Spectral Efficiency of Multiple Access Fading Channels with Adaptive Interference Cancellation

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Abstract

Reliable estimation of users’ channels and data in rapidly time varying fading environments is a very challenging task of multiuser detection (MUD) techniques that promise impressive capacity gains for interference limited systems such as non-orthogonal CDMA and spatial multiplexing MIMO based LTE. This paper analyzes relative channel estimation error performances of conventional single user and multiuser receivers for an uplink of DS-CDMA and shows their impact on output signal to interference and noise ratio (SINR) performances. Mean squared error (MSE) of channel estimation and achievable spectral efficiencies of these receivers obtained from the output SINR calculations are then compared with that achieved with new adaptive interference canceling receivers. It is shown that the adaptive receivers using successive (SIC) and parallel interference cancellation (PIC) methods offer much improved channel estimation and SINR performances, and hence significant increase in achievable sum date rates.

Keywords: Multiuser Detection, Interference Cancellation, Channel Estimation

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1 Introduction

Study of achievable spectral efficiency of realistic interference limited systems such as uplink of CDMA and spatial multiplexing based MIMO systems using different detection techniques is very important from practical points of view. These systems have many similar characteristics: for example, decorrelators or zero forcing detectors used for the both systems are based on the same principle of nulling of interference subspaces and interference cancellation stage is usually added to further enhance the detection performance. Therefore, the techniques and analyses designed for CDMA uplink, are generally applicable to both these systems. There exists many studies on efficient detection techniques for CDMA and MIMO systems, e.g. [1], [2], [3], [4]. It has been noted in [5], an efficient interference cancellation technique combined with multiuser detection is very important to achieve the channel capacity.

The MUD schemes using interference cancellation techniques such as decorrelating decision feedback detectors and V-BLAST usually suffer from error propagation at each stage and requires very accurate channel estimation to maximize the achievable performance [6]. To address this problem, the authors have proposed new blind adaptive SIC [8] and PIC [7] techniques that exploits the interesting amplitude restoring property of the well known constant modulus algorithm (CMA) to minimize the error propagation. Referred to here as BA-SIC and BA-PIC, these schemes have shown to achieve much improved interference estimation and cancellation performances to minimize the error propagation.

Analyzing the spectral efficiency of different receivers under realistic mobile Rayleigh fading channels with estimation errors is very interesting and challenging problem. Asymptotic spectral efficiency analysis of CDMA with different multiuser receivers using random spreading sequences in simple additive white Gaussian noise (AWGN) environment has been shown in [9]. The spectral efficiency analysis of SIC receivers in Rayleigh fading with perfect channel estimation is carried out in [10]. In the case of PIC, signal to interference and noise ratio (SINR) and system capacity analysis of a linear PIC is carried out in [11] for simple AWGN channel environment.

These work have either assumed static channel or fading channels with perfect channel estimation. In practice, users’ channel are often time varying and with high Doppler rates. In such environment, estimation of desired users’ channels/data while fully suppressing interfering users’ signals is very challenging task. In this paper, we investigate channel/interference estimation and cancellation performance of conventional single user and multiuser receivers in comparison with the new adaptive IC receivers [8], [7] in mobile Rayleigh fading channels. We also derive their respective achievable spectral
efficiencies based on output SINR calculations to show the significant gains of the new adaptive approaches.

This paper is organized as follows. A general system model is described Section II. The estimation approaches of the adaptive schemes are described in Section III. The sum rate or spectral efficiency analysis of the schemes to be compared is carried out in Section IV. The performance study in terms of mean squared error of channel estimation and achievable spectral efficiency is provided in Section V. Finally the paper is concluded in Section VI.

2 System Model

An uplink of synchronous DS-CDMA system of $K$ users under Rayleigh flat fading and AWGN channel conditions is considered here. The received composite signal $\mathbf{r} = [r_1, r_2, \ldots, r_N]^T$ can be written as:

$$ r(m) = \sum_{k=1}^{K} \beta_k(m) b_k(m) c_k(m) + \mathbf{v}(m), $$

where $b_k$ is the data signal of period $T_b$ and satisfying condition $E\{b_k^2\} \leq P_k$ where $P_k$ is the $k^{th}$ user’s signal power. The spreading sequence is denoted as $c_k$ with antipodal chips with period $T_c$ and normalized power over a symbol period equal to unity $\int_0^{T_b} c_k^2(t) dt = 1$. The spreading factor is $N = T_b/T_c$ and $\mathbf{v}$ is the AWGN with two sided power spectral density $N_0/2$, $\beta_k(n) = g_k(n)e^{-j\phi_k(n)}$ is sample of time varying complex Rayleigh flat fading channel with zero mean and unit variance and consisting of amplitude $g_k$ and phase $\phi_k$ components, respectively. Note that flat fading channel model is used here for the simplicity; the frequency selective channels can also be considered in proposed system model by using the method in [10] by modeling each multipath signal as independently faded and uncorrelated with the desired path. Where it is also shown that spectral efficiency of a SIC can be higher than that in flat fading case and hence analysis of our schemes can also be extended to multipath channels.

Using the generic system model above, main ideas behind different receiver techniques and the new adaptive methods are briefly described next. For all receivers, it is assumed that coherent phase reference with perfect knowledge of phases of all users’ channels are available and without loss of generality $k^{th}$ user is the desired user. Note that channel phase estimation can be achieved at the receiver by using the known pilot sequences inserted in users’ data streams and is widely used in existing wireless systems.

**Conventional Receivers/Matched Filters (MF):** The MF detectors obtain data and channel estimates using the despreader
output $z_k^{MF}$, also given by

$$z_k^{MF} = \int_0^{T_b} r_k c_k^T = g_k b_k + \sum_{j=1, j \neq k}^K \rho_{kj} g_j b_j + v_k$$

(2)

where the three terms are the desired signal, the sum of interfering users’ data signals, and the AWGN, respectively and $\rho_{ki}$ is the magnitude of cross-correlation between $k^{th}$ and $i^{th}$ users’ spreading sequences. As can be noted here that for the MF, the channel and data estimate are estimated jointly and is simply taken as

$$\hat{g}_k b_k = z_k^{MF}.$$  

(3)

This leads to the SINR $\Gamma$, expression of the receiver as:

$$\Gamma_k^{MF} = \frac{E(z_k^{MF})^2}{\text{var}(z_k^{MF})} = \frac{E\{g_k b_k\}^2}{E\{\sum_{j=k}^K \rho_{kj} g_j b_j\}^2 + N_0}$$

(4)

where $E\{\cdot\}$ and $\text{var}\{\cdot\}$ denote the expectation and variance, respectively of a random variable.

**Conventional PIC:** The PIC receivers employ parallel IC processes one for each user and iterative detection using multiple i.e. $\geq 2$ stages to refine the users’ data estimates. The first stage of a PIC is usually the bank of matched filters hence it’s performance at this stage is the same as that of the conventional MF receivers. The decision variable at $l^{th}, l \geq 0$ stage $z_k^{PIC(l)}$ is obtained by cancelling from $r$ the summation of the other users’ signals as follows:

$$z_k^{PIC(l)} = \int_0^{T_b} \left( r - \sum_{j=1, j \neq k}^K z_j^{PIC(l-1)} c_j \right) c_k^T; \forall k.$$  

(5)

The data and channel estimate at $l^{th}$ stage is taken as

$$\hat{g}_k^{PIC(l)} b_k = z_k^{PIC(l)}.$$  

(6)

As can be seen conventional PIC reduces to MF receiver at $l = 0$. The SINR for a PIC is dependent upon the estimation accuracy of the MF detectors used for each user i.e. by the mean square error (MSE) which will be give later in equation (12). Using the MSE measure, the SINR at $l$ stage can also be obtained as:

$$\Gamma_k^{PIC(l)} = \frac{E\{g_k b_k\}^2}{E\{\sum_{j=1}^K (\hat{g}_j^{PIC(l)} - g_j)^2 \rho_{kj} b_j\}^2 + N_0}.$$  

(7)

**Conventional SIC:** The SIC technique operates on the principles of ordering of users’ signals based on their received power estimates for performing decision on the strongest and then subtracting the estimated signal from the total remaining received signal. The decision variable at each stage $k, 1 \leq k \leq K$, is obtained as

$$z_k^{SIC} = \max_{1 \leq u \leq K} \left\{ \int_0^{T_b} r_k^T c_u \right\}. $$

(8)
The data and channel estimate is taken as

\[ \hat{g}_k b_k = z_{k}^{SIC}. \]  \hspace{1cm} (9)

The signal \( z_{k}^{SIC} \) is respread and subtracted from the remaining received signal to remove its interference as follows:

\[ r_{k+1} = r_k - z_{k}^{SIC} c_k. \]  \hspace{1cm} (10)

These processes are carried out until all users’ data are detected. The SINR for the case of SIC is variable for each user as the earlier detected user sees more interference while the least detected user may not see any interference. Although this assumption is inaccurate due to the imperfect estimation of each users’ signal, we adopt this simplified approach to just reveal the relative SINR gain of the SIC technique. The SINR for a user using the SIC technique can be shown as:

\[ \Gamma_{k}^{SIC} = \frac{E(g_k b_k)^2}{E(\sum_{j=k+1}^{K} \rho_{kj} \hat{g}_j b_j)^2 + N_0}. \]  \hspace{1cm} (11)

The main problem with MF and IC receivers in fading channels is the unreliable estimation and hence cancellation of other users’ interference contributions, particularly when the users’ channels are time varying. It is well known that in CDMA, suppression of interference leads to improved signal estimation of desired users, leading to improved BER and system capacity performance. We can assess the accuracy of the estimation performance of conventional and blind adaptive receivers in terms of average MSE from all \( K \) users, \( \varepsilon^2 \) given by

\[ \varepsilon^2 = \frac{1}{K} \sum_{k=1}^{K} \varepsilon_k^2 = \frac{1}{K} \sum_{k=1}^{K} E\{(\hat{g}_k | \hat{b}_k | - g_k | b_k |)^2\}. \]  \hspace{1cm} (12)

### 3 Blind Adaptive IC Receivers

Blind adaptive approaches to IC as in [7] and [8] reduce the interference effects in two steps:

a) employ adaptive despreading to generate decision variables based on minimum error cost function to preliminarily suppress the interference,

b) use the signal estimates for interference cancellation that are generated blindly from the despreader output and weighted by an adaptive scaling factor at every symbol period \( m, 1 \leq m \leq \infty \).

Unlike the conventional despreaders that multiply the received signal with local copy of fixed amplitude spreading sequence, the adaptive despreaders use weights \( w_k(m) \) are updated every symbol using the CMA algorithm employing a
simple LMS type updating [14] with an objective to minimize the error $e_k(m) = E\{ | b_k(m) | - \sum w_k(m) \}^2$ and is shown below

$$w_k(m + 1) = w_k(m) - \mu \mathbf{r}_k(m)e_k(m)$$  \hspace{1cm} (13)

where $\mu$ is the step size and $e_k$ is the instantaneous error of CMA. Next, the scaling factor $\alpha_k(m)$ is obtained using the despreader weights $w_k(m + 1)$ and the spreading sequence vector $c_k$ as follows

$$\alpha_k(m) = \frac{\tilde{c}_k(m)}{\tilde{w}_k(m)}$$  \hspace{1cm} (14)

where, $\tilde{c}_k(m)$ and $\tilde{w}_k(m)$ are the mean amplitude of chips of user’s spreading sequence and elements of the weight vector, respectively and are given by

$$\tilde{c}_k(m) = \frac{1}{N} \sum_{n=1}^{N} | c_k \{ (m-1)N + n \} |$$  \hspace{1cm} (15)

and

$$\tilde{w}_k(m) = \frac{1}{N} \sum_{n=1}^{N} | w_k \{ (m-1)N + n \} |.$$  \hspace{1cm} (16)

The signal $z_k(m)$ is then scaled with $\tilde{\alpha}_k(m)$ and spread with $c_k$ to generate the cancellation term for $k^{th}$ user for the next stage. Based on above principles, the detection and estimation processes for BA-SIC and BA-PIC are given next.

**BA-PIC** [7]: This scheme operates on the same principles as the conventional PIC, however it obtains the decision variable for the $k^{th}$ user at the $l^{th}$ PIC stage $z_k^{BA-PIC(l)}(m)$ by utilizing the scaling factors as shown above, obtained from the previous stage $\alpha_{j}^{l-1}(m)$ as follows

$$z_k^{BA-PIC(l)}(m) = \int_{0}^{T_b} \left\{ r(m) - \sum_{j=1, j\neq k}^{K} \alpha_{j}^{l-1}(m)z_j^{BA-PIC(l-1)}c_j^T w_k(m) \right\}; \forall k.$$  \hspace{1cm} (17)

The user’s channel estimate at the $l^{th}$ stage can be taken as follows:

$$\hat{g}_k(m)b_k(m) = \alpha_k^l(m)z_k^{BA-PIC(l)}(m).$$  \hspace{1cm} (18)

The SINR of the BA-PIC at the $l^{th}$ stage is obtained as follows:

$$\Gamma_k^{BA-PIC(l)} = \frac{E(\hat{g}_k b_k)^2}{\text{var}\{z_k^{BA-PIC(l-1)}\}}.$$  \hspace{1cm} (19)
Since the BA-PIC employs adaptive despreader at the initial stage, the variance of interference at this stage is expected to lower than conventional PIC. This is given in [7], as follows:

\[
\text{var}\{ z_{BA-PIC}(0) \} = K MR(\Delta t) \sum_{m=1}^{M} \frac{\{e_k^0(m) \pm \mu\}^2}{N}.
\] (20)

From above equation, we observe that the variance \text{var}\{ z_{BA-PIC}(0) \} for a given time period depends upon system parameters such as number of users in the system \( K \), the frame length \( M \) considered (usually \( M >> 1 \)), the channel fading rate defined by the Doppler shift \( R(\Delta t) \), the degree of freedom for weight adaptation \( N \) and the choice of step size \( \mu \) [14].

Note that the choice of step size used in the adaptive algorithms has direct effect on the detection performance. Although the step size can also be varied adaptively to optimize the system performance, this is left for future study. Therefore \text{var}\{ z_{BA-PIC}(0) \} will be minimized, when \( N \) is sufficiently large and \( R(\Delta t) \) does not change significantly for the given frame period \( M \). The following PIC stages improve the detection performance significantly using adaptively weighted interference cancellation using the despreader’s weights leading further reduced variance of interference. The variance at the \( l \)-th stage of the BA-PIC is obtained as follows:

\[
\text{var}\{ z_{BA-PIC(l)} \} = \varepsilon_k^2 + \sum_{j=1, j \neq k}^{K} \varepsilon_j^2 \{ \rho_{kj}^2 - \kappa_{ji} \rho_{ki}^2 \} \text{var}\{ z_{BA-PIC(l-1)} \};
\] (21)

\[
\kappa_{ji} = \begin{cases} 
1 & \text{if } i = j, \\
-1 & \text{if } i \neq j 
\end{cases}; \quad j = 1, ..., K; j \neq k.
\]

**BA-SIC [8]:** The decision variable for the \( k \)-th user, \( z_{BA-SIC}^k(m) \) at each SIC stage is obtained by selecting the despreader output of the strongest user as follows:

\[
z_{BA-SIC}^k(m) = \max_{1 \leq u \leq K} \left\{ \int_0^{T_b} r_k^T(m) w_u(m) \right\}
\] (22)

where \( w_u(m) \) is the weight vector of the adaptive despreader updated every symbol. The data and channel estimate of the \( k \)-th user can be obtained as follows:

\[
g_k(m) b_k(m) = \alpha_k(m) z_{BA-SIC}^k(m).
\] (23)

The signal \( z_{BA-SIC}^k \) is then respread using \( c_k \) and subtracted from remaining received signal to remove its interference similar to that in (10). The SINR analysis for BA-SIC can be obtained following similar approach for the BA-PIC as shown in (17)-(19) and is not shown here for brevity.

It can be observed from the above equations that, the data detection performance of all the receivers are dependent upon the MSE performance of users’ channel estimates. Hence we can conclude that techniques that achieve lower MSE reduce
interference and hence the improved SINR or sum rate performances.

4 Analysis of Achievable Sum Rates

In this section, a simplified analysis of achievable sum rate or spectral efficiency the system in a single isolated cell for the receivers is provided assuming Gaussian distribution of interference \[1\] and calculation of the expected SINR \( E\{\Gamma\} \). Using instantaneous SINR \( \Gamma \) and distribution \( p(\Gamma) \), the multiple access capacity \( C \) in a fading channel can be obtained as follows:

\[
C = \int_{0}^{\infty} B \log_2 \left[ 1 + \Gamma \right] p(\Gamma) d\Gamma
\]

(24)

where \( B \) is the signal bandwidth. In CDMA, the available bandwidth is equally divided among \( K \) users each occupying \( 1/N \) portion. Therefore, the normalized spectral efficiency or sum rate \( R_{sum} \), is obtained as follows

\[
R_{sum} \leq \frac{K}{N} \log_2 \left[ 1 + E\{\Gamma\} \right]
\]

(25)

where \( \overline{\Gamma} = 1/K \sum_{k=1}^{K} \Gamma_k \) is the average SINR. Jensen’s inequality theorem \[12\] is assumed here and hence the obtained spectral efficiency above represents as the upper bound on what these techniques can actually achieve in practice.

**BA-PIC**: The spectral efficiency of BA-PIC at the \( l\)th stage can be obtained as follows

\[
R_{sum}^{BA-PIC(l)} \leq \frac{K}{N} \log_2 \left[ 1 + E\{\Gamma_{BA-PIC(l)}\} \right]
\]

(26)

The lower variance of of BA-PIC compared with conventional PIC for each stage is verified analytically and by simulations in \[7\]. Therefore, it is expected to provide higher spectral efficiency.

**BA-SIC**: The spectral efficiency of system employing BA-SIC can be obtained as follows

\[
R_{sum}^{BA-SIC} \leq \frac{K}{N} \log_2 \left[ 1 + E\{\Gamma_{SIC}\} \right]
\]

(27)

\[
= \frac{K}{N} \log_2 \left[ 1 + \frac{1}{K} \sum_{k} \Gamma_{BA-SIC} \right]
\]

Since \( var\{z^{BA-SIC}\} \) will be much lower compared with conventional SIC, it is anticipated that the sum rate will be higher accordingly.

5 Performance Study

A baseband model of a synchronous uplink DS-CDMA with \( K = 20 \) equal power users i.e. average power control is assumed. Binary Gold sequences of length \( N = 31 \) are used for spreading. The channel is i.i.d Rayleigh flat fading for each
user with normalized Doppler rate $f_d T_b$ of 0.003 corresponding to mobile speed to $\approx 100$ km/hr at the carrier frequency of 2 GHz. A fixed step-size of $\mu_k = 0.0001$, is assumed in the adaptive despreader algorithm for the detection of all users for the case of adaptive receivers. We compare the system performance of conventional techniques, namely the MF, PIC and SIC with the adaptive techniques in terms of MSE of channel estimation and the spectral efficiency next.

**MSE:** Figure 1 shows the MSE of channel estimation of conventional SIC and BA-SIC when users employ binary PSK for data modulation. It can be clearly seen that the adaptive technique offers much improved MSE as the $E_b/N_0$ increases. For example, an MSE of 0.0075 compared with 0.034 at the $E_b/N_0 = 30$ dB is shown to be achieved. In Figure 2 the MSE performances of BA-PIC at different interference cancellation stages are shown and compared to that with conventional PIC. The BA-PIC shows much improved MSE for all stages. For example, at the stage 1 it offers MSE of 0.002 which is better than that of stage 3 of conventional PIC with 0.0025.

**Spectral Efficiency:** Achievable spectral efficiency of BA-SIC using the expression in (27) is shown in Figure 3. The spectral efficiency of a fully loaded synchronous orthogonal CDMA which is equivalent to a single user system [9], is also shown for comparison. As expected, the BA-SIC shows considerable gain compared with MF and conventional SIC giving
Figure 2: MSE of channel estimation for the BA-PIC in Rayleigh flat fading channel: $K = 20$, $N = 31$ (Gold sequence) and $f_d T_b = 0.003$ are used.
≈ 4.4 bits/s at the SNR of 30 dB compared with ≈ 2.7 and ≈ 3 bits/s, respectively. It can also be noted that, the conventional SIC offers only slightly higher spectral efficiency compared with MF. This is due to imperfect estimation and cancellation of the existing SIC methods [13]. The spectral efficiency of BA-PIC using (26) with multiple stages of cancellation is shown in Figure 4. It can be clearly seen from the figure that the scheme offers significant increase in the achievable rate (spectral efficiency) compared with the conventional PIC. For example, even with single stage of cancellation the BA-PIC shows the performance comparable with the conventional PIC with three stage of cancellation, providing ≈ 5 bits/s at the SNR of 30 dB. From the figures [3] and [4] we can also compare the sum rate or the spectral efficiency of BA-SIC and BA-PIC under the same system conditions. It can be seen that the BA-PIC offers much higher rate compared with BA-SIC as the number of cancellation stages \( l \) increases. For example, even with a single stage of cancellation, BA-PIC gives the rate of ≈ 5.5 bits/s at the SNR of 30 db, which is much higher than ≈ 4.5 bits/s of the BA-SIC.

6 Conclusions

We investigated the channel estimation performance of different multiuser detection techniques and its impacts on the SINR and achievable spectral efficiency in highly mobile Rayleigh fading channel environments. Significantly improved MSE
and sum data rates are shown to be achieved with adaptive interference canceling receivers. The adaptive receivers are shown to achieve much higher sum rates, for example, $\approx 7.3$ bits/s compared with $\approx 5.3$ bits/s for the case of using parallel interference cancellation, and also $\approx 4.5$ bits/s compared with $\approx 3.0$ bits/s for successive cancellation architecture, respectively. For the future work, more efficient adaptive interference suppression techniques for joint carrier frequency offset estimation and cancellation for OFDMA will be carried out.

References


