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A B S T R A C T

Proofs of Propositions 6 and 8 of the paper Communication Complexity and Intrinsic Universality in Cellular Automata are formally incorrect. This erratum proves weaker versions of Propositions 6 and 8 and a stronger version of Proposition 9 which are sufficient to get the main results of the paper (Corollary 2) for PREDICTION and INVASION problems. For problem CYCLE, we only prove a weaker version of Corollary 2, essentially replacing a condition of the form ‘\( f \in \Omega(n) \)’ by ‘\( f \notin o(n) \)’. All other statements of the paper are unaffected.

1. Comparison relation

In subsection 4.1 of the paper, a relation \( \prec \) between functions from \( \mathbb{N} \) to \( \mathbb{N} \) is defined. It should be replaced by the following.

\textbf{Definition 1.} \( \phi_1 \prec \phi_2 \) if there are non-constant affine functions \( \alpha, \beta, \gamma, \delta \) from \( \mathbb{N} \) to \( \mathbb{N} \) such that \( \alpha \circ \phi_1 \circ \beta \leq \gamma \circ \phi_2 \circ \delta \).

By a non-constant affine function, we mean a function of the form \( n \mapsto \alpha n + \beta \) for some \( \alpha > 0 \). From now until the end of this erratum, the notation \( \prec \) refers to the above definition.

\textbf{Remark.} If a function \( \phi \) is \( \prec \)-greater than the identity \( n \mapsto n \) then \( \phi \notin o(n) \). However, it is not generally true that \( \phi \in \Omega(n) \).

\textbf{Lemma 1.} Let \( f \) be the identity function \( (f(n) = n) \). Let \( F \) be any CA and let \( g = CC(PRED_F) \) and let \( h = CC(Inv_u^f) \) for some word \( u \). Then we have

- \( \text{if } f \prec g \text{ then } g \in \Omega(n) \);
- \( \text{if } f \prec h \text{ then } h \in \Omega(n) \).

\textbf{Proof.} From the definition of \( \prec \), if a function \( \phi \) verifies \( f \prec \phi \) then

\[ \exists n_0, \alpha > 0, \beta > 0 \text{ such that } \forall n \geq n_0, f(\alpha n) \geq \beta n. \]

Now we claim that \( g \) has the following property:

\[ \exists k_0, \forall k \geq k_0, \exists C_k \text{ such that } \forall n \geq k, g(n) \leq C_k g(n - k). \]
This property is sufficient to prove that $g \in \Omega(n)$. This property is true for $k_0 = 2r + 1$ since, if $w$ is a word of size $n$ and $k \geq k_0$, $\text{Pred}_F(w)$ can be computed from the list of $\text{Pred}_F(w_i)$ (with $0 \leq i \leq k$) where $w_i$ is the subword of $w$ of length $n - k$ starting at position $i$.

To finish the proof it is sufficient to notice that $h$ is an increasing function: indeed, the problem $\text{Inv}_F^{u}$ restricted to inputs of size $n$ is a sub-problem of $\text{Inv}_F^{u}$ restricted to inputs of size $n + 1$ if we add the letter number $n + 1 \mod |u|$ of $u$ at the end of each input of size $n$. □

2. Proposition 6 and 7

Proposition 6 and 7 are true using the new definition of $\prec$ and are proved without changing anything in the original proofs.

3. Proposition 8

Proposition 8 is true if we restrict the simulation relation $\preceq$ to a weaker relation where composition with shifts are not allowed. Precisely, denote by $F \preceq_w G$ if there are parameters $m, m', t, t'$ such that $F^{(m,t,0)} \subseteq G^{(m',t',0)}$. If we replace ‘$F \preceq G$ by ‘$F \preceq_w G$’ in the statement of Proposition 8, then it becomes correct with exactly the same proof.

4. Proposition 9

Let $F$ be the CA used to prove item 3 of Proposition 9. In fact, $F$ has the following stronger property:

$$\forall t, \forall z, \forall k \geq 1, \quad \text{CC} \left( \text{Cycle}_{F^{(1,t,z)}}^k \right) \in \Omega(n)$$

Informally, not only $F$ is hard for the cycle problem, but any finite composition of $F$ and shifts is also hard for this problem. To show this it is sufficient to consider inputs suggested by the proof with the additional restriction that $x_1 = 1, x_2 = 0$ and $y_1 = 0$ and $y_2 = 1$. The problem $\text{Disj}$ can still be encoded into such inputs and the presence of at least one ‘1’ is granted in both $F_1$ and $F_2$ layers. Therefore, whatever the composition of $F$ and shifts we take, we will get a $\Omega(n)$ rotation on at least one of the two components in the case of disjoint inputs ($\bigwedge_{i=1}^n \neg(x_i \land y_i) = 1$).

5. Corollary 2

Item 3 of Corollary 2 is false. We can have a universal CA for which the CYCLE problem is trivial as soon as the input period is odd: just add a layer that checks that two states (say black and white) are alternating everywhere and produces a spreading state as soon as two consecutive black cells or two consecutive white cells are in the neighborhood.

Item 3 should be replaced by the following:

there exists $k$ s.t. $\text{CC} \left( \text{Cycle}_{F}^k \right) \not\in o(n)$.

With all previous modifications, Corollary 2 can be proved as follows.

**Proof.** Items 1 and 2 follow directly from Lemma 1 of this erratum and Propositions 6, 7 and 9.

For item 3, denote by $G$ the CA having property of item 3 of Proposition 9. By definition of $\preceq$, since $G \preceq F$ ($F$ is universal), we have

$$G^{(m,t,z)} \subseteq F^{(m',t',0)}$$

for some parameters $m, m', t, t', z$ (informally, it is always sufficient to use shifts only in the simulated CA). Therefore, we have $G^{(1,t,z)} \preceq_w F$. By Proposition 9 (item 3 modified as above) and Proposition 8, we deduce that there is $k$ such that $\text{CC} \left( \text{Cycle}_{F}^k \right)$ is $\prec$-above some $\Omega(n)$ function. We finally deduce that $\text{CC} \left( \text{Cycle}_{F}^k \right) \not\in o(n)$. □

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