Hanging around and wandering on mobile robots with a unique controller

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Abstract—Patrolling and wandering are two robotics useful behaviours to implement, among others; cleaning, surveillance or exploration tasks. This paper presents an approach to generate these behaviours using the Braitenberg vehicle 2b. After introducing a mathematical model of such vehicles, the resulting trajectories are analysed. Since the model is a non-linear differential equation, attractors appear in the robot’s workspace, in contrast with the random walks usually generated by other wander implementations.

Index Terms—Mobile Robots, Wandering, Patrolling, Braitenberg Vehicles.

I. INTRODUCTION.

Wandering or moving aimlessly, is an interesting behaviour for mobile robots. It can be used as an exploratory mechanism, and inherently implies some kind of obstacle avoidance. Hanging around, or patrolling, has also interesting applications. It consist on a back and forth robot motion restricted to a certain spatial region, though the region could be obstacle free. These two behaviours seem not to have much in common, wandering needs obstacle avoidance with a usually randomly chosen target point, whilst hanging around can be implemented as a simple trajectory control by setting fixed points in the patrolling area. However, this paper presents a unique control mechanism that generates both behaviours depending on a selected function. The proposed controller is derived from a biological model of behaviour, specifically from the work of Braitenberg [3]. The vehicle 2b is used to generate these different behaviours depending on the specific shape of the stimulus. Considering the simplicity of the vehicles proposed [3], we can argue this control mechanism is parsimonious but, at the same time, general enough to produce such different behaviours. On the other hand, no theoretical analysis of Braitenberg vehicles 2b can be found in the literature.

Wandering became an important mobile robot skill after the raising of behaviour based robotics [5] and the incremental building of behaviours on robots. Before that time, all the robots, with rare exceptions, were programed to carefully plan its motion in order to reach some target. Even wandering by itself does not seem a useful skill, it can be actually used as an exploration mechanism [4], foraging in multi-robot systems [11] or even to drive the robot motion while performing SLAM [8]. The first wandering robot was reported in [4], where a random heading generator imposed a moving direction to an obstacle avoidance mechanism. Another early implementation of this behaviour can be found in [2], though both rely in potential field based obstacle avoidance and mixed controllers or control modules. These implementations of the wandering behaviour include a stochastic component which can be undesirable for certain applications. A recent implementation of a wander behaviour in real robots using Braitenberg vehicle 2b has been presented in [14]. A main difference of this mechanism is that it does not rely on a random heading, but an attractor trajectory can appear on the robot’s motion as it will be shown later. Therefore this paper introduces the theoretical background of real world usage of 2b type Braitenberg vehicles.

Patrolling has become recently a topic of interest in robotics because it can be used for surveillance or cleaning task. It can be broadly defined as a cyclic behaviour of walking around a fixed area [6]. Surveillance is performed usually by a set of robots [1] moving around a closed area, though it can be performed by a single robot following a scheduled trajectory [9]. Some optimal criteria can be defined over the trajectory, like covering the whole area in a minimal time, however the mechanism presented in this paper does not deal with optimality and it is used on single robot patrolling. The rest of the paper is organised as follows. Section II presents the aggression model of Braitenberg, both qualitatively and quantitatively. It also states the working assumptions for the analysis of the trajectories and summarises the methodology used to analyse the state space of the derived dynamical system. The generation of the hanging around or patrolling behaviour is presented in Section III. A simple wandering scenario is simulated in Section IV. The paper ends with some conclusions and further research directions in section V.

II. BRAINTENBERG’S MODEL FOR AGGRESSION.

In [3] a series of dual-drive vehicles displaying different behaviours are presented. Starting from simple vehicles, the author explains and models in a qualitative way apparently complex behaviours. He includes the effect of the external stimulus in the analysis of the behaviours. The simplest vehicles consist on directed or crossed functional connections between the sensors and the motors. Some vehicles have increasing connections such that the stronger the stimulus in the sensor gets the faster the wheels turn, making the vehicle increase its velocity. For other vehicles a decreasing connection is used. Since the environment can be populated with different stimuli sources the vehicles can perceive, the resulting behaviour is rich and complex.
One of the vehicles commonly known as vehicle 2b, displays what the author calls aggression. The internal connections are shown in Figure 1. Each motor is linked to the sensor on the opposite side in an increasing way. The sensor on each side of the vehicle captures some stimulus from the environment, and the stronger the stimulus gets the faster the wheel spins, that is why the ‘+’ sing on the figure. For instance, if the stimulus on the right sensor is stronger than the left one, the left wheel will turn faster. The overall effect is making the robot turn in the direction of the stimulus. Moreover, the velocity of the vehicle will increase as it approaches the stimulus source, since both wheels increase their turning speeds. Therefore, the vehicle heads the stimulus source while increasing its velocity. This is why the behaviour of this vehicle is called aggression, the vehicle will reach the source with a maximal speed.

Fig. 1. Internal structure of the Braitenberg vehicle 2b.

If the vehicle can pass through the stimulus source, for instance if the source is virtual, its velocity will decrease as it moves away from it. At the same time, if the stimulus intensity reaching both sensors is equal, it will not turn, i.e., it will follow a straight line. However, in a real world situation it is highly improbable the two sensors having exactly the same value because of the noise. It also could happen that the vehicle does not point exactly on the direction of the source. Either situation results in the vehicle perceiving different sensor intensities, potentially making it turn again towards the source. The whole process could start again. Hence, in the case of noisy environments or if the vehicle does not get the stimulus direction exactly the vehicle can move around the stimulus source.

A. The Mathematical Model and the Assumptions.

Let us assume the stimulus is represented by a scalar positive function of the position $E(x) > 0$, and the relation between the sensors and the motor speed is a functional relation $\omega = F(e)$ where ‘e’ is the stimulus value on the sensor and ‘$\omega$’ is the turning speed of the wheel. Let us impose the function connecting the sensors and the motors the condition $F(e) > 0$. This assumption avoids the vehicle moving backward, a biologically plausible assumption. If both functions $F(e)$ and $E(x)$ are smooth the compound function $F(E(x))$ can be approximated as a first order Taylor series around the midpoint between the sensors of the vehicle ‘$x_0$’ as presented in [13]. Using this approximation to compute the vehicle velocities and decomposing its Cartesian components, the system of differential equations describing the motion of the vehicle can be stated as:

$$\dot{x} = F(x) \cos \theta$$  \hspace{1cm} (1)
$$\dot{y} = F(x) \sin \theta$$  \hspace{1cm} (2)
$$\dot{\theta} = \frac{\delta}{d} \nabla F(x) \cdot \dot{e}_p$$ \hspace{1cm} (3)

where $d$ is the wheelbase of the vehicle, $\delta$ is the distance between the sensors, $(x, y, \theta)$ is the robot pose, $\dot{e}_p = [-\sin \theta \cos \theta]^T$ is a unit length vector orthogonal to the robot heading and $F(x)$ denotes the compound function $F(E(x))$.

We will assume the stimulus source is at the origin, hence the function $E(x)$ has a maximum at the origin. Since the function $F(E)$ is increasing, $F(x)$ has a maximum at the origin of the coordinate system too. Therefore, the gradient of $F(x)$ vanishes at the origin, $\nabla F(0) = 0$, and its Hessian matrix is a negative definite form, $y^T D^2 F(0) y < 0 \forall y \in \mathbb{R}^2$.

Some common stimulus, like light sources, have circular symmetry. This means the stimulus $E(x)$ depends only on the polar coordinate ‘$r$’. Therefore, the overall function will also depend only on the distance to the source, $F(x) = F(r)$. If the system of differential equations (1), (2) and (3) is converted to polar coordinates, under the assumption of circular symmetry, the resulting system of differential equations is:

$$\dot{r} = F(r) \cos(\theta - \psi)$$  \hspace{1cm} (4)
$$\dot{\psi} = \frac{F(r)}{r} \sin(\theta - \psi)$$  \hspace{1cm} (5)
$$\dot{\theta} = -\frac{\delta}{d} \frac{\partial F(r)}{\partial r} \sin(\theta - \psi)$$ \hspace{1cm} (6)

where $(r, \psi)$ are the polar coordinates of the vehicle and $F(r)$ is the compound function $F(E(r))$. The dimension of the system can be reduced by defining a new variable $\eta = \theta - \psi$, such that the above equations can be written as:

$$\dot{r} = F(r) \cos \eta$$  \hspace{1cm} (7)
$$\dot{\eta} = -\left[\frac{F(r)}{r} + \frac{\delta}{d} \frac{\partial F(r)}{\partial r}\right] \sin \eta$$ \hspace{1cm} (8)

Having a system of two differential equations we can perform simple qualitative analysis of the solutions, though an effort must be done to interpret the meaning of the trajectories in terms of the robot’s pose variables. On the other hand, under non-circular symmetric stimulus the qualitative analysis of the trajectories generated by equations (1), (2) and (3) is much more complex and no general characteristics of the solutions can be obtained easily.

B. Qualitative Trajectory Analysis.

This section presents a qualitative analysis of the trajectories of the vehicle 2b obtained from equations (7) and (8). In order to analyse the trajectories of the aggression controller we will
consider the vector flow generated by the right hand side of the system of differential equations. The analysis technique is described in [10] and has been used in [12] on the taxis model. This method relies on the zero isoclines, the set of points of the phase space where the derivative with respect to time vanishes. The intersection of the zero isoclines for all the variables is the equilibrium set of the system of differential equations. Since all the functions involved in the flow are continuous, if one of the component becomes zero a change on the sign of the component can appear. This means the corresponding component of the vector field changes its direction on both sides of this set.

In order to obtain the isoclines for equation (7) we have to find the set of points where \( F(r) \cos \eta = 0 \). According to our working assumptions the function \( F(r) \) is always positive, \( F(0) > F(r) > 0 \forall r \in \mathbb{R}^n \). This will later translate in the absence of an equilibrium point in the systems of differential equations. Both functions \( F(r) \) and \( \cos \eta \) are at least \( C^1 \) but only the second one vanishes. Therefore the zero ‘r’ isoclines are the lines on the state space defined by equations \( \eta = \pm \pi /2 \).

The flow ‘r’ component will have the sign of \( \cos \eta \), meaning the direction of the vector flow will be positive for \( |\eta| < \pi /2 \) and negative for \( |\eta| > \pi /2 \). The radial flow component in the reduced state space is drawn qualitatively if Figure 2.

Due to the circular symmetry assumption, the gradient has only radial component and it points in the direction of the origin, the stimulus source. Therefore, the direction in which the function value increases is the opposite direction to the polar angular variable \( \psi \). Hence, when the robot is aligned with the gradient direction, i.e. \( \theta = \psi - \pi \), the difference angular variable will be \( \eta = \psi - (\psi - \pi) = \pi \). Contrarily, if the the robot heads the opposite direction to the gradient, its heading will be equal to the polar angular variable, \( \theta = \psi \), and the value of the angular difference will be zero \( \eta = 0 \). When the heading of the vehicle points in the direction of the stimulus source within a range of \( \pm \pi /2 \) the flow points towards the origin, and therefore the ‘r’ variable value will decrease. The distance will increase if the vehicle has the source behind. These behaviours can be deduced from the flow component in Figure 2. We can also analyse the module of the radial vector flow component. Since the function \( F(r) \) has its maximum at the origin, the radial velocity of the robot will increase as it approaches the origin. In fact, the velocity of the vehicle will be maximal at that point, the stimulus source. As Figure 2 shows the length of the vector increases for small values of \( \psi \), and decreases for large distances to the source or when the robot’s heading is orthogonal to the stimulus gradient (the zero r-isoclinc).

The zero isoclines for the angular variable \( \eta \) is obtained by setting equation (8) to zero. This generates two different conditions, \( \sin \eta = 0 \) and \( G(r) = \frac{F(r)}{r} + \frac{\delta \partial F(r)}{\delta r} = 0 \). Assuming \( G(r) \) keeps its sign constant for all ‘r’, the first condition represents two horizontal lines on the phase space, specifically \( \eta = 0 \) and \( \eta = \pi \). Therefore the angular component of the flow changes its sign at these values. However, the flow direction depends on the sign of \( G(r) \). Figure 3 represent the whole flow components depending on the sign of \( G(r) \). It is worth noting that \( G(r) \) can have a singularity at the origin \( r = 0 \), and the reduced angular velocity \( \dot{\eta} \) goes to infinity, but this must be analysed for each specific \( F(r) \) function.

1) Aggression simulation: In order to test the results expected from the analysis of the flow generated by equations (7) and (8) we simulated the aggression behaviour using as function \( F(r) = \frac{a}{1 + r} \). Since we do not want a change in the angular flow component, the condition \( G(r) > 0 \) must be imposed. This reduces to the condition \( 1 + ar(1 - \frac{d}{\delta}) > 0 \), which is fulfilled for any \( \frac{d}{\delta} < 1 \). Figure 4 represents one trajectory obtained for this stimulus simulated for 150 time units and initial conditions \( (3,0,10\pi/11) \). The values of the parameters for the figure simulation were \( g_0 = 5 \), \( a = 10 \) and \( \frac{d}{\delta} = 0.85 \). As deduced from the flow in Figure 3(a), the angular variable has an attractor at \( \eta = 0 \), i.e. \( \theta = \psi \), the robot heading the outward radial direction. Since the function \( F(r) \) has circular symmetry with a maximum at the origin, the direction of the gradient is contrary to the polar angular variable, hence it ends in the contrary direction of the source. Even the robot at first moves in the direction of the gradient, after approaching the source it will change its heading to point in the opposite direction. On the other hand, for the selected value, the flow \( \dot{\eta} \) component has a singularity point at \( r = 0 \), making the angular variables change faster as the vehicle approaches the source, as can be seen in Figure 4.

Even it cannot be inferred from the figures, since the linear velocity of the vehicle equals the value of the function \( F(r) \), the robot first increases its speed until it reaches the closest point to the source and then slows down while it moves away.

III. HANG AROUND (PATROLLING AROUND A POINT).

In this section we will analyse the case of the discriminant function \( G(r) \) changing its sign for some ‘r’ value. Interesting behaviours appear under these conditions. One of this behaviours is what we call hang around. If there is a
In order to verify the analysis of the resulting behaviour we simulated the system of equations (1), (2) and (3) with $F(r) = g_0 - ar$ as the stimulus function, where ‘$g_0$’ and ‘$a$’ are positive real values. For this function, the discriminant $G(r)$ of the equation (8) becomes $G(r) = \frac{5 a}{r} - a(1 + \frac{\pi}{2})$, and the change of sign occurs for $r_0 = \frac{a}{5(1 + \frac{\pi}{2})}$. It is worth noting that the selected function does not verify the condition of being positive for all the range of ‘$r$’, but only for $r < \frac{2a}{\pi}$, however the sign change on the flow will occur while the values are positive, because $r_0 < \frac{2a}{\pi}$. Since the function is continuous, the robot will slow down as it approaches $r = \frac{2a}{\pi}$ and it will eventually stop. In terms of the differential equations (7) and (8) this means that the trajectories will not go further than $r = \frac{2a}{\pi}$ since $r$ vanishes at this point, even the direction of the robot can change since $\eta \neq 0$ at $r = \frac{2a}{\pi}$.

Figure 6 represents the resulting trajectories for $g_0 = 5$, $a = 0.25$ and $\frac{\pi}{2} = 0.75$, simulated during 275 time units with the vehicle starting at the point $(1.5, 0)$ in Cartesian coordinates and a heading $\theta = 2\pi/3$. As it can be seen in Figure 6(b), the angular component of the flow changes its sign for $r = 20/1.75 \approx 11$, whilst the radial component increases for $\eta < \pi/2$ and decreases for $\pi/2 < \eta < \pi$. The arrows in the figure represent the direction of the trajectory in the reduced state space. The vehicle never goes further than $\frac{5 a}{\pi} = 20$ distance units since its linear velocity decreases as it approaches that distance, though the angular velocity makes the robot turn again towards the stimulus source. Figure 6(a) draws the actual vehicle trajectory in Cartesian coordinates. Even the trajectory in the reduced phase space is closed, in the workspace the robot will perform a hang around behaviour with the stimulus source at its centre. The trajectory in the workspace does not come back during the simulation time to the original vehicle pose, though as it can be seen in the figure the vehicle passed close to its initial position. In sum, a hang around behaviour for a stimulus source can be implemented if the selection of $F(E)$ is such that the join function looks like an inverted parabolic function of the position.

### IV. WANDERING.

The aggression Braitenberg vehicle has already been used to implement a wandering mechanism in real robots [14]. In this section some conclusions about this behaviour will be drawn out of simple simulations of the equations describing
Since the stimulus shows circular symmetry we can analyse the qualitative vector flow generated by equations (7) and (8). The flow completely vanishes for \( r < r_0 \) and \( r > r_1 \), since the function has a zero constant value in this range, but the angular flow component changes its sign for some radius which can be obtained as the solution of the equation \( G(r) = 0 \), leading to \( (1 + 2\alpha^2)\alpha r^2 + (1 + 2\alpha)br + c = 0 \). Since this is a second order equation two solutions appear, though obviously only the solution fulfilling \( r_0 \leq r_{sol} \leq r_1 \) must be considered.

The flow generated by the equations is presented in the Figure 10(b). It is similar to the flow of Figure 5 though it vanishes outside the range \( r_0 < r < r_1 \) and therefore the vehicle will never approach closer than \( r = r_0 \). Something interesting occurs for \( r = r_{sol} \), since it is a zero isocline of the reduced angular variable ‘\( \eta \)’. If the initial conditions of the robot are such that \( y_0^2 + y_0^2 = r_{sol}^2 \) and the heading is \( \theta = \arctan \frac{y_0}{x_0} \pm \pi \) both dynamic equations become zero simultaneously, since \( \cos(\pm\pi/2) = 0 \) and \( G(r) = 0 \). The trajectory in the reduced phase space is \( r(t) = r_{sol} \) and \( \eta(t) = \pm\pi \), therefore the robot stays at a constant distance of the origin and the heading is orthogonal to the angular polar coordinate, \( \eta = \theta - \psi = \pm\pi/2 \). The result is the robot describing a circular trajectory around the origin. For the case at hand the distance at which the robot turns is \( r_{sol} = 3.38 \) as can be seen in Figure 9, and since the initial pose was \( (r_{sol}, 0, \pi/2) \) it turned in the counter clockwise direction. As \( \eta(t) = \pi/2 \) and \( r(t) = r_{sol} \) the state space trajectory uniquely consist on the intersection point between the ‘\( r \)’ zero isocline, \( \eta = \pi/2 \) and \( r = r_{sol} \), see Figure 8.

\[
F(r) = \begin{cases} 
0 & \text{if } r \leq r_0 \\
\alpha r^2 + br + c & \text{if } r_0 < r < r_1 \\
0 & \text{if } r_1 \leq r
\end{cases}
\]

with \( r_0 < r_1 \). The values of the parameters ‘\( \alpha \)’, ‘\( b \)’ and ‘\( c \)’ have been selected such that the function is continuous and has a maximum at the midpoint between the maximum and minimum radius \( r_m = \frac{r_0 + r_1}{2} \). Specifically, \( r_0 = 1 \), \( r_1 = 5 \), \( a = -2 \), \( b = 12 \) and \( c = -10 \), and the shape of the resulting stimulus in the robot workspace is presented in Figure 7. This shape could represent some potential between two circular obstacles, for instance a function of the distance to the round shaped walls defined by two obstacles. Since the linear velocity of the robot is \( F(r) = F(x, y) \), it will always move forward unless the function \( F(r) \) vanishes, but this only happens when the robot reaches one of the circumferences defined by \( x^2 + y^2 = r_0^2 \) and \( x^2 + y^2 = r_1^2 \). In general the aggression behaviour will generate a proper wandering mechanism if the stimulus is related to the closest distance to an obstacle in a closed environment. Contrary to other wandering mechanism where the target is set at random an attractor trajectory appears on the robot behaviour.

Fig. 6. Example of hang around behaviour \((F(r) = 5 - 0.25r)\).

Fig. 7. Function \( F(x) \) to generate wander behaviour.

Fig. 8. Reduced state space flow for \( F(r) \) given by Equation (9)
generated for different stimulus. Even these vehicles display a behaviour named aggression by the author, they can be used to generate patrolling (hanging around) and wandering behaviours using the appropriate stimulus function. These robotic behaviours are useful for vigilance or exploration tasks respectively. Contrary to other wandering behaviours, using Braitenberg vehicles to implement it does not imply a random selected target but a deterministic attractor on the trajectory of the robot appears. The concrete attractor trajectory depends on the shape of the stimulus and on the robot morphology (δ).

An interesting future task is trying to find the attractor based on the stimulus and robot morphology for a general stimulus, which will mean finding the limit cycle of a system of differential equations. On the other hand, the connecting function could be adapted to obtain a desired attractor, such that the robot follows a specified trajectory. Such a wandering behaviour will be much more efficient for some tasks that a pure random walk. Also, this means a trajectory can be defined for the vehicle to follow. A potential application of the hanging around behaviour could be the generation of images from the patrolling arena using a camera on top of a robot [7].

REFERENCES


V. CONCLUSIONS AND FURTHER WORK.

This paper fills the existing lack of mathematical models of the Braitenberg vehicle 2b and analyses the trajectories