Time-Varying FIR Decision Feedback Equalization for MIMO Transmission over Doubly Selective Channels

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Abstract—In this paper we propose time-varying decision feedback FIR equalization techniques for multiple-input multiple-output (MIMO) transmission over doubly selective channels. The doubly selective channel is approximated using the basis expansion model (BEM), and equalized by means of time-varying FIR filters. The time-varying FIR filters are modeled using the BEM. By doing so, the time-varying deconvolution problem is converted into a two-dimensional time-invariant deconvolution problem in the time-invariant coefficients of the channel BEM and the time-invariant coefficients of the equalizer BEM. The DFE can be designed according to two different scenarios. In one scenario, the DFE is based on feeding back previously estimated symbols on one particular antenna to cancel/reduce inter-symbol interference (ISI) on that antenna, while in the other scenario, the previously estimated symbols on all transmit antennas are made available, and hence allows to cancel/reduce ISI in a joint fashion. The performance of the proposed equalizers in the context of MIMO transmission is demonstrated by numerical simulations.

I. INTRODUCTION

The wireless communication industry has experienced a rapid growth in recent years, and digital cellular systems are currently designed to provide high data rates at high terminal speeds. High data rates give rise to inter-symbol interference (ISI) due to the so-called multi-path fading. Such an ISI channel is called frequency-selective. On the other hand, due to terminal mobility and/or receiver frequency offset the received signal is subject to frequency shifts (Doppler shifts). The Doppler shift induces time-selectivity characteristics. The Doppler effect in conjunction with ISI give rise to the so-called doubly selective channel (frequency- and time-selective).

In this paper we propose time-varying FIR decision feedback equalization (DFE) for MIMO transmission over doubly selective channels. Decision feedback equalizers (DFE) employ previously detected symbols to compensate for ISI, and hence they are superior to their counterpart linear equalization techniques. For time-invariant channels DFEs are discussed in [1], [2]. Utilizing FIR filters for DFEs is investigated in [3]. The extension to MIMO transmission over time-invariant channels is proposed in [4]. For CDMA Downlink, the DFE is proposed in [5].

In this paper, we extend the results of [6], where time-varying FIR DFE equalization for single-input multiple-output (SIMO) transmission over doubly selective channels, to the case of MIMO transmission over doubly selective channels. For the MIMO transmission we consider spatial multiplexing transmission technique.

This paper is organized as follows. In Section II, the system model is introduced. The time-varying FIR decision feedback equalization is presented in Section III. Our findings are confirmed by numerical simulations introduced in Section IV. Finally, our conclusions are drawn in Section V.

Notation: We use upper (lower) bold face letters to denote matrices (vectors). Superscripts $H$, $T$, and $\ast$ represent Hermitian, transpose, and conjugate respectively. To simplify notations and save space, the double summation over the subscripts $i$ and $j$ is denoted as $\sum$, where the ranges of $i$ and $j$ should be clear from the context. We denote the $N \times N$ identity matrix as $I_N$, the $M \times N$ all-zero matrix as $0_{M \times N}$. The $\otimes$ denotes Kronecker product, and $\oplus$ denotes the direct sum. Finally, $\text{diag}(x)$ denotes the diagonal matrix with vector $x$ on its diagonal.

II. SYSTEM MODEL

We consider a multiple-input multiple-output (MIMO) system with $N_t$ transmit antennas and $N_r$ receive antennas. The input data stream is spatially multiplexed across the $N_t$ transmit antennas, and transmitted over the time-varying multi-path fading channel at a rate of $1/T$ symbols/s. The time-varying channel characterizing the link between the $t$th transmit antenna and the $r$th receive antenna at time-index $n$ is denoted as $g^{(r,t)}[n;r]$. The system under consideration is depicted in Figure 1. The baseband received symbol at the $r$th receive antenna at time-index $n$, $y^{(r)}[n]$ is obtained as

$$
y^{(r)}[n] = \sum_{t=1}^{N_t} \sum_{l=0}^{L} g^{(r,t)}[n;l] x^{(l)}[n-l] + v^{(r)}[n], \tag{1}
$$

where $x^{(l)}[n]$ is the QAM symbol transmitted from the $l$th transmit antenna at time-index $n$, and $v^{(r)}[n]$ is the additive noise at the $r$th receive antenna at time-index $n$. The basis expansion model (BEM) is used to approximate the doubly selective channel $g^{(r,t)}[n;l]$ for $n \in \{0, \ldots, N + L' - 1\}$ ($L'$ will be the time-varying equalizer order). In this model, the channel is specified as a time-varying FIR filter, where each tap is expressed as a superposition of time varying complex exponential basis functions with frequencies on the DFT grid. The $t$th tap of the time-varying FIR channel between the $t$th transmit antenna and the $r$th receive antenna at time-index $n$ is expressed as [7], [8], [9]

$$
h^{(r,t)}[n;l] = \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r,t)} e^{j2\pi q n/K}, \tag{2}
$$

where $Q$ is the number of time-varying basis functions satisfying $Q/(2KT) \geq f_{\text{max}}$, with $f_{\text{max}}$ is the channel maximum Doppler spread, and $K$ is the BEM resolution. The coefficients $h_{q,l}^{(r,t)}$ are kept invariant over a block of $N + L'$ symbols. Substituting the BEM channel model in (1), we obtain

$$
y^{(r)}[n] = \sum_{t=1}^{N_t} \sum_{q,l} e^{j2\pi q n/K} h_{q,l}^{(r,t)} x^{(l)}[n-l] + v^{(r)}[n], \tag{3}
$$

with a block level formulation the received block of length $N + L'$ at the $r$th receive antenna can be written as

$$
y^{(r)} = \sum_{t=1}^{N_t} h_{q,l}^{(r,t)} D_q Z_l x^{(l)} + v^{(r)}, \tag{4}
$$
where $y^{(r)} = \begin{bmatrix} y^{(r)}[-L'], \ldots, y^{(r)}[N - 1] \end{bmatrix}^T$, $x^{(t)} = \begin{bmatrix} x^{(t)}[-L - L'], \ldots, x^{(t)}[N - 1] \end{bmatrix}^T$, and $v^{(r)}$ is similarly defined as $y^{(r)}$. The diagonal matrix $D_q$ representing the $q$th basis function is defined as $D_q = \text{diag}\{[1, \ldots, e^{2\pi q(N+L'-1)/K}]^T\}$, and the $(N + L') \times (N + L' + L)$ Toeplitz matrix $Z_l$ is defined as $Z_l = [0_{N+L'\times(L-1)}1_{N+L'}1_{(N+L')\times L}]$. 

III. DECISION FEEDBACK EQUALIZATION

The DFE consists of a feed-forward part and a feedback part. The feed-forward part consists of a bank of $N_t$ time-varying FIR filters that are applied at each receive antenna. For the feedback part, two different scenarios are considered. In the first scenario, only the previously estimated symbols of the data stream of a particular transmit antenna are fed back to cancel/reduce ISI of that particular transmit antenna data stream. Therefore, for each data stream, only one feedback filter is used. In the second scenario, the previously estimated symbols on all transmit antennas are made available and fed back to cancel/reduce ISI in a joint fashion. Hence, for each data stream $N_t$ time-varying FIR feedback filters are used. The latter is referred to as joint DFE.

A. DFE Scenario 1

In this scenario, the feedback part consists of one time-varying FIR filter that feeds back previously estimated symbols of the transmit antenna of interest. The decision feedback equalizer with the feedforward part and feedback part is shown in Figure 2. An estimate of the transmitted symbol from the $a$th antenna at time-index $n$ subject to some decision delay $d$ is obtained as

$$\hat{x}^{(a)}[n-d] = \sum_{r=1}^{N_r} \sum_{l'=0}^{L'} w^{(a,r)}[n;l'] y^{(r)}[n - l'] \sum_{r'=0}^{L''+\Delta} b^{(a,r)}[n;l'';l'] \hat{x}^{(a)}[n-l''] \quad (5)$$

where $\Delta \geq d + 1$, and $Q(\cdot)$ is the quantizer used by the decision device. The feed-forward filter $w^{(a,r)}[n;l']$ corresponding to the $a$th transmit antenna and $r$th receive antenna and the feedback filter $b^{(a,r)}[n;l'';l']$ corresponding to the $a$th transmit antenna are designed according to the BEM. The feed-forward filters are designed to be of order $L'$, and have $Q’$ time-varying basis functions, while the feedback filters are designed to be of order $L''$ and have $Q''$ time-varying basis functions. The time-varying feed-forward filter on the $(l')$th tap at time-index $n$ can be written as

$$w^{(a,r)}[n;l'] = \sum_{q'=0}^{Q’/2} w_{q',l'}^{(a,r)} e^{-j2\pi q'n/K}, \quad (6)$$

and similarly the time-varying feedback FIR filter on the $(l'')$th tap at time-index $n$ can be written as

$$b^{(a)}[n;l''] = \sum_{q''=0}^{Q''/2} b_{q'',l''}^{(a)} e^{-j2\pi q''n/K}, \quad (7)$$

Substituting (7) and (6) in (5) and assuming past decisions are correct, we obtain

$$\hat{x}^{(a)}[n-d] = \sum_{r=1}^{N_r} \sum_{l'=0}^{L'} w^{(a,r)}[n;l'] \bar{Z}_{l'}^{r} \hat{y}^{(r)}[n - l'] - \sum_{q''=0}^{Q''/2} \sum_{l''=0}^{L''+\Delta} b^{(a)}[n;l'';l'][n - l''] \hat{x}^{(a)}[n-l''] \quad (8)$$

On a block level formulation, (8) can be written as

$$\hat{x}^{(a)} = \sum_{r=1}^{N_r} \sum_{q',l'} w^{(a,r)}[n;l'] \bar{D}_{q'}^{r} \hat{Z}_{l'}^{r} \hat{y}^{(r)}[n - l'] \sum_{r=1}^{N_r} \sum_{q'',l''} b^{(a)}[n;l'';l'] \bar{D}_{q''}^{r} \hat{Z}_{l''}^{r} \hat{x}^{(a)} \quad (9)$$

where the vector of the estimated symbols $\hat{x}^{(a)} = [\hat{x}^{(a)}[-d], \ldots, \hat{x}^{(a)}[N - d + 1]]^T$, the $N \times N$ diagonal matrix $D_{q'}^{r} = \text{diag}\{[1, \ldots, e^{2\pi q'(N+L'-1)/K}]^T\}$, and the $N \times N + L'$ Toeplitz matrix $Z_{l'}^{r} = [0_{N \times (L'-1)}1_N, 0_N]$. Substituting (4) in (9), we obtain

$$\hat{x}^{(a)} = \sum_{r=1}^{N_r} \sum_{q',l'} \sum_{q''} \sum_{l''} \sum_{q',l'} w^{(a,r)}[n;l'] h^{(r)}_{l',l''} \bar{D}_{q'}^{r} \bar{Z}_{l'}^{r} \hat{x}^{(a)} - \sum_{q''=0}^{Q''/2} \sum_{l''=0}^{L''+\Delta} b^{(a)}[n;l'';l'][n - l''] \hat{x}^{(a)}[n-l''] \quad (10)$$

Using the property $\bar{Z}_{l'}^{r} \bar{D}_{q'}^{r} = e^{2\pi q'(L'-1)/K} \bar{D}_{q'}^{r} \bar{Z}_{l'}^{r}$, (10) can be written as

$$\hat{x}^{(a)} = \sum_{r=1}^{N_r} \sum_{q',l'} \sum_{q''} \sum_{l''} e^{2\pi q'(L'-1)/K} w^{(a,r)}[n;l'] h^{(r)}_{l',l''} \bar{D}_{q'}^{r} \bar{Z}_{l'}^{r} \hat{x}^{(a)} \quad (11)$$
where \( p = q + q', k = l + l' \), and the \( N \times (N + L + L') \) matrix \( \mathbf{Z}_k = [\mathbf{I}_N \times (L + L' - k), \mathbf{I}_N, 0_{N \times k}] \). Defining the two-dimensional (2-D) \( p_{f, p, k}^{(a)}(t) \) as

\[
f_{p, k}^{(a)}(t) = \sum_{r=1}^{N_r} \sum_{q'=q}^{2^{p-1}(2q'-q)} \sum_{r'=q'}^{2^{p-1}(2q'-q')} \mathbf{D}_p^{(a, r)} \mathbf{Z}_k x(t) + \sum_{r=1}^{N_r} \sum_{q'=q}^{2^{p-1}(2q'-q)} \sum_{r'=q'}^{2^{p-1}(2q'-q')} \mathbf{D}_q^{(a, r)} \mathbf{Z}_k x(t),
\]

and substituting (12) in (11) we arrive at

\[
\mathbf{x}(t) = \sum_{r=1}^{N_r} \sum_{q'=q}^{2^{p-1}(2q'-q)} \sum_{r'=q'}^{2^{p-1}(2q'-q')} \mathbf{D}_p^{(a, r)} \mathbf{Z}_k x(t) + \sum_{r=1}^{N_r} \sum_{q'=q}^{2^{p-1}(2q'-q)} \sum_{r'=q'}^{2^{p-1}(2q'-q')} \mathbf{D}_q^{(a, r)} \mathbf{Z}_k x(t).
\]

Note that, \( f_{p, k}^{(a)}(t) \) is a 2-D function representing a weighted 2-D convolution in the time-invariant BEM coefficients of the equalizer and the time-invariant BEM coefficients of the channel. Assuming that the feedback filter is designed such that \( Q'' \leq Q + Q' \) and \( L'' \leq L + L' \), then we can write (13) as

\[
\mathbf{x}(t) = \sum_{r=1}^{N_r} \sum_{q'=q}^{2^{p-1}(2q'-q)} \sum_{r'=q'}^{2^{p-1}(2q'-q')} \mathbf{D}_p^{(a, r)} \mathbf{Z}_k x(t) + \sum_{r=1}^{N_r} \sum_{q'=q}^{2^{p-1}(2q'-q)} \sum_{r'=q'}^{2^{p-1}(2q'-q')} \mathbf{D}_q^{(a, r)} \mathbf{Z}_k x(t).
\]

where \( f_{p, k}^{(a)}(t) = b_{p, k}^{(a)} \mathbf{Z}_k x(t) \).

The feedback filter and feed-forward filters’ coefficients can be obtained by minimizing the mean-squared error (MSE) across the decision device. The MSE is defined as

\[
\text{MSE} = \sum_{n} \left[ \mathbf{x}(n) - \tilde{\mathbf{x}}(n) \right]^2,
\]

where the multiplication by \( \tilde{\mathbf{Z}}_d \) is to count for the decision delay. Hence, the minimum mean-squared error (MMSE) DFE can be obtained as

\[
\text{arg } \min_{w_{(a), (b)}} \text{MSE}.
\]
where $\mathbf{R}_\perp^{(t)}$ is given as

$$\mathbf{R}_\perp^{(t)} = \mathbf{R}_A - \mathbf{I}_Q^{(t)T} (Q^{-1} + \mathbf{H}^H\mathbf{R}_B^{-1}\mathbf{H})^{-1} \mathbf{H}^H\mathbf{R}_B^{-1}\mathbf{H}Q^{(t)}\mathbf{I}_Q^{(t)}.$$  

(24)

The feedback BEM coefficients are then obtained by solving the following equation

$$\arg\min_{\mathbf{b}^{(t)}} MSE$$  

(25)

To solve for the feedback filter BEM coefficient, we define $\mathbf{u}^{(a)} = [b_{Q^T/2}^{(a)}, \ldots, b_0^{(a)}]^T$, with $b_q^{(a)} = \{b_q^{(a)}, \ldots, b_0^{(a)}\}^T$. Hence, (25) is equivalent to the following constrained quadratic minimization problem

$$\arg\min_{\mathbf{u}^{(a)}} \mathbf{u}^{(a)H}\mathbf{R}_\perp^{(t)}\mathbf{u}^{(a)}, \text{ s.t. } \mathbf{u}^{(a)H}\mathbf{e}_0 = 1,$$  

(26)

where $\mathbf{R}_\perp^{(t)} = \tilde{\mathbf{P}}^T\mathbf{R}_\perp^{(t)}\tilde{\mathbf{P}}$ with $\tilde{\mathbf{P}}$ is a $(Q + Q' + 1)(L + L' + 1) \times ((Q' + 1)(L' + 1) + 1)$ selection matrix defined as

$$\tilde{\mathbf{P}} = \begin{bmatrix} \mathbf{0}_{L+1} & \mathbf{I}_{L+1}^{(t)} + \mathbf{J} \end{bmatrix} \mathbf{0}_{(Q' + 1)(L' + 1) + 1}$$  

(27)

with $\mathbf{J}$ defined as

$$\mathbf{J} = \begin{bmatrix} \mathbf{0}_{L+1} & \mathbf{I}_{L+1}^{(t)} + \mathbf{J} \end{bmatrix} \mathbf{0}_{(Q' + 1)(L' + 1) + 1}$$  

and $\mathbf{e}_0$ is a $(Q' + 1)(L' + 1) + 1$ dimensional unitary vector with 1 in the $(Q'/2)(L' + 1) + 1$st position. The feedback filter coefficients are then obtained by solving the quadratic constraint problem (26) as

$$\mathbf{u}^{(a)} = \frac{\mathbf{R}_\perp^{(t)−1}\mathbf{e}_0}{\mathbf{e}_0^T\mathbf{R}_\perp^{(t)}−1\mathbf{e}_0}.$$  

(28)

### B. Joint DFE

In the derivation of the decision feedback equalizer obtained in the previous subsection, the data streams of the different antennas are treated separately, i.e. only past decisions of an antenna of interest are fed back to cancel/reduce ISI. However, more degrees of freedom can be utilized in the system provided naturally by the MIMO structure. In this sense, a joint decision feedback equalizer can be derived, where past decisions of the data streams on the other transmit antennas are also made available and fed back to cancel/reduce ISI. As such, for each transmit antenna $N_t$ feedback equalizers are applied, and feedback the past decisions of the estimated symbols of the data streams of all transmit antennas. This allows to cancel/reduce ISI in a joint fashion. This joint structure of the DFE is depicted in Figure 3. An estimate of the data symbol on the $t$th antenna subject to the decision delay $d$, and assuming past decisions are correct, is obtained as

$$\hat{x}^{(a)}[n] = \sum_{r=1}^{L_t} y^{(a,r)}[n−r] + \sum_{r=1}^{L_t} \hat{\theta}_q^{(a,r)}y^{(a,q')}[n−r−l']$$  

(29)

Assuming the feedback filters are designed such that $Q'' \leq Q + Q'$ and $L'' + \Delta \leq L + L'$, on a block level formulation (29)

$$\hat{\mathbf{z}}^{(a)}[n \pm d] = \frac{\sum_{r=1}^{L_t} w^{(a)}[n−d] \hat{\theta}_q^{(a,r)}y^{(a,q')}[n−r]}{\sum_{r=1}^{L_t} w^{(a)}[n−d] \hat{\theta}_q^{(a,r)}y^{(a,q')}[n−r]}$$  

(30)

Substituting for the feed-forward coefficients $\partial MSE/\partial w^{(a)} = 0$, we obtain

$$w^{(a)} = R_\perp^{−1} (Q^{−1} + \mathbf{H}^H\mathbf{R}_B^{−1}\mathbf{H})^{−1} (\hat{b}^{(a)} + e^{(a)})$$  

(31)

where $\hat{b}^{(a)} = \{b_0^{(a)}, \ldots, b_{Q^T/2}^{(a)}\}^T$, and $b_q^{(a)} = \{b_q^{(a)}, \ldots, b_0^{(a)}\}^T$. Assuming past decisions are correct, the MSE across the decision device can then be written as

$$MSE = ||\hat{x}^{(a)} - \hat{\mathbf{z}}^{(a)}||^2$$  

(32)

where $\hat{\mathbf{z}}^{(a)} = \hat{\mathbf{z}}^{(a)}[n \pm d]$. Solving for the feed-forward coefficients $\partial MSE/\partial w^{(a)} = 0$, we obtain

$$w^{(a)} = R_\perp^{−1} (Q^{−1} + \mathbf{H}^H\mathbf{R}_B^{−1}\mathbf{H})^{−1} (\hat{b}^{(a)} + e^{(a)})$$  

(33)

Substituting for the feed-forward in (32) we obtain

$$MSE = (\hat{b}^{(a)} + e^{(a)})^T R_\perp (\hat{b}^{(a)} + e^{(a)})$$  

(34)

where the matrix $\mathbf{R}_\perp$ is given as

$$\mathbf{R}_\perp = Q^{−1} + \mathbf{H}^H \mathbf{R}_B^{−1}\mathbf{H} Q^{−1}$$  

(35)
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