MAP channel estimation for Alamouti-based cooperative networks

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Abstract—In this paper, we consider a cooperative network with one source, one destination and two relays to bring cooperative diversity. We assume the link between the source and the destination is so bad that we do not consider it. We propose a Decode-And-Forward (DAF) strategy based on the Alamouti space-time (ST) code. The two relays must recover the transmitted sequence and forward it to the destination. We consider fast time varying channels between the cooperative users and the destination and our objective is to estimate these two channels in order to recover the information sequence at the destination. We propose a data-aided decision-directed Maximum A Posteriori (MAP) iterative channel estimation algorithm. It can profit from an optimum initialization by means of the MAP data-aided channel estimation. Besides, we adapt this iterative receiver to the case where the destination is equipped with two receive antennas to bring more diversity. The validity of the proposed algorithms is highlighted by simulation results.

I. INTRODUCTION

Recently, wireless communications with the cooperation of single-antenna terminals have received tremendous attention [1] [2] [3] [4]. The terminals share their antennas to exploit space-time diversity. Hence, they can be seen as a multiple antenna transmitter. Several strategies have been proposed to bring cooperative diversity [1]. In this paper we use the same setup as considered by Laneman et al. in [1]. Hence, we impose the half-duplex constraint (either transmit or receive, but not both) on the cooperating terminals and propose a DAF cooperative strategy based on the Alamouti ST code [5]. For the DAF transmission, the relays must decode the received message and then forward it.

In our study, we assume the link between the source and the destination to be bad and uses two relays to reach the destination while bringing diversity. We consider slowly time-varying channels between the source and the cooperative relays and fast time varying channels between these relays and the destination. Our objective is to find a robust receiver which efficiently estimates the two channels seen at the destination in order to recover the transmitted information sequence. In previous works, we investigated a similar problem but for a MIMO system using two transmit antennas and one receive antenna in [6] or two receive antennas in [7]. In [6] and [7] the channels between transmit and receive antennas are characterized by the same Doppler spread. In this paper we propose to generalize these results for the DAF protocol where the two links seen at the destination are characterized by different Doppler spreads and each relay must recover the transmitted sequence before forwarding it to the destination. We derive a data-aided decision-directed algorithm which performs an iterative channel estimation according to the MAP criterion, using the Expectation-Maximization (EM) algorithm. This algorithm will be derived for the estimation of the two links seen at the destination but the formulation for the link source-relay is straightforward. Besides, we adapt this iterative receiver for a cooperative network which considers a similar DAF protocol but with two receive antennas at the destination. The rest of the paper is organized as follows. In section II, we introduce the system model with one receive antenna at the destination. In section III, we develop a convenient representation of the two discrete multi-path fading channels seen at the destination. In section IV, we derive the iterative receiver based on a data-aided decision-directed MAP channel estimation algorithm and give some simulations results to highlight the validity of the algorithm. In section V, we adapt this iterative receiver for the case of two receive antennas at the destination.

II. SYSTEM MODEL

In this section, we propose a DAF protocol based on the Alamouti ST code [5]. We consider one source, two relays and one antenna receive destination. In our aim to estimate the two channels seen at the destination, we consider a block-by-block estimation using PSK modulated symbols. Each block is composed of an even number \( K = 2N \) of data and pilot symbols, \( S_k \) at time positions \( p_k = kT \), where \( T \) denotes the symbol period. As schematized in Table I, in the first phase, the source sends the symbols \( S_{2k} \) and \( S_{2k+1} \), while the relays listen.

<table>
<thead>
<tr>
<th>Source ( S_{2k} )</th>
<th>( S_{2k+1} )</th>
<th>Relay 0</th>
<th>( Y_{0,2k} )</th>
<th>( Y_{0,2k+1} )</th>
<th>( S_{2k} )</th>
<th>( -S_{2k+1} )</th>
<th>Relay 1</th>
<th>( Y_{1,2k} )</th>
<th>( Y_{1,2k+1} )</th>
<th>( S_{2k+1} )</th>
<th>( S_{2k} )</th>
<th>Destination</th>
<th>( R_{2k} )</th>
<th>( R_{2k+1} )</th>
</tr>
</thead>
</table>

In the second phase, the first relay sends a decoded version of the symbols \( S_{2k} \) and \( -S_{2k+1} \) (denoted with \( \tilde{y} \)) while...
the second relay sends the decoded versions of the symbols $S_{2k+1}$ and $S_{2k}^*$ (denoted with (·)). We notice that this protocol requires 4 channel uses to send 2 symbols. Hence, the symbol rate is 1/2 symbols per channel use (pcu). All channels are assumed to be non dispersive in time. Hence, they can be be modelled by complex multiplicative distortions. The components of the received vector $Y_{rl}$ at the $l^{th}$ relay, $l = 0, 1$ are 

$$Y_{rl,k} = \sqrt{E} h_{l,k} S_k + b_{l,k}$$

(1)

for $k = 0, 1, ..., K - 1$, where $h_{l,k}$ denotes the channel coefficient between the source and the $l^{th}$ relay and $E$ is the transmitted symbol energy at the source since we assume that the symbols $S_k$ are drawn from a unit energy constellation. The additive noises $b_{0,k}$ and $b_{1,k}$, representing complex noise and interference, are assumed to be Gaussian distributed with zero mean and variances $\sigma_0$ and $\sigma_1$ respectively. We introduce for each transmitted block the vector $R = (R_0, R_1, ..., R_{2N-1})^T$ of $2N$ received samples at the destination in the second phase of transmission, where $(·)^T$ denotes transposition. The components of the received vector $R$ can be written as 

$$R_{2k} = \sqrt{E_0} c_{0,2k} \tilde{S}_{2k} + \sqrt{E_1} c_{1,2k} \tilde{S}_{2k+1} + n_{2k}$$

(2)

for even indices and 

$$R_{2k+1} = -\sqrt{E_0} c_{0,2k+1} \tilde{S}_{2k+1} + \sqrt{E_1} c_{1,2k} \tilde{S}_{2k} + n_{2k+1}$$

(3)

for odd indices, where $E_0$ is the average radiated energy per symbol at the $l^{th}$ relay, $l = 0, 1$ and $c_{l,k}$ denotes the channel coefficient between the $l^{th}$ relay and the destination. The additive noises $n_{2k}$ and $n_{2k+1}$ are assumed to be Gaussian distributed with zero mean and variance $N_0$. Next, we consider time-varying channels between the relays and the destination and our objective is to estimate them in order to recover information sequence at the destination. We denote the link between the first relay and the destination by link0 and the link between the second relay and the destination by link1.

We assume that the fading is independent between the two links 0 and 1. Each multiplicative distortion vector $c_0 = (c_{00}, c_{01}, \cdots, c_{0,2N-1})^T$ or $c_1 = (c_{10}, c_{11}, \cdots, c_{1,2N-1})^T$ is characterized by its average power as well as its Doppler power Spectrum (DPS). We consider a classical DPS which is typically met in outdoor environments. The corresponding autocorrelation function, for the $l^{th}$ link path, $l = 0, 1$ with average power $\phi_l(0)$, is given by 

$$\phi_l(\tau) = \phi_l(0) J_0(\pi B_{Dl} \tau)$$

(4)

where $B_{Dl}$ is the Doppler spread of the channel of the $l^{th}$ link and $J_0(·)$ is the $0^{th}$-order Bessel function of the first kind. Alamouti’s scheme assumes the channel to be constant over each pair of consecutive even and odd indexed symbols. Consequently, we can use, up to a multiplicative factor $\sqrt{E_l}$, the decimated version of the channel $C_l = \sqrt{E_l} (c_{l0}, c_{l2}, \cdots, c_{l,2N-2})^T$ for $l = 0, 1$.

III. CONVENIENT REPRESENTATIONS OF THE CHANNELS

In our study, we need a convenient representation of the discrete channels seen at the destination during each received block. This representation is based on a discrete version of the Karhunen Loève (KL) orthogonal expansion theorem [6]. The decimated channels vectors $C_l$, $l = 0, 1$ can be expressed as

$$C_l = \sum_{k=0}^{N-1} G_{lk} U_{lk}$$

(5)

where $\{U_{lk}\}_{k=0}^{N-1}$ are the normalized eigenvectors of the covariance matrix $\Sigma_l = E [C_l (C_l)^H]$ of $C_l$, $\{G_{lk}\}_{k=0}^{N-1}$ are independent complex zero-mean Gaussian coefficients and $(·)^H$ denotes the Hermitian transposition. The variances of $\{G_{lk}\}_{k=0}^{N-1}$, arranged in decreasing order, are equal to the eigenvalues $\{\lambda_{lk}\}_{k=0}^{N-1}$ of the Hermitian matrix $\Sigma_l$.

The vectors $\{G_l\}_{l=0}^{N}$, where $G_l = (G_{l0}, G_{l1}, \ldots, G_{l,(N-1)})^T$, are referred to as the convenient representations of the two discrete channels seen at the destination during each received block.

IV. MAXIMUM A POSTERIORI CHANNEL ESTIMATION

Next, we use interchangeably $G_l$ and $C_l = \sum_{k=0}^{K-1} G_{lk} U_{lk}$, $l = 0, 1$ to specify each of the two links.

The MAP estimates $\{\hat{G}_l\}_{l=0}^{1}$ of the discrete channels $\{G_l\}_{l=0}^{1}$ seen at the destination are defined as

$$\hat{G}_l = \arg \max_{\{G_l\}_{l=0}^{1}} p \left( \{G_l\}_{l=0}^{1} \mid R \right).$$

(6)

We use the iterative EM algorithm to reach the solution. This algorithm starts with an initial guess $\{G_l^{(0)}\}_{l=0}^{1}$ of $\{G_l\}_{l=0}^{1}$. The evolution from the estimate $\{G_l^{(d)}\}_{l=0}^{1}$ to the new estimate $\{G_l^{(d+1)}\}_{l=0}^{1}$ is performed via a maximization of an auxiliary function defined by

$$Q \left( \{G_l\}_{l=0}^{1}, \{G_l^{(d)}\}_{l=0}^{1} \right) = \sum_s p \left( R, S, \{G_l^{(d)}\}_{l=0}^{1} \right) \log p \left( R, S, \{G_l\}_{l=0}^{1} \right)$$

(7)

where the latter sum is operated over all possible transmitted data vectors during one block denoted with $S = (S_0, S_1, \ldots, S_{2N-1})^T$. The destination assumes perfect recovery of $S_k$ at the two relays.

A. Expression of the algorithm

First, the logarithmic term in the expression of $Q(·)$ can be expressed as :

$$\log p \left( R, S, \{G_l\}_{l=0}^{1} \right) = \log P(S) + \log p \left( \{G_l\}_{l=0}^{1} \right)$$

$$+ \log p \left( R\mid S, \{G_l\}_{l=0}^{1} \right).$$

(8)
When we develop the three terms of $Q(.,.)$, we obtain

$$Q\left(\left\{G_t^{(d)}\right\}_{t=0}^1, \left\{G_t^{(d)}\right\}_{t=0}^1\right) = c + 2p(\mathbf{R}, \left\{G_t^{(d)}\right\}) \sum_{n=0}^{N-1} \Re\left\{b_k\right\}$$

$$-p(\mathbf{R}, \left\{G_t^{(d)}\right\}) \sum_{n=0}^{N-1} \left(\frac{1}{N_0} + \frac{1}{\lambda_{nk}}\right)\left(kG_0 P\right)^2$$

(9)

where $G_t^{(d)} = \left\{G_t^{(d)}\right\}_{t=0}^1$, $b_k = \Lambda_{t,k}^d \left(\hat{S}_{2k}^{(d)}\right)^* + \Lambda_{t,k+1}^d \left(\hat{S}_{2k+1}^{(d)}\right)^*$, $c$ is an additive factor independent of $\left\{G_t^{(d)}\right\}_{t=0}^1$,

$$\Lambda_{t,k}^d = \frac{R_{2k}(C_{0k}^\dagger) + R_{2k+1}C_{0k}}{N_0},$$

(10)

$$\Lambda_{t,k+1}^d = \frac{R_{2k}(C_{0k}^\dagger) - R_{2k+1}C_{0k}}{N_0},$$

(11)

$$\hat{S}_{2k+1}^{(d)} = \sum_{s} S_{2k+1}P \left(\mathbf{S}R, \left\{G_t^{(d)}\right\}_{t=0}^1\right),$$

(12)

for $i = 0, 1$. When we calculate the derivatives of $Q(.,.)$ with respect to $\left\{G_t^{(d)}\right\}_{t=0}^1$ and we equate them to zero we obtain

$$C_{0k}^{(d+1)} = \omega_{zk} \sum_{m=0}^{K-1} \left(R_{2m} \left(\hat{S}_{2m+1}^{(d)}\right)^* + R_{2m+1} \hat{S}_{2m+1}^{(d)}\right) U_{0k,m},$$

(13)

$$C_{1k}^{(d+1)} = \omega_{1k} \sum_{m=0}^{K-1} \left(R_{2m} \hat{S}_{2m+1}^{(d)}\right)^* + R_{2m+1} \hat{S}_{2m}^{(d)} U_{1k,m},$$

(14)

for $k = 0, 1, ..., N - 1$, with $\omega_{zk} = 1/1 + N_0/\Lambda_{t,k}$, $l = 0, 1$. The calculation of the general expressions of $\hat{S}_{2k+1}^{(d)}$ and $\hat{S}_{2k+1}^{(d)}$ needed in the determination of $\left\{G_t^{(d+1)}\right\}_{t=0}^1$ yields [6]

$$\hat{S}_{2k+1}^{(d)} = \frac{1}{2} \left(\tanh\left(\Re\left\{\Lambda_{t,k}^{2d+1}\right\}\right) + j \tanh\left(\Im\left\{\Lambda_{t,k}^{2d+1}\right\}\right)\right)$$

(15)

for QPSK modulated symbols, with $i = 0, 1, \Lambda_{t,k}^{2d}$ and $\Lambda_{t,k+1}^{2d}$ are given in (10) and (11) by replacing $C_{1k}^d$ by $C_{1k}^d$ for $l = 0, 1$ and $k = 0, 1, ..., N - 1$.

### B. Initialization of the algorithm

Let $A$ be the set of pilot symbols indices within a block and by $D_k$ the value taken by the normalized pilot symbol $S_k$, $k \in A$. The pilot symbols are also considered in pairs and space-time encoded in the same manner as data symbols. At the start of the algorithm, the receiver can use for the determination of the initial guess $\left\{G_t^{(0)}\right\}_{t=0}^1$, the components $G_{t,k} = \omega_{zk} \sum_{m \in A} \left(R_{2m}D_{2m+l}^* + (-1)^{l+1}R_{2m+1}D_{2m-l+1}U_{1k,m}^*\right)$, for $l = 0, 1$ and $k = 0, 1, ..., N - 1$. As proposed and investigated thoroughly in [6], a possible solution to optimize the initialization of the algorithm is to use exclusively and optimally the observations corresponding to pilot symbols. In this case, we define the restrictions $C_t^p = \left(C_{t,0}^p, C_{t,1}^p, ..., C_{t,N-1}^p\right)^T$ of the decimated versions of the two channel vectors $C_t$, $l = 0, 1$. We denote the number of pilot symbols by $2N_p$ and introduce the one-to-one mapping $k = \delta(k)$ between the index set $\{k\}_{k=0}^{2N_p-1}$ and the pilot symbols index set $A$ for notational convenience.

Expressed in terms of the equivalent channel vectors $\left\{C_t^{(d)}\right\}_{t=0}^1$, we show that the optimum initial guesses are given by

$$\hat{C}_t^{(0)} = \sum_{k=0}^{N_p-1} B_{t,k} \hat{V}_{1k},$$

(16)

where

$$B_{t,k} = \omega_{zk} \sum_{m \in A} \left(R_{2m}D_{2m+l}^* - R_{2m+1}D_{2m-l+1}\right) V_{1k,m}^*,$$

for $l = 0, 1$, $V_{1m} = \left(V_{1m,0}, V_{1m,1}, ..., V_{1m,(N-1)}\right)^T$ for $m = 0, 1, ..., N_p - 1$. The extended orthonormal base vectors of the eigenvectors of the Hermitian covariance matrix $H_t^p = E \left[C_t^p C_t^{pT}\right]$. The corresponding eigenvalues are denoted by $\{\lambda_{t,k}\}_{k=0}^{2N_p-1}$ and assumed to be arranged in decreasing order. The weighting coefficients $\omega_{zk}$ are given by $\omega_{zk} = 1/1 + N_0/\Lambda_{t,k}$ for $l = 0, 1$ and $k = 0, 1, ..., N_p - 1$.

### C. Decoding information symbols

The iterative algorithm we have proposed leads to a joint improvement of channel estimation and symbol detection through iterations. After a fixed number of iterations $D$ we obtain the estimate $\left\{\hat{G}_t^{(d)}\right\}_{t=0}^1$ of the discrete multipath channel relative to link $l$, $l = 0, 1$. This number $D$ is chosen so that the reached estimate $\left\{\hat{G}_t^{(d)}\right\}_{t=0}^1$ guarantees an unnoticeable degradation in performance with respect to the optimum estimate $\left\{\hat{G}_t^{(d)}\right\}_{t=0}^1$.

Based on this estimate, the receiver provides the soft outputs $\Lambda_{t-k}^{(d)}$ which can be potentially used by an error-correction Viterbi decoding algorithm for recovering the transmitted information sequence. For uncoded QPSK modulated data and training symbols, the decision on normalized symbols $S_k$ is simply given by

$$\frac{1}{2} \left(\text{sgn} \Re\left\{\hat{S}_k^{(d)}\right\} + j \text{sgn} \Im\left\{\hat{S}_k^{(d)}\right\}\right)$$

(17)

### D. Simulation results

For the characterization of the performance of the proposed MAP channel estimation, we assume that the transmissions over the two links suffer from the effects of fast or slowly time-varying flat fading. We consider transmitted blocks of $K = 128$ QPSK modulated symbols with $2N_p = 12$ pilot symbols per block. We assume an equal power allocation among the two relays and $E_0 = E_1 = E/2$. Based on numerical results, we show that only a small number of eigenvectors contribute significantly to the convenient representation of each channel. This number is approximately proportional to the product of the corresponding normalized Doppler spread.
$B_D T$ and the number $K$ of symbols per transmitted block. Hence, for complexity reduction all negligible coefficients can be removed from the expression of the algorithm with unnoticeable degradation in performance. In Figure 1, we show the behaviour of the raw Binary Error Rate (BER), averaged over all block data symbols, as a function of $E_r / N_0$ where $E_r = 1/2 (\phi_0^T (0) + \phi_1^T (0)) E$. We consider the two cases of similar and different Doppler spreads of the two channels seen at the destination. As benchmarks, we also show in this figure the raw BER for the proposed Alamouti DAF protocol with Perfect Channel State Information (PCSI) and the raw BER obtained when we use the direct link source-destination (no use of DAF cooperative strategy) with PCSI. For the DAF strategy we assumed a perfect detection at the two relays. We add in this figure the raw BER for the proposed Alamouti DAF protocol when the two relays estimate the channels seen from the source using the same iterative algorithm adapted for a SISO (Single-Input Single-Output) link, detect the transmitted sequence and forward it to the destination (curve labelled with MAP-R). For that, the source-relay link is assumed to be slowly time-varying channel and characterized by a normalized Doppler spread $B_D T = 1/256$ and a signal to noise ratio fixed at $25 dB$.

![Figure 1](image1.png)

**Fig. 1. Raw BER as a function of $E_r / N_0$ with sub-optimum initialization**

We notice that the best achievable simulated raw BER for the links characterized is reached after 3 or 4 iterations depending on the Doppler spread value. We can see also that the Alamouti DAF protocol has better performance than the scheme that doesn’t use the cooperative strategy. Figure 1 shows also that for low normalized Doppler spreads, the performance of the proposed iterative receiver approaches that of the PCSI receiver. It shows also that the performance of this receiver taking into account the MAP detection at the two relays is close to the one obtained with perfect detection at these relays. However, for large values of the normalized Doppler, we notice the instability of the algorithm and its possible convergence to a local maximum rather than the global one. We show in Figure 2 the behaviour of the raw BER, averaged over all block data symbols, as a function of $E_r / N_0$. Sub-optimum initialization and optimum initialization are considered for the case where the two links are characterized by the same normalized Doppler spread $B_D T = 1/32$ and for the case where they are characterized by two different Doppler spreads. We assume a perfect detection of the transmitted sequence at the two relays. Figure 2 shows that for the case when the one or the two links are characterized by a large value of the normalized Doppler spread, an important enhancement in binary error rate performance is noticed when the initialization is optimized.

**V. ALAMOUTI DAF SCHEME WITH TWO RECEIVE ANTENNAS AT THE DESTINATION**

In this section, we consider a wireless network with a destination equipped with two receive antennas. Our aim is to find a robust receiver which efficiently estimates the four distorting channels seen at the two antennas of the destination in order to recover the transmitted information sequence. We propose the Alamouti 2x2 DAF protocol schematized in Table II.

![Figure 2](image2.png)

**Fig. 2. Raw BER as a function of $E_r / N_0$. SOI and OI stand for sub-optimum initialization and optimum initialization**

<table>
<thead>
<tr>
<th>Source</th>
<th>$S_{2k}$</th>
<th>$S_{2k+1}$</th>
<th>$S_{2k+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relay 0</td>
<td>$Y_{0,2k}$</td>
<td>$Y_{0,2k+1}$</td>
<td>$S_{2k}$</td>
</tr>
<tr>
<td>Relay 1</td>
<td>$Y_{1,2k}$</td>
<td>$Y_{1,2k+1}$</td>
<td>$S_{2k+1}$</td>
</tr>
<tr>
<td>Dest(Ant0)</td>
<td>$R_{0k}^k$</td>
<td>$R_{0k}^{k+1}$</td>
<td></td>
</tr>
<tr>
<td>Dest(Ant2)</td>
<td>$R_{1k}^k$</td>
<td>$R_{1k}^{k+1}$</td>
<td></td>
</tr>
</tbody>
</table>

All channels considered in the two phases of the transmission are assumed to be non dispersive in time. We model the four channels seen at the destination by complex multiplicative distortions $c_{1,l}^q$ where $l = 0$ or 1 represents the cooperative relay and $q = 0$ or 1 represents the receive antenna index at the destination. Let $c_{1,l}^q = (c_{1,l,0}^q, c_{1,l,1}^q, \cdots, c_{1,l,2N-1}^q)^T$, the multiplicative distortion vector for $l, q = 0, 1$. From Table II, the components of the received vectors at the two relays are the same as the ones given for the DAF protocol described in Table
I. In the second phase of the transmission, the components of the received vectors $\mathbf{R}^q$, $q = 0, 1$ can be written as

$$R^q_{2k} = \sqrt{E_d} c^q_{1,2k} S_{2k} + \sqrt{E_d} c^q_{1,2k} S_{2k+1} + n^q_{2k}$$  \hspace{1cm} (18)$$

for even indices and

$$R^q_{2k+1} = -\sqrt{E_d} c^q_{1,2k+1} S_{2k+1} + \sqrt{E_d} c^q_{1,2k+1} S_{2k+2} + n^q_{2k+1}$$  \hspace{1cm} (19)$$

for odd indices, where $E_d$ is the average radiated energy per symbol at the $l^{th}$ relay, $l = 0, 1$, and $n^0_{2k}$ and $n^1_{2k}$, representing complex noise and interference at the two receive antennas of the destination, are assumed to be Gaussian distributed with zero mean and variances $N_{0,0}$ and $N_{0,1}$ respectively. As described in section IV, we can use, up to a multiplicative factor $\sqrt{E_d}$, the decimated versions of the channel vectors $\mathbf{C}^q_{i} = \sqrt{E_d} \left( c^q_{i,0}, c^q_{i,2}, \ldots, c^q_{i,2N-2} \right)^T$, $l, q = 0, 1$. We represent also these four channels with convenient representations (denoted by $\mathbf{G}^q_{i}$, $l, q = 0, 1$) based on the KL theorem as explained in section III with the same notations of the eigenvectors and the eigenvalues of the matrices $\mathbf{F}^q_l = E \left[ \mathbf{C}^q_{i} \mathbf{C}^q_{j}^T \right]$. We note that $\mathbf{F}^q_l$ is independent of $q$. After some calculations, omitted here for lack of space, we show that the $k^{th}$ component of the vector $\mathbf{G}^q_{i}$, $l, q = 0, 1$, is explicitly given by

$$\left( \mathbf{g}^q_{i,k} \right)^{ls} = \frac{1}{1 + N_{0,l} / \lambda_{bk}}$$

where $\lambda_{bk} = \frac{1}{(1 + N_{0,l} / \lambda_{bk})}$ for $k = 0, 1, \ldots, N - 1$ and $l, q = 0, 1$ and

$$\tilde{S}^q_{2k+1} = \sum \mathbf{P}(S|\mathbf{R}, \mathbf{G}^q_{i})$$

for $i = 0, 1$. The explicit expressions of $\tilde{S}^q_{2k}$ and $\tilde{S}^q_{2k+1}$, for QPSK modulated symbols, are given by

$$\tilde{S}^q_{2k+1} = \frac{1}{2} \tanh \left( \frac{1}{2} \sqrt{\mathbf{b}^q} \mathbf{R}^q \right) + \frac{1}{2} j \tanh \left( \frac{1}{2} \sqrt{\mathbf{b}^q} \mathbf{R}^q \right)$$

for $i = 0, 1$, where $\lambda_{0,2k}$ and $\lambda_{1,2k+1}$ are given by

$$\lambda_{0,2k} = \frac{R_{2k} c^q_{0,k}}{N_{0,0}} + \frac{R_{2k+1} c^q_{1,k}}{N_{0,1}}$$

and

$$\lambda_{1,2k+1} = \frac{R_{2k} c^q_{0,k}}{N_{0,0}} - \frac{R_{2k+1} c^q_{1,k}}{N_{0,1}}$$

The iterative receiver proposed in this section can also profit from an optimum initialization by means of the MAP data-aided channel estimation algorithm as explained in section IV. We show in Figure 3 the performance of the proposed receiver when $E_0 = E_1$ and $N_{0,0} = N_{0,1} = N_0$. It plots the raw BER, as a function of $E_r/N_0$ where $E_r = 1/2(\phi_0(0) + \phi_1(0))E$. As benchmarks, we show also in this figure the raw BER for A2x2 DAF protocol with PCSI and the raw BER obtained with A2x1 DAF protocol.

Figure 3 shows that the performance of the system proposed in this section is greatly improved with respect to the one proposed in section II. We see also that the performance of the iterative receiver approaches that of the PCSI receiver and that an optimum initialization leads to an important enhancement in bit error rate performance.

VI. CONCLUSION

We have proposed two cooperative schemes using one source, one destination and two relays to bring cooperative diversity. We assumed the link between the source and the destination is so bad that we didn’t consider it. The destination can be equipped with one or two receive antennas. We proposed to use a DAF strategy at the two relays. We have proposed a MAP iterative receiver at the destination which can profit from an optimum initialization. We noticed that the degradation in performance presented by this algorithm with respect to perfect channel state information is very small.

REFERENCES