laser was made parallel to one of the induced principal axes of the CMT, and thus in this case the crystal could be used to measure only the magnetic field of the transmission line. A half-waveplate and linear polarizer that change the input linear polarization angle were used before the probe beam propagating through the CMT. The analyzer was placed at 45° relative to the polarizer after the probe beam reflected back from the CMT, allowing the MO signal to be maximized according to Malus’ Law.

4. Experimental results

Figure 3 shows the measured $E_z$ electric field when the CMT was mounted out of the plane of the microstrip, as in Fig. 2(a). The $E_z$ electric-field amplitudes for part of the area over the microstrip for both the open-termination and matched-load conditions are plotted in Fig. 3(a) and (b), respectively using the same amplitude scale. Along the center strip of the transmission line, a standing-wave pattern with three nodes at 4.8 mm, 25 mm, and 46 mm can be seen in Fig. 3(a), as indicated by the large periodic changes in field amplitude. A travelling wave is clearly observed in Fig. 3(b), where there is little modulation of the electric field along the strip. The periodic small field peaks that do appear in Fig. 3(b) arise from the imperfect match of the load to the line impedance. Similarly, the $E_z$ electric field phases for the open-termination and matched-load conditions are plotted on a scale from −180 degrees to + 180 degrees in Fig. 3(c) and (d), respectively. Again looking along the center strip of the transmission line, the phase in Fig. 3(c) alternates 180 degrees with the adjacent rectangular parts of the field pattern, indicating that the amplitude in Fig. 3(a) represents peaks and valleys in the standing wave generated by the open termination. In contrast, the phase in Fig. 3(d) varies from −180 degrees to 180 degrees continuously for both the field over the top strip as well as outside the strip, demonstrating again that the amplitude in Fig. 3(b) represents a travelling wave allowed by the matched-load termination.
Setting the conditions in Fig. 2(a) to MO sensing by rotating the input linear polarization by 45 degrees and removing the quarterwave plate, the measured $H_y$ magnetic field is obtained as shown in Fig. 4. The $H_y$ magnetic field amplitudes for the open-termination and matched-load conditions are plotted with the same amplitude scale in Fig. 4(a) and (b), respectively. Here, a standing-wave pattern with two nodes at 14.8 mm and 35.4 mm can be seen in Fig. 4(a), whereas a travelling wave is clearly present in Fig. 3(b). Compared to Fig. 3(a), Fig. 4(a) can be seen to exhibit the magnetic field only, since the node positions of the magnetic field correspond to the peak positions of the electric field. The $H_y$ magnetic-field phase for the open termination and matched load plotted in Fig. 4(c) and (d), respectively, have similar behaviors compared to Fig. 3(c) and (d), where the phase changes 180 degrees from the adjacent peak for the open termination, and the phase changes continuously along the transmission line for the matched load. The signal-to-noise ratio (SNR) for the maximum magnetic field measured was ~30 dB, while that for the electric field was ~65 dB. The difference in magnitude of the SNR values is expected because of the wave impedance of the guiding structure supporting the fields. The dynamic range for both of the EO and MO sensing techniques is expected to be somewhat larger than these values, as the linearity of the measurements will be essentially maintained for fields up to, say, 10% of the half-wave field of the modulator embodiments. Thanks to the narrow-band detection methods employed and the extremely low noise floors achievable for cw RF measurements, an analysis yields dynamic ranges for both EO and MO field-mapping that are on the order of 120 dB [20].
Fig. 4. Experimental results of $H_y$ amplitude in (a) open termination and (b) matched-load termination; $H_y$ phase in (c) open termination and (d) matched-load termination.

Possessing the measured data of Figs. 3 and 4, the first term on the right hand side of Eq. (2) can be used to calculate a partial experimentally determined Poynting vector, $S_x$. Multiplying the amplitudes of the $E_z$ electric field in Fig. 3 and the $H_y$ magnetic field in Fig. 4, the partial $S_x$ amplitude for the open-termination and matched-load conditions are plotted with the same amplitude scale in Fig. 5(a) and (b), respectively. At the edge of the transmission line, for the distances 7 mm and 13 mm on the vertical scale, there is no electric field that exists along the $z$ direction, and no magnetic field along the $y$ direction. Therefore, the partial Poynting vector shows a discontinuity. Subtracting the phases of the $H_y$ magnetic field in Fig. 4 from the phase of the $E_z$ electric field in Fig. 3, the partial $S_x$ phase for the open and matched loads are plotted from $-180$ degrees to $180$ degrees in Fig. 5(c) and (d), respectively. The phase alternates $180$ degrees at the nodes of the electric and magnetic field, which further substantiates that the energy does not flow along the open-terminated transmission line. On the other hand, for the matched load, the energy appears to flow along the transmission line with a phase that remains essentially constant. The phase in Fig. 5(d) does vary slightly due to the imperfect load match.
As their trends follow those already shown for $E_z$ and $H_y$, the measured field-pattern results for $E_y$ and $H_z$ are not given here. Both the amplitudes for the $E_y$ electric and $H_z$ magnetic fields are strongest at the edge of the transmission line. Compared to Fig. 3(a), a standing wave pattern of $E_y$ with three nodes at 4.8 mm, 25 mm, and 46 mm can also be observed. Similarly, a standing wave pattern of $H_z$ with two nodes at 14.8 mm and 35.4 mm can also be observed. The field amplitudes at the center of the transmission line, however, show discontinuity since most of the electric and magnetic fields are $E_z$ and $H_y$. The Poynting vector sum, $S_x$, is plotted in Fig. 6 by combining the partial Poynting vectors from Fig. 5 and the product of $E_yH_z^*$ (not shown). The amplitude distribution clearly indicates that the “matched” load condition in Fig. 6(b) allows energy to flow along the microstrip in the $x$ direction, whereas the open-terminated transmission line in Fig. 6(a) has no continuous energy flow. The results in Fig. 6 quantitatively match a FEM simulation (run with ANSOFT HFSS software). Both simulation and experiment show that the Poynting vector of the open-terminated microstrip has a standing wave pattern that is strongest at the edge of the transmission line, with the Poynting vector as one moves away from the center of the top strip of the transmission line decreasing continuously for the case with the matched load.
In order to better quantify the Poynting-vector maps and to solidify how best to understand their significance, the phase (blue curve and circles) and amplitude (black curve and squares) along the center of the microstrip line are plotted in Fig. 7. Figure 7(a) shows that the phase of the measured Poynting vector changes by 180 degrees for each of the adjacent peaks, which is representative of a standing wave pattern with no energy flow along the x direction. The 180° phase difference appears at the positions of the electric and magnetic fields nodes in Figs. 3 and 4. On the other hand, a propagating Poynting vector along the transmission line with a perfect matched load should have a constant phase and uniformly distributed amplitude. Although the 50-Ω load has exhibited previously above that it is not perfectly matched to the microstrip, the traces for the matched-load termination in Fig. 7(b) show that the Poynting vector has a relatively uniform amplitude and only a slightly varying phase along the x direction.

5. Conclusions

In this paper, a single <110> CMT crystal with a high-reflection coating on one side has been employed as a Poynting vector sensor. The use of this multi-birefringent CMT crystal demonstrates the ability to independently measure the electric and magnetic fields by precisely controlling the polarization angle of the probe beam. Mounting the CMT properly allows for the measurement of all components of an electric and a magnetic field. When all of the electric and magnetic field components are obtained, the energy flow of an EM wave can be subsequently mapped out without any transformational calculations based on the definition.
of the Poynting vector. The energy-flow conditions for a \(~50\Omega\) microstrip transmission line in two different situations, the open-termination and matched-load cases, have been successfully mapped. The open-termination case shows no energy flow according to both the acquired amplitude and phase information, whereas the matched-load case indicates energy flow along the microstrip transmission line.

The transmission spectrum of this CMT crystal in the terahertz (THz) electromagnetic wave region has also been measured via time-domain THz spectroscopy, with the result revealing a lack of resonances for frequencies below 2 THz. Therefore, CMT can presumably also be employed as a Poynting-vector probe for THz electromagnetic waves. Refinement of this technique, such as with a fiber based probe, could also be used for the near-field measurement of any device that radiates RF-to-THz energy.