Generalized State-Plane Analysis of Soft-switching DC-DC Converters

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ABSTRACT—Based on Generalized switching-cell approach, a generalized state-plane analysis for families of soft-switching dc-to-dc converters will be presented. It is shown the analysis of soft-switching dc-to-dc converter families can be analyzed in terms of simple two-dimensional state-plane diagram. Complete generalized design equations will also be given. It will be shown that one generalized transformation table will be given for the analyzed families. The basic generalized equations will be summarized. The concept of generalized state-plane analysis will be applied to selected soft-switching families such as ZVS-QRC, ZCS-QRC, QSW-CC, QSW-CV, ZCT-PWM, and ZVT-PWM. Other switching families can also be analyzed using the state-plane method.

1. Introduction

Research in soft-switching dc-dc converters continue to generate numerous new soft-switching families operating at higher and higher switching frequencies. The advantages of high frequency include smaller size and lighter weight of magnetic components. Over the last ten years, several soft-switching dc-dc families where analysis thoroughly in the open literature [4,5,6,7]. Moreover, the analysis of soft-switching converters is normally accomplished by identifying the circuit modes of the converter during a switching cycle along with the boundary conditions associated with the transitions between these modes. This is a time consuming process for obtaining a steady state solution. A more simplified technique is to use the state-plane diagram where the state variables in each mode are sketched on a two-dimensional state-plane diagram. In this paper, it will be shown that the steady-state analyses of the soft-switching dc-dc converters can be generalized for a given switching networks family. As a result, instead of analyzing each converter topology in a given family separately, only one switching network for each family is needed to be analyzed [1]. Once the state-plane diagram is obtained, the steady state solution in the time domain can be found easily. Moreover, the state-plan analysis method gives more insight into the converter cell operation [2,3]. Furthermore, it will be shown by using the desired converter parameters, the control characteristics of soft-switching converters can be derived from the state-plane diagram. In particular, the converter design gain as function of frequency and load value can be determined. Finally, the component’s peak values can be also obtained directly from such diagram.

In Section 2, generalized switching cells are derived for selected dc-dc soft-switching PWM families including the conventional hard-switching PWM topologies. The generalized cells were derived for the following well-known soft-switching families: 1) Zero-Voltage-Switching (ZVS) and Zero-Current-Switching (ZCS) – Quasi-Resonant Converter (QRC) families [4], 2) ZVS-Clamped Voltage (CV) Quasi-Square-Wave (QSW) family [5,6], 3) ZCS-Clamped-Current (CC) QSW family [7], and 4) Zero-Voltage-Transition (ZVT) and Zero-Current-Transition (ZCT) PWM families [8,9].

2. The Generalized Switching Cells and Their Corresponding State-Planes

Figure (1) shows the six soft-switching cells for the above mentioned families, while Figure (2) shows the corresponding state-plane for the soft switching cells. Since each family uses the same switching network (same modes of operation) and the same state-plane, the analysis can be generalized. As mentioned in [1], steps can be used in the analysis generalization process. Various orientations of any of the cells in Figure (1) result in a family of converters. Using the three terminals $a$, $b$, and $c$ in the switching-cells, generalized parameters can be defined and their values can be determined from the orientation of
the cell in a specific converter. Let us define the following parameters:

\( M \): The overall output-to-input converter voltage gain, \( M = \frac{V_o}{V_{in}} = \frac{I_{in}}{I_o} \).

\( V_{ng} \): The normalized cell-input voltage to the converter input voltage, \( V_{ng} = \frac{V_g}{V_{in}} \).

\( I_{nF} \): The normalized cell-output current to the converter input current, \( I_{nF} = \frac{I_F}{V_{in} / Z_o} \).

\( V_{nF} \): The normalized cell-output voltage to the converter input voltage, \( V_{nF} = \frac{V_F}{V_{in}} \).

\( V_{ncF} \): The normalized filter capacitor \((C_F)\) voltage to the converter input voltage, \( V_{ncF} = \frac{V_{cF}}{V_{in}} \).

\( I_{nb} \): The normalized cell-current entering node \( b \) to the converter input current, \( I_{nb} = \frac{I_b}{V_{in} / Z_o} \).

\( I_{nT} \): The normalized filter inductor \((L_T)\) current to the converter input current, \( I_{nT} = \frac{I_T}{V_{in} / Z_o} \).

\( Z_o \): The characteristic impedance, \( Z_o = \frac{1}{\sqrt{L_T C_T}} \).

\( Q \): The normalized load, \( Q = \frac{R_o}{Z_o} \), where \( R_o \) is the converter load resistor.

\( f_s \): The switching frequency, \( f_s = \frac{1}{T_s} \), where \( T_s \) is the switching period.

\( f_o \): The natural frequency, \( f_o = \frac{1}{2\pi\omega_o} \).

Where \( \omega_o = \frac{1}{\sqrt{L_T C_T}} \).

\( f_{as} \): The normalized frequency, \( f_{as} = \frac{f_s}{f_o} \).

\( D \): The duty ratio of the main switch.

\( D_t \): The duty ratio of the auxiliary switch.

\( \alpha, \beta, \delta, \eta, \rho, \lambda, \sigma \): The intervals for the modes of operation.

It must be noted that in the state-planes given in Fig. 2, the voltages are normalized with respect to \( V_o \), while all the current are normalized with respect to \( V_{in} / Z_o \). The generalized equations for each cell shown in Figure (1) will include one or more of the following previous defined normalized parameters \( V_{ng}, I_{nF}, V_{nF}, I_{nb}, V_{ncF}, I_{nT}, V_{as}, I_{as}, V_{as} \), depending on the topology of the switching cell itself. By applying the switching cells of Figure (1) to the conventional DC-DC converters (Buck, Boost, Buck-Boost, Cuk, Zeta, and Sepic), a transformation table for each family can be generated. It can be shown that the single transformation table given in Table (1) is complete and applies to all the cells given in Figure (1).

<table>
<thead>
<tr>
<th>Normalized Parameters</th>
<th>Buck</th>
<th>Boost</th>
<th>Buck-Boost, Cuk, Zeta, and Sepic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{ng} )</td>
<td>1</td>
<td>M</td>
<td>1+M</td>
</tr>
<tr>
<td>( V_{nF} )</td>
<td>M</td>
<td>M-1</td>
<td>M</td>
</tr>
<tr>
<td>( V_{ncF} )</td>
<td>1-M</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( I_{nF} )</td>
<td>( \frac{M}{Q} )</td>
<td>( \frac{M^2}{Q} )</td>
<td>( \frac{M(1+M)}{Q} )</td>
</tr>
<tr>
<td>( I_{nb}, I_{nT} )</td>
<td>( \frac{M(1-M)}{Q} )</td>
<td>( \frac{M}{Q} )</td>
<td>( \frac{M}{Q} )</td>
</tr>
</tbody>
</table>

3. Generalized Analysis Results For The Selected Families

Table (2) shows the basic generalized equations (the intervals and gain equations) for the selected soft-switching cells. By substitute for the normalized parameter in table (2) by the value in table (1) for the selected converter; the interval and gain equation can be obtained for that converter.

4. Component Stresses

The state-plane method is very useful in predicting the capacitor and inductor maximum values as well as the device ratings. It can be shown that, using one generalized state-plane diagram, component stresses can be obtained relatively easy for the entire soft-switching converter family. Table (3) is summarizing the stresses on the resonant capacitor and inductor for the selected families while table (4) summarize the stresses on the main switch for the mentioned above families.
Figure (1): Switching-Cells: (a) ZVS-QRC Cell, (b) ZCS-QRC Cell, (c) ZVS-QSW CV Cell, (d) ZCS-QSW CC Cell, (e) ZVT-PWM Cell, and (f) ZCT-PWM Cell

Figure (2): Switching-Cells Basic state-plane: (a) ZVS-QRC Cell, (b) ZCS-QRC Cell, (c) ZVS-QSW CV Cell, (d) ZCS-QSW CC Cell, (e) ZVT-PWM Cell, and (f) ZCT-PWM Cell
Where,

$$a = \sqrt{(I_{nF} - I_{n,l0})^2 + V_{ncF}^2} \quad b = \sqrt{(I_{nF} - I_{nF})^2 + (V_{ng} - V_{na})^2}$$

$$I_{n,l0} = I_{nF} - \sqrt{(V_{ncF})^2 - (V_{ng} - V_{ncF})^2 + (I_{nF} - I_{n,F})^2}$$

$$V_{na} = V_{ng} + \eta(I_{nF} - I_{nF}) + I_{nF} \sin \delta$$

$$V_{n,CI} = V_{ng} - \sqrt{(V_{ng} - V_{na})^2 + (I_{nF} - I_{nF})^2 + (I_{nF})^2}$$

### Table (2): The Basic Generalized Equations Summary

<table>
<thead>
<tr>
<th>CELL</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \delta )</th>
<th>( \eta )</th>
<th>( \rho )</th>
<th>( \lambda )</th>
<th>( \sigma )</th>
<th>( \text{GAIN EQUATION} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZVS</td>
<td>( \frac{V_{ag}}{I_{na}} )</td>
<td>( \sin^{-1}\left(-\frac{V_{ag}}{I_{na}}\right) )</td>
<td>( I_{ag} \left(1 - \cos \beta\right) )</td>
<td>( \frac{\pi}{f_{ns}} )</td>
<td>( -\alpha - \beta - \delta )</td>
<td>( \text{N/A} )</td>
<td>( \text{N/A} )</td>
<td>( V_{ag} = \frac{-f_{ns}}{2\pi} \left[\left(V_{ag} - V_{ag}\right) + V_{ag} \beta + I_{ag} \left(1 - \cos \beta\right)\right] + V_{ag} )</td>
</tr>
<tr>
<td>ZCS</td>
<td>( I_{ag} )</td>
<td>( \sin^{-1}\left(-\frac{I_{ag}}{V_{ng}}\right) )</td>
<td>( V_{ag} \left(1 - \cos \beta\right) )</td>
<td>( \frac{\pi}{f_{ns}} )</td>
<td>( -\alpha - \beta - \delta )</td>
<td>( \text{N/A} )</td>
<td>( \text{N/A} )</td>
<td>( V_{ag} = \frac{-f_{ns}}{2\pi} \left[\left(V_{ag} - V_{ag}\right) + V_{ag} \left(\beta - \sin \beta - \delta \cos \beta\right) - \frac{I_{ag}^2}{2}\right] )</td>
</tr>
</tbody>
</table>

### Table (3): Resonant Capacitor and Inductor Generalized Stress Equations

<table>
<thead>
<tr>
<th>Converter</th>
<th>( V_{ncr,q} )</th>
<th>( I_{nr,q} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZVS-QRC</td>
<td>( I_{nr,F} + V_{ng} )</td>
<td>( I_{nr,F} )</td>
</tr>
<tr>
<td>ZCS-QRC</td>
<td>( 2V_{ng} )</td>
<td>( I_{nr,F} + V_{ng} )</td>
</tr>
<tr>
<td>ZVS-CV</td>
<td>( V_{ng} )</td>
<td>( I_{nr,F} + \left(V_{ng} - V_{ncF}\right) )</td>
</tr>
<tr>
<td>ZCS-CC Q</td>
<td>( V_{ng} + i_{nT} )</td>
<td>( I_{nr,F} )</td>
</tr>
<tr>
<td>ZVT-PWM</td>
<td>( V_{ng} )</td>
<td>( I_{nr,F} + V_{ng} )</td>
</tr>
<tr>
<td>ZCT-PWM</td>
<td>( I_{nF} / \cos(\beta_d) )</td>
<td>( I_{nF} / \cos(\beta_d) )</td>
</tr>
</tbody>
</table>

### Table (4): Some of the Main Switch Generalized Stress Equations

<table>
<thead>
<tr>
<th>Converter</th>
<th>( V_{n,sw-p} )</th>
<th>( I_{n,sw-p} )</th>
<th>( V_{n,sw-ave} )</th>
<th>( I_{n,sw-ave} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZVS-QRC</td>
<td>( I_{nF} + V_{ng} )</td>
<td>( I_{nF} )</td>
<td>( V_{ng} - V_{nf} )</td>
<td>( I_{nF} - I_{nb} )</td>
</tr>
<tr>
<td>ZCS-QRC</td>
<td>( V_{ng} )</td>
<td>( I_{nF} + V_{ng} )</td>
<td>( V_{ng} - V_{nf} )</td>
<td>( I_{nF} - I_{nb} )</td>
</tr>
<tr>
<td>ZVS-CV</td>
<td>( V_{ng} )</td>
<td>( I_{nF} - I_{n,F,0} )</td>
<td>( V_{ng} - V_{nf} )</td>
<td>( I_{nF} - I_{nb} )</td>
</tr>
<tr>
<td>ZCS-CC Q</td>
<td>( V_{ng} - V_{nx} )</td>
<td>( I_{nF} )</td>
<td>( V_{ng} - V_{nf} )</td>
<td>( I_{nF} - I_{nb} )</td>
</tr>
<tr>
<td>ZVT-PWM</td>
<td>( V_{ng} )</td>
<td>( I_{nF} )</td>
<td>( V_{ng} - V_{nf} )</td>
<td></td>
</tr>
<tr>
<td>ZCT-PWM</td>
<td>( V_{ng} )</td>
<td>( I_{nF} (1 + 1/\cos(\beta_d)) )</td>
<td>( V_{ng} - V_{nf} )</td>
<td></td>
</tr>
</tbody>
</table>
5. ZCS-CC Cell as an Example
An appropriate cell can be chosen as an example here is ZCS-CC. A design for 8.1 W ZCS-CC buck boost dc-dc converter with $V_{in}=30V$ and $V_o=9V$ as shows below. Also the design curves can be generated by substituting for the generalized parameter from Table (1) to the generalized equations in Table (2). Figure (3) shows the voltage gain versus normalized frequency for selected family.

$$M = \frac{9}{30}$$

From figure3(c) for $Q=0.5$ and $M=0.3$ which yields to $f_{ns} = 0.218$

$$R_o = \frac{(9)^3}{8.1} = 10\Omega, I_o = \frac{9}{10} = 0.9A, Z_o = \frac{10}{0.5} = 20\Omega$$

By choosing $f_r = 100k, f_o = \frac{f_r}{f_{ns}} = 458.72KHZ$

By solving the following two equations:

$$Z_o = \sqrt{L_o\over C_o} = 20 \text{ And } f_o = \frac{1}{2\pi\sqrt{L_o C_r}} = 458.72KHZ$$

This yield to:

$L_o = 6.94\mu H$ and $C_r = 17.35nF$ .

To achieve ZCS operation, the duty ratio should be $0.25 < D < 0.48$ as show in figure 3 (d); by choosing $D=0.3$ will satisfy the condition. Based on these control characteristic curves, ZCS-CC converters can be designed. Moreover, the state-plane diagram is a great tool to observe the component stresses directly from the state-plane as the line and load vary. For example the stress on the capacitor can be directed obtained form the sate plane as shown in figure 4 where the simulation results match the derived equation in table (3) as follows:

Fig. 3 (a) ZCS-CC Buck, (b) ZCS-cc Boost, (c) ZCS-CC Buck-Boost, Cuk, Zeta, and Sepic; (d) ZCS-cc Buck_Boost Duty ratio.
As shown in table (3), the stress on capacitor and inductor is given by:

\[ V_{ncr}, \overline{p} = 1.9 \Rightarrow V_{cr}, \overline{p} = (1.9)(30) = 57V \]

\[ I_{nLr}, \overline{p} = 0.78 \Rightarrow I_{nLr} = 1.17A \]

Also the stress on the main stress can be calculated using the derived equations in table (4) as following:

\[ V_{nsnP} = 3.27 \Rightarrow V_{sn}, \overline{P} = (3.27)(30) = 98.1V \]

\[ I_{nsnP} = 0.78 \Rightarrow I_{snP} = 1.17A \]

6. CONCLUSION

A generalized state-plane analysis method for families of soft-switching dc-dc converters was presented in this paper. The generalization technique is applied to several well-known switching families including QRC, QSW, and PWM converters. It is shown that there exists a single Generalized Transformation Table for all the families. This leads to several advantages such as improving the computer-aided analysis and design, simplified mathematical modeling, and gives more insight into the converter-cell operation. The generalized equations for each family can be easily used in the analysis of any new converter that uses the same switching cell. This is done by finding the generalized parameters for the new converter and then substituting in the generalized equations.

7. REFERENCES

[3]. Issa Baterseh, C.Megalemos and M.Sznaier “Small Signal Analysis of the LCC-Type Parallel Resonant Converter” IEEE Transactions On Aerospace and Electronics system VOL.32, No.2