Morphological contrast measure and contrast enhancement: One application to the segmentation of brain MRI

Jorge D. Mendiola-Santibañez\textsuperscript{a,}\textsuperscript{*}, Iván R. Terol-Villalobos\textsuperscript{b}, Gilberto Herrera-Ruiz\textsuperscript{a}, Antonio Fernández-Bouzas\textsuperscript{c}

\textsuperscript{a}Doctorado en Ingeniería, Universidad Autónoma de Querétaro, 76010 Querétaro, México
\textsuperscript{b}CIDETEQ, S.C., Parque Tecnológico Querétaro S/N, San Fandilla-Pedro Escobedo, 76700 Querétaro, México
\textsuperscript{c}Instituto de Neurobiología, UNAM Campus Juriquilla, 76001 Querétaro, México

Received 19 March 2006; received in revised form 20 December 2006; accepted 20 February 2007
Available online 4 March 2007

Abstract

In this paper a morphological contrast measure is introduced. The quantification of the contrast is based on the analysis of the edges, which are associated with substantial changes in luminance. Due to this, the contrast measure is used to detect the image that presents a high visual contrast when a set of output images is analyzed. The set of output images is obtained by application of morphological contrast mappings with size criteria. These contrast transformations are defined under the notion of partitions generated by the set of flat zones of the image; therefore, they are connected transformations. In addition, an application to the segmentation of white and grey matter in brain magnetic resonance images (MRI) is provided. The detection of white matter is carried out by means of a contrast mapping with specific control parameters; subsequently, white and grey matter are separated and their ratio is calculated and compared with manual segmentations. Also, an example of segmentation of white and grey matter in MRI corrupted by 5% noise is presented in order to observe the performance of the morphological transformations proposed in this work.

© 2007 Elsevier B.V. All rights reserved.

Keywords: Morphological contrast measure; Contrast mappings; Connected transformations; MRI segmentation

1. Introduction

In mathematical morphology (MM) contrast enhancement is based on morphological contrast mappings as described by Meyer and Serra [1,2].

\textsuperscript{*}Corresponding author. Tel./fax: +52.442.192.1200x6023.
E-mail addresses: mendijor@uaq.mx
(J.D. Mendiola-Santibañez), famter@ciateq.net.mx
(I.R. Terol-Villalobos), gherrera@uaq.mx (G. Herrera-Ruiz),
fabouzas@servidor.unam.mx (A. Fernández-Bouzas).

The main idea of these transformations is to compare each point of the original image with two primitives; as a result, the nearest value with respect to the original image is selected. The reported primitives in the literature can be openings and closings [1,2], erosions and dilations [3,4], or white and black top-hats [3,5], in addition such primitives can be connected or morphological. In Fig. 1, two primitives, the original function and the final result of certain contrast mapping are illustrated.

0165-1684/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.sigpro.2007.02.008
On the other hand, a special class of contrast mappings denominated morphological slope filters (MSFs) was introduced in [4,6,7]. The images processed by MSFs have a well-defined contrast; this is so because in each point of the output image, the gradient value is greater than the filter parameter or has a zero value. The original idea of this proposal from a practical point of view, is to modify the gradient image by working on the original image, without imposing markers, as is the case of the watershed transformation [8].

Subsequently, a family of sequential MSFs was introduced in [6,7]. The application of sequential MSFs allows a better control on the image contrast; however, the major drawback of using these filters can be seen around contours, since new information is generated in some configurations of blurred edges. In Fig. 2, we observe this behavior; here “p” is a high contrast point. In Fig. 2(b) the high contrast contour is preserved; while a new edge appears during the processing because the flat zone (region with the same grey level) is broken.

Due to this situation, in [9] a class of connected MSF was proposed. These contrast transformations involve the connectivity concept [10–12], i.e., they preserve or remove connected components (flat zones) without generating new contours during the

---

**Nomenclature**

- \( \mu, \lambda, \alpha, \beta \): scalars (i.e. positive numbers)
- \( B \): structuring element
- \( A, E, X, Z \): euclidean or digital space under study
- \( Z^n \): euclidean space
- \( x, y \): points in \( Z^n \)
- \( \emptyset \): set of all subsets of \( Z^n \)
- \( P(x) \): element of the partition containing \( x \)
- \( f(x) \): numerical function of \( x \)
- \( C \): connected set
- \( F_x(f) \): flat zone of a function \( f \) at point \( x \)
- \( (f', P_f)(x) \): grey level value of the connected component obtained by \( F_x(f) \)
- \( e_\mu(f, P_f)(x) \): erosion on the partition of size \( \mu \) induced by \( f \)
- \( \delta_\mu(f, P_f)(x) \): dilation on the partition of size \( \mu \) induced by \( f \)
- \( \gamma_\mu(f, P_f)(x) \): opening on the partition of size \( \mu \) induced by \( f \)
- \( \varphi_\mu(f, P_f)(x) \): closing on the partition of size \( \mu \) induced by \( f \)

\( \text{grad}_\mu(f, P_f)(x) \): morphological gradient on the partition of size \( \mu \) induced by \( f \)

\( \text{Thw}_\mu(f, P_f)(x), \text{Thb}_\mu(f, P_f)(x) \): white and black top-hat on the partition of size \( \mu \) induced by \( f \)

\( \rho(x) \): proximity criterion

\( W_{\lambda, \alpha, \beta}^z(f, P_f)(x) \): morphological three states contrast mappings with size criteria on the partition induced by \( f \)

\( \eta_\mu(f, P_f)(x) \): opening spectrum on the partition induced by \( f \)

\( \text{Thwb}_{\lambda, \alpha, \beta}(f, P_f)(x) \): self-complementary top-hat

\( \text{vol}(\xi, \mu, \lambda) \): ratio between the volume of the top-hat spectrum and the total of the bright and dark regions

\( C, L \): contrast and luminance

\( VC_p \): variation of contrast in function of certain parameters \( p \)

\( e_\mu(f)(x) \): morphological erosion of size \( \mu \)

\( \delta_\mu(f)(x) \): morphological dilation of size \( \mu \)

\( \hat{B} \): transposed of the structuring element \( B \), i.e., \( \hat{B} = \{ -x \cdot x \in B \} \)

\( \tilde{\gamma}_\tau(f) \): opening by reconstruction of size \( \tau \)

\( \tilde{\varphi}_\tau(f) \): closing by reconstruction of size \( \tau \)

---

Fig. 1. Example of contrast mapping.
processing. Here, the flat zone and partition concepts are fundamental to introduce the connectivity notion, as well as the basic morphological transformations at partition level in the numerical case [9].

Once the morphological transformations at partition level were defined, the morphological contrast mappings with size criteria were introduced as connected transformations in [14,15]. Here, the sizes of the primitives (opening and closing on the partition of size $\mu$) were considered variable parameters. This originates a problem, since adequate values for the sizes of the primitives and the parameters involved in the proximity criterion [1] must be calculated. In particular, in this class of contrast mappings, the sizes of the primitives are a very important point, because the proximity criterion is obtained as a function of the primitives (opening and closing on the partition of size $\mu$). In addition, the proximity criterion compares a function $f$ with the primitives of variable size; in a way similar to that proposed in [1].

In this paper, the problem of obtaining adequate sizes for the primitives associated in the morphological contrast mappings is solved. In order to compute such parameters, a morphological contrast measure, as well as a method that works similarly to the granulometric density [16–19] is employed.

On the other hand, in the literature, several methodologies have been provided to quantify the contrast in the spatial domain. For example, in Morrow et al. [20], the contrast in an image is measured by using the mean grey values in two rectangular windows centered on a given pixel. In [21], Peli briefly describes other examples of contrast measures, among them, the root mean square (rms) contrast, which is used to compare the contrast of two different images employing a statistical method. From the point of view of visual contrast, the Weber–Fechner law is a psychophysical model widely used to quantify the contrast in accordance to human visual perception [21–26].

In our case, the morphological contrast measure is obtained from a local analysis of the enhanced image trying to detect important changes in luminance in a way that is psychophysically valid, i.e., representative of the apparent or perceived contrast. In this way, luminance is associated with the contours of the image, since changes in the contours produce modifications in the contrast. Moreover, the morphological contrast measure introduced in this work will be useful to select adequate parameters involved in morphological contrast mappings with size criteria. About this proposal, there are two main aspects to mention. First, notice that in MM several transformations have been proposed in order to enhance the contrast, for example, morphological contrast mappings [1–3,5,7], morphological slope filters [4,6], morphological center [3,27], and morphological top-hat [28], among others. However, there is not a defined morphological contrast measure capable of quantifying contrast in the processed images.

Second, although the study is directed to obtain some parameters involved in morphological contrast mappings with size criteria, an important contribution of this work is the introduction and formalization of the method to quantify contrast from the point of view of mathematical morphology.

On the other hand, the proposals given in this paper will be useful to perform the segmentation of white and grey matter in a frontal lobule area from slices of brain magnetic resonance images (MRI). In order to better appreciate the segmentation of white and grey matter, some MRI slices belonging to a simulated brain database are segmented; these analyzed slices have the characteristic of being corrupted by the presence of 5% noise [29].

Finally, this paper is organized as follows. In Section 2, a background of different transformations and concepts related to mathematical morphology is presented. The connectivity notion,
the flat zone concept and the basic transformations on the partition are provided in Section 2.1. In Section 2.3, morphological contrast mappings are defined at partition level. A method to obtain adequate sizes for the closing involved in contrast mappings on the partition is presented in Section 3. A method to quantify the contrast in the processed images is introduced in Section 4. A practical example is given in Section 5.1.3, in which the grey and white matter are segmented in a frontal lobe area from brain MRI. While in Section 5.2, a segmentation of white and grey matter is carried out on slices of brain MRI in the presence of noise. In Section 6, a brief explanation about the computing complexity of the transformations on the partition is discussed.

2. Background on morphological connected transformations

2.1. Connectivity and connected transformations

Serra [27] established connectivity by means of the connected class concept.

Definition 1 (Connected class). A connected class $C$ in $\varphi(E)$ is a subset of $\varphi(E)$ such that: (i) $\emptyset \in C$; (ii) $\forall x \in E, \{x\} \in C$; (iii) for each family $\{C_i\}$ in $C$, $\bigcap C_i \neq \emptyset \Rightarrow \bigcup C_i \in C$.

where $\varphi(E)$ represents the set of all subsets of $E$. An element of $C$ is called a connected set. An equivalent definition to the connected class notion is the opening family expressed by the next theorem [27].

Theorem 1 (Connectivity characterized by openings). The definition of a connectivity class $C$ is equivalent to the definition of a family of openings $\{\gamma_x, x \in E\}$ such that: (a) $\forall x \in E, \gamma_x(x) = \{x\}$; (b) $\forall x, y \in E$ and $A \subseteq E, \gamma_x(A) = \gamma_y(A)$ or $\gamma_x(A) \cap \gamma_y(A) = \emptyset$; (c) $\forall A \subseteq E \ and \ \forall x \in E, \forall x \notin A \Rightarrow \gamma_x(A) = \emptyset$.

When the transformation $\gamma_x$ is associated with the usual connectivity (arcwise) in $Z^2$ ($Z$ is the set of all integers), the opening $\gamma_x(A)$ can be defined as the union of all paths containing $x$ that are included in $A$. When a space is equipped with $\gamma_x$, the connectivity can be expressed using this operator. A set $A \subset Z^2$ is connected if and only if $\gamma_x(A) = A$.

In Fig. 3, the behavior of this opening is illustrated. The connected component of the input image $X$ where point $x$ belongs (Fig. 3(a)) is the output of the opening $\gamma_x(X)$, while the other components are eliminated (Fig. 3(b)).

In order to introduce the morphological contrast mappings with size criteria in the following subsection, the next definitions are presented [9,15,30]:

Definition 2. Given a space $E$, a function $P : E \rightarrow \varphi(E)$ is called a partition of $E$ if: (a) $x \in P(x)$, $x \in E$; (b) $P(x) = P(y)$ or $P(x) \cap P(y) = \emptyset$ with $x, y \in E$.

$P(x)$ is an element of the partition containing $x$. If there is a connectivity defined in $E$ and, $\forall x$ the component $P(x)$ belongs to this connectivity, then the partition is connected.

Definition 3. The flat zones of a function $f : Z^2 \rightarrow Z$ are defined as the connected components (largest) of points with the same value of the function.

Fig. 3. Extraction of connected components. (a) Binary image $X$, (b) the opening $\gamma_x(X)$ extracts the connected component in $X$ where point $x$ belongs.
The operator $F_x(f)$ will represent the flat zone of a function $f$ at point $x$.

**Definition 4.** An operator is connected if and only if it extends the flat zones of the input image.

The term “extends” in latter definition means that the flat zones of the image are enlarged during the processing by merging contiguous flat zones.

**Definition 5.** Let $x$ be a point of $Z^2$ equipped with $\gamma_x$. Two flat zones $F_x(f)$ and $F_y(f)$ in $Z^2$ are adjacent if $F_x(f) \cap F_y(f) = \gamma_x(F_x(f) \cup F_y(f))$.

**Definition 6.** Let $x$ be a point in $Z^2$ equipped with $\gamma_x$. The set of flat zones $A_x$ adjacent to the component extracted by $F_x$ is given by $A_x(f) = \{F_x(f) \cap F_{x'}(f) = \gamma_x(F_x(f) \cup F_{x'}(f))\}$.

In Definition 6, the element extracted by $F_x(f)$ belongs to the set of adjacent flat zones, since $A_x(f)$ fulfills the criterion of being reflexive.

The notion of adjacent flat zones is illustrated in Fig. 4. The original image is located in Fig. 4(a). In Figs. 4(b) and 4(c) two adjacent flat zones are shown, while in Fig. 4(d), the adjacency is exemplified according to the expression $F_x(f) \cup F_y(f) = \gamma_x(F_x(f) \cup F_y(f))$.

The introduction of the morphological transformation using the flat zone notion for the numerical case is provided in [9]. In [9] a new representation for the grey level image was provided. Such representation is given by the original function $f$ and the partition $P_f$ induced by $f$ using the notion of flat zone. This implies that the operators are going to act on the pair $(f, P_f)$, where the element $(f, P_f)(x)$ is taken as the grey level value of the connected component obtained by $F_x(f)$. Thus, the morphological dilation and erosion on the partition induced by $f$ are given by [9]:

$$
\delta(f, P_f)(x) = \max\{(f, P_f)(y): F_y \in A_x\}, \quad (1)
$$

$$
\varepsilon(f, P_f)(x) = \min\{(f, P_f)(y): F_y \in A_x\}. \quad (2)
$$

Fig. 4. Adjacent flat zone concept. (a) Image $f$ with 14 flat zones; (b) flat zone in point $x$, $F_x(f)$; (c) flat zone in point $y$, $F_y(f)$; and (d) two adjacent flat zones, i.e., $F_x(f) \cup F_y(f) = \gamma_x(F_x(f) \cup F_y(f))$. 
The dilation $\delta_\mu$ and erosion $\varepsilon_\mu$ of size $\mu$ are obtained iterating $\mu$ times the elemental dilation and erosion given in Eqs. (1) and (2):

$$\delta(f, P_f)(x) = \delta \cdot \delta(f, P_f)(x),$$

$$\varepsilon(f, P_f)(x) = \varepsilon \cdot \varepsilon(f, P_f)(x).$$

The opening and closing on the partition of size $\mu$ induced by $f$ are:

$$\gamma_\mu(f, P_f)(x) = \delta_\mu(\varepsilon(f, P_f), P_f)(x),$$

$$\varphi_\mu(f, P_f)(x) = \varepsilon_\mu(\delta(f, P_f), P_f)(x).$$

The morphological gradient transformations of size $\mu$ on the partition are presented as follows:

$$\text{gradm}_\mu(f, P_f)(x) = \delta_\mu(f, P_f)(x) - \varepsilon_\mu(f, P_f)(x).$$

On the other hand, the top-hat transformation was proposed by Meyer [28]. This operator allows the detection of peaks (respectively, valleys) of certain height (respectively, depth) and certain thickness, having also granulometric (anti-granulometric) characteristics. This approach allows the classification of regions of the image by size and height. The top-hat transformation is divided in white and black top-hat. In the case of dealing with clear regions the white top-hat is employed, and it is obtained as the subtraction between the original image and the opening transformation. Whereas for the dark regions the black top-hat is used, this transformation is determined by the subtraction between the closing transformation and the original image. As follows, top-hat expressions build with the opening and closing on the partition are presented:

$$\text{Thw}_\mu(f, P_f)(x) = (f, P_f)(x) - \gamma_\mu(f, P_f)(x),$$

$$\text{Thb}_\mu(f, P_f)(x) = \varphi_\mu(f, P_f)(x) - (f, P_f)(x).$$

Generally, transformations (8) and (9) are followed by a thresholding operation in order to obtain a binary image containing certain bright structures with a given size and contrast. Indeed, the top-hat approach leads to a size distribution involving contrast of the image. This contrast operator is widely used in image segmentation [8,28].

### 2.2. Transformations by reconstruction

Into MM there are defined two connected morphological filters called opening and closing by reconstruction. These morphological filters have next characteristics [31,32]: (i) the generation of new features is avoided; and (ii) several details are eliminated without considerably modifying the remainder structures of the image.

When filters by reconstruction are built, the basic geodesic transformations, the geodesic erosion and the geodesic dilation of size 1 are iterated until idempotence is reached [32]. The geodesic dilation $\delta^1(g)$ and the geodesic erosion $\varepsilon^1(g)$ of size one are given by $\delta^1(g) = f \land g$ with $g \leq f$ and $\varepsilon^1(g) = f \lor g$ with $g \geq f$, respectively. When the function $g$ is equal to the erosion or the dilation of the original function, we obtain the opening and the closing by reconstruction [31,32], i.e.:

$$\tilde{\gamma}(f) = \lim_{n \to \infty} \delta^n(\varepsilon(f)) \quad \text{and} \quad \tilde{\varphi}(f) = \lim_{n \to \infty} \varepsilon^n(\delta(f)),$$

where the morphological erosion $\varepsilon_B$ and dilation $\delta_B$ are expressed by

$$\varepsilon_B(f)(x) = \lor\{f(y) : y \in B\}$$

and

$$\delta_B(f)(x) = \lor\{f(y) : y \in B\}.$$
used in separate ways, i.e., two contrast mappings are built. One of them uses the original function and the morphological erosion, while the other employs the original function and the morphological dilation. Due to this, the degradation in the output image is less marked, than if it were processed by the Kramer and Bruckner transformation. In morphological slope filters, the proximity criterion corresponds to a criterion given in function of the internal and external morphological gradients.

However, when the erosion or dilation is used as primitive to build contrast mappings, a risk exists of degrading the processed image. This problem was considered by Serra [2], who solved it by using idempotent transformations as primitives. But, in the case of working at pixel level, if the size of the structuring element is large, the use of idempotent transformations does not ensure that the degradation in the processed image will be avoided, unless the primitives are connected transformations. This situation is illustrated in [5]. Here the opening and closing by reconstruction [10] are utilized as primitives, but instead of using a proximity criterion for selecting the primitives, a contrast criterion given by the top-hat transformation is used.

In this work, contrast mappings have the characteristic of being connected, since the transformations employed are defined on the partition generated from the set of flat zones of the image. As follows, three-state contrast mappings with size criteria on the partition induced by $f$ are considered. These transformations are composed of three primitives, opening on the partition, closing on the partition and the original image (see Eqs. (5) and (6). The proximity criterion [1] presented in Eq. (11) considers the bright and dark regions of the image:

$$\rho(x) = \frac{\varphi_2(f, P_f)(x) - (f, P_f)(x)}{\varphi_2(f, P_f)(x) - \gamma_\mu(f, P_f)(x)}.$$  \hspace{1cm} (11)

Eq. (12) establishes a three-state contrast mapping with size criteria on the partition

$$W_{\beta, \mu, \sigma}^3(f, P_f)(x) = \begin{cases} 
\varphi_2(f, P_f)(x), & 0 \leq \rho(x) < \beta, \\
(f, P_f)(x), & \beta \leq \rho(x) < \alpha, \\
\gamma_\mu(f, P_f)(x), & \alpha \leq \rho(x) \leq 1.
\end{cases}$$  \hspace{1cm} (12)

Images in Fig. 6 illustrate the performance of some contrast mappings mentioned above. The original image is displayed in Fig. 6(a), while the output image in Fig. 6(b) corresponds to the Kramer and
Bruckner transformation after 20 iterations at pixel level. The image in Fig. 6(c) is obtained after applying MSF with slope $\phi = 40$ at pixel level; the image processed with the morphological contrast mapping on the partition is presented in Fig. 6(d). In this case the parameters are $\mu = 5$, $\lambda = 7$, $\alpha = 0.388$, and $\beta = 0.686$ at the partition level. (c) image contours of (a); (f) image contours of (b); (g) image contours of (c); (h) image contours of (d); (i), (j), (k) and (l) thresholding operation between 95–255 sections of image in Figs. 6(e–h).

The former situation occurs because the employed transformations are connected. On the other hand, notice from Eqs. (11) and (12) that there are four parameters to be determined: $\mu$, $\lambda$, $\alpha$ and $\beta$. In this work, the parameters $\mu$ and $\lambda$ are obtained by a graphic method described in Section 3, whereas suitable values for the parameters $\alpha$ and $\beta$ are obtained from a quantifying contrast method introduced in Section 4.

3. Opening and closing size determination

In the case of working at partition level, the notion of the structuring element disappears (see Eqs. (1)–(12)). In particular, it is necessary to propose a method that enables to know the sizes of the homotetic parameters $\mu$ and $\lambda$ involved in the proximity criterion and the contrast mappings as expressed in Eqs. (11) and (12). In these equations,
the sizes of opening and closing on the partition must be calculated for every particular case.

In this work, a graphic method is employed in order to find adequate values for the homotetic parameters. The graphic method consists of drawing information from the top-hat spectrum which is obtained from the opening spectrum concept. The opening spectrum is the image sequence created by computing the difference between successive openings fulfilling the granulometry definition [28,36–38], thus the opening spectrum on the partition is:

$$\eta_{\mu}(f, P_f)(x) = \gamma_{\mu}(f, P_f)(x) - \gamma_{\mu+1}(f, P_f)(x)$$

for $\mu = 1, \ldots, N - 1.$  \hspace{1cm} (13)

Classically, the opening spectrum given in Eq. (13), enables to obtain information of the changes of size distribution of the different structures detected in the processed image. Notice that expression (13) can be rewritten in terms of the white top-hat notion (see Eq. (8)), i.e.:

$$\eta_{\mu}(f, P_f)(x) = \text{Thw}_{\mu+1}(f, P_f)(x) - \text{Thw}_{\mu}(f, P_f)(x)$$

for $\mu = 1, \ldots, N - 1.$  \hspace{1cm} (14)

Eq. (14) is known as the top-hat spectrum [38]. Notice that this expression yields information about the contrast of the image, since the top-hat spectrum gives information concerning changes in the white regions as the size of the opening increases. On the other hand, in the literature, the normalized opening spectrum [18,39] is calculated from the ratio between the area of the opening spectrum and the area of the original image. In this work, the ratio between the top-hat spectrum and the total of black and white regions detected by top-hat transformations is denoted by $\xi_{\mu,\lambda}$. The total of black and white regions is obtained from the sum of black and white tophats. The final result is also a top-hat known as self-complementary top-hat [3], which is expressed as follows:

$$\text{Thwb}_{\lambda,\mu}(f, P_f)(x) = \text{Thb}_{\lambda}(f, P_f)(x) + \text{Thw}_{\mu}(f, P_f)(x).$$  \hspace{1cm} (15)

If $\text{vol}(f)$ represents the volume detected in the function $f$, the ratio between the volume of the top-hat spectrum and that of the sum of bright and dark regions in the processed image is written as follows:

$$\text{vol}(\xi_{\mu,\lambda}) = \frac{\text{vol}(\text{Thw}_{\mu+1}(f, P_f)(x)) - \text{vol}(\text{Thw}_{\mu}(f, P_f)(x))}{\text{vol}(\text{Thwb}_{\lambda,\mu}(f, P_f)(x)) + 1}. \hspace{1cm} (16)$$

Notice that the unit has been introduced in the denominator of Eq. (16) to avoid any indetermination.

On the other hand, parameters $\mu$ and $\lambda$, involved in the morphological three-state contrast mapping (See Eq. (12)), will be detected using a graphic method obtained from Eq. (16). This expression works similar to the granulometric density, and allows the obtention of adequate sizes for the opening or closing on the partition from a graph, when one of these parameters is fixed. In this work two examples are provided; in the first, the closing size is maintained without change, while the opening size varies within a certain interval. In the second example, the closing size varies within a certain interval, while the opening size is maintained unchanged.

The objective of plotting $\text{vol}(\xi_{\mu,\lambda})$ vs $\mu$ or $\text{vol}(\xi_{\mu,\lambda})$ vs $\lambda$ is to determine the interval in terms of $\mu$ or $\lambda$ sizes in which the main structure of clear or black regions of the processed image is situated.

3.1. First example: closing size is maintained without change, while the opening size varies within a certain interval

The graphic method is illustrated in Fig. 7. The original image is displayed in Fig. 7(a), while in Fig. 7(b) a graph obtained from Eq. (16) is presented. In this graph, the closing size is fixed at $\lambda = 8$, while the opening size $\mu$ changes within the interval $[40,41]$. It is important to mention that, experimentally, a large size for the closing and opening on the partition (greater than 5) allows adequate segmentations on MRI of the brain; this is the reason for selecting $\lambda = 8$. Nevertheless, if an arbitrary value of $\lambda$ is selected, an adequate size $\mu$ for the opening on the partition may be obtained from the graph drawn from Eq. (16). In the present example, the main structure of clear regions in Fig. 7(b) is detected for $\mu$ values comprised in the interval 1 to 7. Therefore, an adequate value for the size of the opening on the partition may be $\mu = 7$ [5,15,42].

In Fig. 7(c), the morphological three-state contrast mapping with parameters $\lambda = 8$, $\mu = 7$, $\alpha = 0.196$, $\beta = 0.392$ is obtained. In this example, parameters $\alpha$ and $\beta$ were selected without following a particular method; however, an estimation of these parameters will be done by means of the morphological contrast method proposed in Section 4.
3.2. Second example: closing size varies within a certain interval, while the opening size is maintained unchange

This example is illustrated in Fig. 8. Here, the opening size is maintained with \( \mu = 5 \), and \( \lambda \) changes within the interval \([1,12]\). The opening size is selected empirically, as well as the variable interval. One must bear in mind that assigning large sizes for the primitives allows a better detection of white and grey matter. Note that different values of \( \mu \) and \( \lambda \) are used in the examples provided in the paper to illustrate the performance of the proposals given in the article.

On the other hand, if an arbitrary size \( \mu \) for the opening on the partition is selected, the closing size \( \lambda \) will be detected from the graph derived from Eq. (16).

In Fig. 8(b), the graph of Eq. (16) is obtained with the established parameters. Note that there are two intervals in which important structures of dark regions are detected: \([1,7]\) and \((7,12)\) \([5,15,42]\). Therefore, two adequate values for the size of the closing on the partition may be \( \lambda = 7 \) and 12. Some output images are presented in Figs. 8(c)–8(e). These were obtained by fixing the same values for \( a \) and \( b \) as in the first example and applying a morphological three-state contrast mapping (see Eq. (12)). Whereas the image in Fig. 8(f) was obtained for different values of \( a \) and \( b \), as mentioned above, there are two values for the \( \lambda \) parameter. Note that if \( \lambda = 7 \) is selected, the morphological three-state contrast mapping is not capable of modifying all “black” components, since some of these can only be modified for values of lambda greater than 7. However, if \( \lambda = 12 \), practically all “black” components are modified.
In Figs. 8(e) and 8(f) this effect can be appreciated. In the three-state morphological contrast mappings, the proximity criterion determines which primitive acts base on \(a\) and \(b\) parameters, as expressed in Eq. (12). In Fig. 8(e) the behavior of the black regions is observed. Here, black regions are merged due to the closing on the partition as the \(b\) value decreases, while in Fig. 8(f) the opening on the partition and the original function predominate as the \(a\) value increases.

4. Contrast measure

The application of contrast transformations is a common practice for enhancing characteristics of interest in the processed images; however, it is not common to have a quantification of the contrast to select the enhanced image presenting the best visual contrast. This situation occurs because the improvement of images after being processed by some contrast transformation is often quite difficult to measure. In the literature, some definitions of contrast measure have been reported, see for example [20,21,40,41,43]. These methodologies are not based on techniques of mathematical morphology; therefore, the introduction of a morphological contrast measure is important, since the improvement of contrast using morphological transformations is widely used.

In this work, a morphological contrast measure is proposed and treated from a point of view of visual contrast. In psychovisual studies, the contrast \(C\) of an object with luminance \(L\) against its surrounding luminance \(L_s\) is defined as follows [20]:

\[
C = \frac{L - L_s}{L_s}.
\]  

(17)
A perceptive contrast measure is a complex task, since several conditions must be considered, for example, state of adaptation of the observer, nature of existent contours between adjacent areas, relation between adjacent areas, size of the internal structures of the image, spatial frequency of the stimuli, among others. In order to model the way the eye perceives luminance changes, several contrast models have been introduced, for example, Weber law, power law, Michelson law, just to mention a few [23]. However, there is no universal measure which can specify both the objective and subjective validity of the enhancement method [41]. This situation is illustrated with some attempts where a local contrast measurement in the spatial domain is obtained [40]. For example, the local contrast proposed by Gordon and Rangayan [44] is defined by the obtention of the average of intensity values detected in two rectangular windows centered on a current pixel. In order to improve the method proposed by Gordon and Rangayan [44], Baghdadi and Negrate proposed a local contrast measure based on the local edge information of the image [43]. In Agaian et al. [40], a quantifying contrast method was proposed, in which the maximum intensity and minimum intensity inside the block were analyzed to calculate the measure of the enhancement. Another example of contrast measure can be found in Morrow et al. [20]; this approach uses statistical quantifications based on the contrast histogram.

Notice that each one of the contrast measures mentioned above attempts to quantify contrast enhancement in different ways, confirming the absence of a universal method to measure contrast in processed images.

In this paper, the morphological contrast measure is oriented toward the analysis of contours of the image. The modification of the contours in a processed image will produce changes in the contrast. For this reason, important changes in luminance are associated with the contours of the image. From the point of view of visual perception, local visual information is combined to form a global representation. In our case, each edge of the image is analyzed as local information, in such a way that a global representation yielded by all contours of the processed image provides information about contrast changes in the processed image.

In the next section, a morphological contrast measure is proposed. The morphological contrast measure will be useful to determine the best parameters associated with certain morphological contrast transformation. The parameters of interest are obtained from a graphical method which involves a set of output images. In particular, these output images are generated by the application of the morphological three-state contrast mappings (see Section 2.3).

4.1. Morphological contrast measure based on image edges

Edges are defined as significant local intensity changes in the image; usually, they are considered step discontinuities. These important transitions of local intensity are associated with significant variations of luminance. Due to this, edges are important features to be analyzed since they produce changes in the contrast. On the other hand, in psycho visual studies, the finding of contours is the first step in vision processing, since edges often coincide with important boundaries in the visual scene [17]; this process is called primal sketch. In order to detect the edges of the image, there are for example first derivative techniques, second derivative techniques, among others [23]. In general, the implementation of these approaches includes the convolution of the signal with some form of linear filter. On the other hand, in mathematical morphology, the extraction of contours is carried out by the implementation of morphological gradients [45].

In this work, the morphological gradient presented in Eq. (7) is used. The analysis of image contours consists in quantifying in local mode the variations of the maximum and minimum intensities of the detected contours of the image into a window $B$ containing its origin. Formally, this is expressed as follows.

For the sake of simplicity, let us consider $\max \text{gradm}(x) = \max \{\text{gradm}(f, P_f)(x+b) : b \in B\}$ and $\min \text{gradm}(x) = \min \{\text{gradm}(f, P_f)(x+b) : b \in B\}$, where $x$ belongs to the domain of definition of $f$, denoted by $D_f$. Expression (18) is proposed in order to have an indirect measure of the variations of the contrast. This quantification is denoted as $\text{VC}_p$, where $p$ represents the parameters involved in the contrast enhancement of the processed image,

$$\text{VC}_p = \sum_{x \in D_f} [\max \text{gradm}(x) - \min \text{gradm}(x)].$$ (18)

$\max \text{gradm}(x)$ and $\min \text{gradm}(x)$ represent the maximum and minimum intensity values of the morphological gradient on the partition around
point $x$. These values are taken from one set of pixels contained in a window $B$ of elemental size ($3 \times 3$ elements) that contains its origin (notice that the window $B$ corresponds to the structuring element). From Eq. (18), and in accordance with the order-statistical filters [34,35], the maxima and minima intensity values of the image in certain neighborhood are the morphological dilation \(\delta_\mu B(f)(x)\) and the morphological erosion \(e_\mu B(f)(x)\) (where \(\inf\) is the inf operator, \(\sup\) is the sup operator, and \(\hat{B}\) the transposed of the structuring element $B$, i.e., \(\hat{B} = \{-x : x \in B\}\)), in such a way that Eq. (18) is rewritten as follows:

\[
VC_p = \sum_{x \in B} \left[ \delta_\mu B(\text{gradm}(f, P_f)(x))(x)
- e_\mu B(\text{gradm}(f, P_f)(x))(x) \right].
\] (19)

However, the morphological gradient at pixel level is expressed as follows:

\[
\text{gradm}_\mu B(f)(x) = \delta_\mu B(f)(x) - e_\mu B(f)(x).
\] (20)

From Eq. (20), Eq. (19) is written as:

\[
VC_p = \sum_{x \in B} \text{gradm}_\mu B(\text{gradm}(f, P_f(x))(x)).
\] (21)

Notice from Eq. (21) that the edges of the processed image are obtained by the application of the morphological gradient on the partition, whereas important variation of the detected edges are obtained through the morphological gradient at pixel level. Therefore, the morphological contrast measure introduced in this work is a measure of the intensity variations of the detected contours. Notice that important differences between the maximum and minimum intensities of the morphological gradient in certain window $B$ concern with substantial changes in the luminance of the image.

On the other hand, the main purpose of the morphological quantitative contrast method in this work consists in detecting the output image that presents a good visual contrast from a set of output images generated by some parametric contrast transformation. For each one of the output enhanced images, the $VC_p$ values are calculated and plotted. The analysis of the obtained graph is focused on its global maxima, since they provide information about important changes in the analyzed edges. The following steps are employed for selecting the maximum producing a good visual contrast:

**Step 1:** Calculate and draw the graph $VC_p$ vs parameters for a set of output enhanced images.

**Step 2:** A higher visual contrast will correspond to $VC_p$ value associated with the global maximum in the graph $VC_p$ vs parameters.

Notice that in the graph $VC_p$ vs parameters it is possible to find more than one maximum, though the largest change in luminance will be detected by the global maximum (the maximum with highest altitude), hence only the global maximum is employed.

Also it is important to mention that images with high $VC_p$ values not necessarily mean good visual contrast; for example, if an input image is processed with a contrast enhancement transformation producing a large degradation in the output image, then high $VC_p$ values can produce an output image without good visual contrast. In consequence, the election of the parametric values directly controlling contrast transformations must be done with care in order to avoid such problems. The performance of this contrast measure is illustrated in the next section.

5. One application to the segmentation of brain MRI

In this section, two examples of the proposals given in this work are presented. The first example deals with white and grey matter segmentation in a frontal lobe area, while in the second example white and grey matter are also segmented, but in this case MRI are corrupted by introducing 5% noise.

As follows we describe the procedure used to segment white and grey matter in a frontal lobe region. This procedure is similar to that followed for segmenting white and grey matter in MRI corrupted by noise.

5.1. Segmentation of white and grey matter in frontal lobe

The brain MRI-T1 presented in this example belongs to the MRI-T1 bank of the Institute of Neurobiology, UNAM Campus Juriquilla, Querétaro México. The file processed and presented in this article comprises 120 slices, from these 22 belong to a frontal lobule. The selection of the different frontal lobule slices was carried out by a specialist in the area of the same institute. The segmentation of the skull for each brain slice is done by means of the transformation proposed in [46]; our sole interest at this point is the segmentation of
white and grey matter. The first three slices of a frontal lobule without skull are presented in Fig. 9(a).

The general idea is to apply contrast mappings on the partition as a pre-processing step to enhance clear regions, i.e., enhance the zones where white matter is located. Note that during the enhancement process, several clear regions will be merged, thus white matter is obtained for certain grey levels.

The procedure followed to obtain white and grey matter in frontal lobe is explained in the next sections; however, in order to simplify the procedure, the same contrast mapping on the partition with specific parameters $\lambda$, $\mu$, $\alpha$, and $\beta$ will be applied to all slices. This approximation is made for two reasons: first, the intensities of white matter are similar in all slices; second, to avoid a large and inadequate process. Thus, the parameters $\lambda$, $\mu$, $\alpha$, and $\beta$ are obtained solely for the first slice of the frontal lobe and applied to the remaining slices.

5.1.1. Determination of the opening size on MRI

The analysis carried out in this section corresponds to the first slice of frontal lobe presented in Fig. 9(a) as was mentioned previously.

The morphological contrast mappings on the partition will be used to enhance the clear regions in frontal lobe. This is achieved by attenuating the dark regions, while the clear regions are maintained or “hardly” modified. Subsequently, adequate sizes for the opening and closing on the partition involved in the contrast mappings must be determined.

By means of Eq. (16), the size of the opening is calculated. The size of the closing on the partition is fixed at $\lambda = 15$. This value is empirical, since, experimentally a large size for the closing will give adequate segmentations.

An adequate size for the opening on the partition is calculated graphically from Eq. (16). The aim of plotting the graph $\text{vol}(\zeta_{\mu,\lambda})$ vs $\mu$ is to determine an interval given in terms of $\mu$ sizes, where the main structures of clear regions of the processed image are located. In Fig. 9(b) the graph $\text{vol}(\zeta_{\mu,\lambda})$ vs $\mu$ is presented. Note that $\mu$ takes values within the interval 1–12, while $\lambda = 15$ is a fixed value. The interval for $\mu$ is considered within these values given that the opening on the partition originates larger modifications than the traditional morphological opening [9,15].

The main structure of clear regions in Fig. 9(b) is detected for $\mu$ values in the interval 1–6. For this reason an adequate value for the size of the opening on the partition may be $\mu = 6$ [5,15,42]. Hence, a contrast mapping on the partition with parameters $\lambda = 15$ and $\mu = 6$ will be applied for all slices of the analyzed frontal lobe area.

5.1.2. Determination of parameters $\alpha$ and $\beta$

The analysis presented in this section also corresponds to the first slice of the frontal lobe shown in Fig. 9(a). The determination of parameters $\alpha$ and $\beta$ involved in the contrast mappings on the partition is carried out by means of the morphological contrast measure introduced in Section 4.1. In other words, parameters $\alpha$ and $\beta$ are associated with the image presenting the “best” visual contrast.

The methodology consists in generating a set of output images by means of a contrast mapping on the partition with specific parameters $\lambda$ and $\mu$, while $\alpha$ and $\beta$ take values within the interval $[0,1]$. Subsequently, the contrast measure $VC_{\alpha,\beta}$ is calculated for each image of the set by means of Eq. (21). The image with the best visual contrast is obtained from the graph $VC_{\alpha,\beta}$ vs $\alpha, \beta$; such image allows the adequate determination of $\alpha$ and $\beta$ values. In this work, a set of 12 images is generated from a contrast mapping on the partition with parameters $\lambda = 15$ and $\mu = 6$, while $\alpha$ and $\beta$ take values within the interval $[0,1]$. The values for $VC_{\alpha,\beta}$ are then calculated for each output image; the graph $VC_{\alpha,\beta}$ vs $\alpha, \beta$ is presented in Fig. 9(c).

The image presenting the best visual contrast is associated with the global maximum located in the graph $VC_{\alpha,\beta}$ vs $\alpha, \beta$. In this example, the maximum corresponds to the image with parameters $\alpha = 0.137$ and $\beta = 0.627$. Hence, $\lambda = 15$, $\mu = 6$, $\alpha = 0.137$ and $\beta = 0.627$ will be used as the specific parameters in a contrast mapping for enhancing the clear regions in all slices of the frontal lobe. This situation is illustrated in Fig. 9(d).

5.1.3. Segmentation of white and grey matter in a frontal lobe area

In order to illustrate the different steps of the proposed algorithm to segment white and grey matter in a frontal lobe region, Fig. 10 will be used. The original image is located in Fig. 10(a1), while the image processed by the contrast mapping (see Eq. (12)) using the parameters calculated previously is presented in Fig. 10(a2).

Algorithm to segment white and grey matter:

(i) Compute the threshold of the image in Fig. 10(a2) between sections 90–255. The sections
Fig. 9. MRI segmentation. (a) First three slices of a frontal lobule; (b) graph of the volume calculated from Eq. (16), the opening size varies within the interval [1, 12], while closing size is fixed to $\lambda = 15$; (c) graph $VC_{\alpha, \beta}$ vs $\alpha, \beta$; and (d) contrast mapping applied to images in Fig. 9(a) with $\lambda = 15$, $\mu = 6$, $\alpha = 0.137$ and $\beta = 0.627$. 
90–255 are obtained approximately from the normalized histogram presented in Fig. 10(a3); the output image is presented in Fig. 10(a4).

(ii) Obtain from the original image (Fig. 10(a1)) the grey level values where the binary image in step (i) takes the value of 1. At this point the white matter is segmented (See Fig. 10(a5)).

(iii) Compute point by point the arithmetic difference between the original image in Fig. 10(a1) and the image in step (ii). Here the grey matter and other structures are detected. The output image is presented in Fig. 10(a6).

(iv) Compute the threshold of the image obtained in step (iii) between sections 70–255; the output image is presented in Fig. 10(a7). The sections 70–255 are obtained approximately from the normalized histogram presented in Fig. 10(a3). For values greater than 70 levels of intensity, the cerebrospinal fluid is eliminated.

Fig. 10. White and grey matter segmentation. (a1) Original image; (a2) enhanced image; (a3) normalized histogram of image in Fig. 10(a2); (a4) thresholding between 90–255 sections of image in Fig. 10(a2); (a5) detection of white matter; (a6) difference between image in Fig. 10(a1) and image in Fig. 10(a5); (a7) thresholding between 70–255 sections to detect grey matter of image in Fig. 10(a6); (a8) detection of grey matter; (a9–a11) first three slices in which white matter is segmented; and (a12–a14) first three slices in which grey matter is segmented.
(v) Obtain from the original image the grey level values where the binary image in step (iv) has the value of 1. In this step the grey matter is segmented (see Fig. 10(a8)).

In Figs. 10(a9)–(a11), the white matter of images in Fig. 9(a) are presented, as well as the grey matter in Figs. 10(a12)–(a14).

On the other hand, the specialist in neuroanatomy in the Institute of Neurobiology, UNAM, Campus Juriquilla, Querétaro México, identifies white and grey matter by intensity, and quantifies white and grey matter by selecting different regions of interest. In order to illustrate the difficulties found to segment manually white and grey matter in frontal lobe, Fig. 11 is presented. In Fig. 11(a) the original images are shown (this images have been used throughout the paper). In Fig. 11(b) we have the output images after applying Eq. (12) with parameters obtained in Sections 5.1.2 and 5.1.1, i.e., \( \lambda = 15, \mu = 6, \alpha = 0.137 \) and \( \beta = 0.627 \).

In Fig. 11(c), white matter has been detected by means of the algorithm introduced previously. Finally, in Fig. 11(d) a similar procedure followed by the specialist in neuroanatomy is provided. In this case a thresholding operation is carried out between sections 90–255. The idea is to separate clear regions. Subsequently, the thresholded image is superposed to the original image to get a reference. By comparing Fig. 11(c) and Fig. 11(d) notice that in the latter, the thresholding operation detects several clear regions that were not detected in the images of Fig. 11(c). Nevertheless, many of them do not correspond to white matter. This is the main problem in manual segmentations, since the specialist has to decide whether the regions correspond to white or grey matter. Moreover, manual segmentations are time-consuming procedures, and many errors are introduced in the measurements, because the selection of white or grey matter is subjective. In the images of Fig. 11(c), white matter

---

**Fig. 11.** Detection by intensity. (a) Original images; (b) enhanced images after applying the contrast mapping with the parameters obtained in Section 5; (c) output images where white matter is segmented by using the algorithm introduced in Section 5.1.3; (d) thresholding operation between 90–255 sections to the images in Fig 11(b).
is obtained as flat zones of regions that fulfill the thresholding operation. The objective of applying contrast mappings on the partition is to enhance and merge clear regions; further on, white matter is separated by a thresholding operation.

On the other hand, in the Institute of Neurobiology, UNAM, Campus Juriquilla, Querétaro México, several specialists study the problem of memory impairment related to aging. They compare the index given by the ratio between white and grey matter in the frontal lobe for different brains. In this paper, the segmentation of the frontal lobe called e2397 is presented; however, the same procedure was applied for segmenting four other frontal lobes. In Fig. 12(a) the set of slices that form the frontal lobe of the brain e2397 is presented, whereas in Fig. 12(b) we have the enhanced images after applying Eq. (12) with parameters $\lambda = 15$, $\mu = 6$, $\alpha = 0.137$ and $\beta = 0.627$. In Fig. 13(a), the segmentation of white matter is presented. White matter in frontal lobe was obtained by the application of the algorithm given in this section. Finally, grey matter is segmented following the algorithm provided also in this section; output images can be observed in Fig. 13(b).

Once that white and grey matter are segmented, all the pixels different from zero in Figs. 13(a) and (b) are counted. The volume of white matter amounts to 24,731 pixels and that of grey matter to 33,169 pixels; the ratio between grey and white matter is equal to 1.341. The relation between white and grey matter is compared with a manual segmentation performed by an expert in the area; the comparison gives a variation of $+5\%$ with respect to the manual procedure.
Likewise, in our remaining segmentations of frontal lobes, the ratios between white and grey matter presented a variation of $\pm 5\%$ with respect to the manual method. The segmentation of white and grey matter, as well as the ratios between white and grey matter was validated by an expert of the Institute of Neurobiology, UNAM, Campus Juriquilla, Querétaro México.

5.2. Segmentation of white and grey matter in the presence of noise

In the last three sections a methodology to segment white and grey matter in frontal lobe was provided. However, in such images (see for example Fig. 12) the noise level is less than 1%; therefore $\alpha$ and $\beta$ parameters can be obtained from the morphological contrast measure without any problem. Nevertheless, if the processed image is corrupted by noise, the methodology followed to compute $\alpha$ and $\beta$ parameters fails, because the contrast measure involves the morphological gradient, as expressed in Eq. (21).

Noise is an undesirable characteristic in MRI. It reduces image quality and makes the segmentation process troublesome. In our particular case, a solution to this problem is a pre-filtering step on the corrupted images. The segmentation quality is conditioned by this step, and more precisely by preserving the useful information. As follows, an example where white and grey matter are separated in some slices of a volume of MRI in presence of noise is provided; here the reconstruction transformations (see Section 2.2) will be used as a pre-processing step to the noisy images. The MRI data volume was obtained from a simulated brain image.

Fig. 13. White and grey matter segmentation in frontal lobe of e2397 brain. (a) Segmentation of white matter from the images in Fig. 12(b); (b) segmentation of grey matter from the images in Fig. 12(b).
database [29], which has the characteristic of being corrupted by 5% noise.\(^2\) In Fig. 14 eight slices belonging to the file *t1_icbm_normal_1mm_pn5_rf20[1].mnc* are presented. While in Figs. 15 and 16, some output images illustrate the methodology followed in this paper to segment white and grey matter. In Fig. 15(a) we have the original image corrupted by noise. This image corresponds to slice 107. In order to appreciate the noise in the original image; a threshold was obtained between sections 150–255 (Fig. 15(b)). In Fig. 15(c), the normalized log-histogram of the image in Fig. 15(a) is obtained.

---

\(^2\)In accordance with the information given in http://www.bic.mni.mcgill.ca/brainweb/, the noise in the simulated images has Rayleigh statistics in the background and Rician statistics in the signal regions. The “percent noise” number represents the percent ratio of the standard deviation of the white Gaussian noise versus the signal for a reference tissue.
Notice the different regions where cerebrospinal fluid, grey matter and white matter are detected. The intensity levels where these regions are located are relevant, given that the thresholding operations in the algorithm proposed to segment white and grey matter use this information. On the other hand, the pre-processing step using the filter \( \tilde{\phi}_{t=1}(\tilde{\gamma}_{t=1}(f)) \) is presented in Fig. 15(d). Here the opening and closing by reconstruction are applied with a size \( \tau = 1 \); the intention is to suppress noise components without affecting the remaining structures of the image. This characteristic is very important, since the image has been simplified without the introduction of new contours. In Fig. 15(e) a thresholding operation between sections 124–177 is shown to appreciate the noise in the image. While in Fig. 15(f), the normalized log-histogram of the image in Fig. 15(d) is displayed. By comparing histograms in Figs. 15(c) and (f), it is evident that in the latter some dark and clear regions were eliminated. Several intensity levels were stretched as well, resulting in enhancement of the image. In Figs. 16(a) we obtain the opening size when the closing size is fixed at \( \lambda = 15 \). In this example, a high value for the closing is selected, because we want to suppress or merge the noise with the grey levels of the image when it is processed by the contrast mapping. As mentioned previously, the flat zones that form the image will be extended or merged when a transformation on the partition is applied (see Fig. 18). From the graph in Fig. 16(a), notice that an important structure of white regions can be found in the interval \( \mu \in [1, 11] \), thus \( \mu \) is selected with a value of \( \mu = 11 \). On the other hand, parameters \( \alpha \) and \( \beta \) are associated with the image presenting the best visual contrast. Parameters \( \alpha \) and \( \beta \) were deduced from the global maximum located in the graphic presented in Fig. 16(b), in this case, \( \alpha = 0.078 \) and \( \beta = 0.313 \). Furthermore, Fig. 16(c) shows the output image after applying Eq. (12) with parameters, \( \mu = 11, \lambda = 15, \alpha = 0.078 \) and \( \beta = 0.313 \). While in Fig. 16(d) its normalized log-histogram is presented. In this histogram notice that white matter is formed by regions with intensity levels around 150. The value of 150 will be useful in the algorithm as the inferior threshold level to separate white matter. The segmentation of white and grey matter is then obtained by following the algorithm given previously in Section 5.1.3; the output images are presented in Figs. 16(e) and (f).

Although the algorithm to segment white and grey matter works properly, some noise components appear in the borders, as presented in Figs. 16(e) and (f). Hence, a better segmentation will be obtained if an efficient transformation to suppress noise is utilized. Finally, in Fig. 17 a set of images corrupted with 5% noise is processed following the same procedure. Each row displays the original image, output image following the pre-processing step, enhanced image, white and grey matter.

6. Implementation of the transformations on the partition

Though the bidimensional array (pixel matrix) is a common way to represent an image, it does not allow to deal effectively with the concept of partition based on the notion of flat zone, since there is not an easy access to the regions and to its vicinity relations. The structure of data better adapted to this problem is the graph made up of nodes and arcs, where nodes represent flat zones and arcs the adjacency between flat zones. This structure is useful for the description and implementation of connected transformations on images [47]; it is known as region adjacency graph.

Using this representation, connected transformations can be obtained in terms of this graph though it is necessary to change the grey level of the merging nodes. A graph structure was implemented for the transformation introduced in this work. Here the nodes represent the partition components of the original image and the arcs describe the adjacency between components. Fig. 18(a) illustrates the adjacency graph of the image presented in Fig. 18(b), while Fig. 18(c) shows the image obtained from the erosion on the graph.

The values to the interior of the nodes correspond the grey level of the flat zones of the original image (see Fig. 18(a)) and the grey levels of the transformed image by the erosion (see Fig. 18(c)). The value on the top and in the interior correspond to the grey levels of the original image, while the emphasized value at the bottom is the grey value of the eroded image. The representation of the adjacency graph allows the implementation of a sequence of transformations in an efficient way. The algorithm for the creation of the graph is relatively fast. An image of the kind presented in Fig. 7(a) with size 256 × 256 (65,536 pixels) containing 7353 flat zones (nodes) requires approximately 6 s to have its adjacency graph on a computer Pentium III at 600 MHz. After creating the graph, a contrast mapping (see Eq. (12)) of size \( \lambda = \mu = 15 \) is carried
out in 5 s approximately, i.e., 60 basic operations (erosion and dilation of size 1) are undertaken during this time.

7. Discussion and conclusions

In this paper we present a morphological contrast mapping defined at partition level and a morphological contrast measure based on edge analysis. The contrast measure is useful to find adequate values for the parameters $a$ and $b$ involved in the contrast mapping. Moreover, a graphic method is proposed to obtain the size of one of the primitives, the opening or closing on the partition, when the size of the closing or the opening on the partition is fixed. The purpose of detecting suitable values for the sizes of the primitives and the parameters $a$ and $b$ that appear in the contrast mapping is to get an adequate contrast enhancement in the processed image. Contrast enhancement not only serves to

![Graph to obtain the opening size](image)

![Graph to obtain the $a$ and $b$ parameters](image)

![Enhanced image](image)

![Normalized log-histogram](image)

![Detection of white matter](image)

![Detection of grey matter](image)

Fig. 16. (a) Graph to obtain the opening size; (b) graph to obtain the $a$ and $b$ parameters; (c) enhanced of image in Fig. 15(d) by means of the contrast mapping with parameters $\mu = 11, \lambda = 15, a = 0.078$ and $b = 0.313$; (d) normalized log-histogram of image in Fig. 16(c); (e) detection of white matter; and (f) detection of grey matter.
improve the image, but it is also useful in segmenting the image. This approach was mainly used to detect white matter in frontal lobe sections as is illustrated in Figs. 12 and 13. The segmentation of white matter consists in the separation of all clear regions that are enhanced by the contrast mapping with specific parameters. In this case, a thresholding operation was used to separate white matter. However, in Section 5.1.3, we mention that manual segmentation is based basically on a thresholding operation (see Fig. 11). The difference between the two methods consists in the following: we separate merged and enhanced flat regions, while in manual segmentations that is not possible. Nevertheless,
when an image is processed at the partition level, the main disadvantage is that it is strongly modified and adjacent flat zones are merged as the size of the primitives increases (see Fig. 18). Hence, the practical problem solved in this paper, grey matter regions having similar intensities than white matter areas will be merged. To separate similar intensities of grey and white matter is also a hindrance in manual segmentations. That is the reason of ±5% variations when our method and manual segmentations are compared. On the other hand, the segmentation of white and grey matter takes 20 min per lobe (more or less 17 slices are analyzed); this includes the calculation of all the parameters involved in the contrast mappings. The time spent by the neuroanatomist is dramatically reduced, for she/he may employ almost three days per lobe. However, the obtainment of opening and closing sizes, as well as alpha and beta parameters that control the contrast mapping are time consuming, mainly alpha and beta parameters, since they are derived from a set of images which must be analyzed to select the image presenting the best visual contrast related to the contrast measure. In this case, the time employed for detecting such parameters is approximately 15 min. This is the main drawback when white and grey matter are segmented.

On the other side, when noisy MRI is processed, our method fails. This is because the quantifying contrast method is defined as a function of the morphological gradient (see Eq. (21)). However, by introducing a pre-processing step to suppress noise components, followed by the procedure discussed in this paper to segment white and grey matter, acceptable segmentations were obtained. Nevertheless, some noise borders can be appreciated in the output images. Better segmentations will be obtained if an improved pre-processing step to suppress noise is applied.

Acknowledgments

The author Jorge D. Mendiola Santibañez thanks CONACyT México for financial support. The author I. Terol would like to thank Diego Rodrigo
References


