Kinematic Calibration of a 3-DOF Planar Parallel Robot

Abstract

Purpose – The purpose of this paper is to describe a calibration method developed to improve the absolute accuracy of a novel three degrees-of-freedom planar parallel robot. The robot is designed for the precise alignment of semiconductor wafers and, even though its complete workspace is slightly larger, the accuracy improvements are performed within a target workspace, in which the positions are on a disc of 170 mm in diameter and the orientations are in the range ±17°.

Design/methodology/approach – The calibration method makes use of a single optimization model, based on the direct kinematic calibration approach, while the experimental data are collected from two sources. The first source is a measurement arm from FARO Technologies, and the second is a Mitutoyo coordinate measurement machine (CMM). The two sets of calibration results are compared.

Findings – Simulation confirmed that the model proposed is not sensitive to measurement noise. An experimental validation on the CMM shows that the absolute accuracy inside the target workspace was improved, by reducing the maximum position and orientation errors from 1.432 mm and 0.107°, respectively, to 0.044 mm and 0.009°.

Originality/value – This paper presents a calibration method which makes it possible to accurately identify the actual robot’s base frame (base frame calibration), at the same time as identifying and compensating for geometric errors, actuator offsets, and even screw lead errors. The proposed calibration method is applied on a novel planar robot, and its absolute accuracy was found to improve to 0.044 mm.

Keywords: kinematic calibration, parallel robot, absolute accuracy

Paper type: Research paper

Introduction

It is often claimed that parallel robots are more precise than serial robots, because they do not suffer from error accumulation. While this might be true in theory (Briot and Bonev, 2007), the real reason is that parallel robots can be built to be more rigid without being bulkier. No matter how rigid they are, parallel robots, too, should be calibrated in order to improve their accuracy. Depending on the sources of inaccuracy (geometric vs. non geometric errors), robot calibration is classified in two main categories: kinematic and non kinematic (Elatta et al., 2004; Greenway, 2000; Schroer, 1994). Some researchers refer to them as level 2 and level 3 calibrations respectively, while level 1 calibration represents the compensation of only the actuator lead errors or offsets (Roth et al., 1987). A kinematic calibration, which is the subject of this paper, considers that the robot’s links are perfectly rigid and that the robot is not in dynamic mode. This calibration is more appropriate for parallel robots since they present good rigidity, which implies that their non geometric errors are generally small.

The kinematic approach to calibrating parallel robots has been used by many authors (such as Oliviers and Mayer, 1995; Masory et al., 1997; Zhuang et al., 1998; Ren et al., 2009), mainly for the 6 degrees-of-freedom (DOF) Gough-Stewart platform. However, few works involve the calibration of parallel robots with less than 6 DOFs, especially planar ones. Some of these identify only actuator offsets (Pashkevich et al., 2009), or only the standard geometric parameters (e.g., link lengths), like (Last et al., 2006), who present a calibration method for a five-bar parallel robot based on crossing singularities. For the same type of robot, Durango et al. (2010) propose a divide-and-conquer strategy to identify both geometric parameters and joint offsets. Also, Kim (2005) presents a geometric calibration of a Cartesian parallel manipulator using a ballbar to identify geometric parameters and offsets. Finally, Joubair et al.
(2011) propose a simple partial calibration approach for directly identifying (without any optimisation) some of the geometric parameters of the robot studied in this paper.

This paper presents a contribution to the kinematic calibration of parallel robots by developing a method based on the direct kinematic approach using pose (position and orientation) measurement of the end-effector applied to a novel prototype of a planar parallel robot. The measurements were processed using two probing devices: a FaroArm Platinum measurement arm and a Mitutoyo coordinate measurement machine (CMM). The aim of using both devices is to determine how far each of the two instruments is capable of improving the robot’s absolute accuracy (while it is obvious that the CMM will lead to better results, it is not evident how much that improvement will be). Identification is achieved using a single non linear model based on the minimization of the square residuals of the platform’s position and orientation (i.e., we use the direct kinematic approach).

With respect to the traditional calibration method, our approach makes it possible to identify and compensate for geometric errors at the same time as identifying actuator offsets and lead errors. Also, it makes possible the accurate determination of the actual mobile reference frame and the base reference frame (BF) with respect to the world reference frame (WF). This in turn makes it possible to identify the end-effector’s position and orientation with respect to the actual BF. As a result, our method improves the absolute accuracy of the robot and not only its relative accuracy. Knowing the precise location of the BF with respect to the WF is highly advantageous as it permits the accurate interaction between the robot and other devices located in the same work area, such as a vision system or other robots (multi-robot systems). In contrast, robots calibrated without identifying their BF (e.g., calibrated using a ballbar) are able to perform precise displacements only relative to a given pose.

Moreover, the proposed method offers the following advantages: (a) it can be applied to other types of parallel robots; (b) it can be applied with a minimum time lapse (with only two poses to identify the WF and eight calibration poses); and (c) measurements can be performed using any 3D coordinate measurement instrument.

This paper is organized as follows: the next section provides a description of the nominal kinematic model, followed by presentation of the robot prototype. Then, the calibration model and method are explained. Finally, the simulation study and experimentation results are given, and conclusions are drawn.

Nominal kinematic model

The PreXYT (for Precision XY-Theta table), shown in Figure 1, is a planar parallel robot with two PRP legs and one PPR leg (Bonev, 2010). Referring to Figure 1, the directions of the actuators in the PRP legs are parallel to the y axis of the BF, while the direction of the actuator in the PPR leg is parallel to the x axis. The two passive prismatic joints on the mobile platform are parallel, and the axes of the three revolute joints are parallel and coplanar. The directions of the two prismatic joints in the PPR leg are normal. Consequently, if the two parallel actuators move in conjunction with one another at the same rate, the mobile platform is only translated along the y axis. If the two move in opposite directions, a pure rotation about the z axis (not shown) could occur. Finally, the other actuator directly controls the x coordinate of the platform’s center.

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1 It is customary to refer to parallel robots using the symbols P and R, which stand for prismatic and revolute joints respectively. When a joint is actuated, its symbol is underlined.
Figure 1. Schematic diagram of the nominal PreXYT model.

The BF Oxy is fixed at the base, so that the axis of the revolute joint of leg 2 always intersects the y axis, and a mobile reference frame Cx'y' is fixed to the mobile platform so that the axes of the revolute joints of legs 2 and 3 always intersect the x' axis. The origin C lies on the axis of the revolute joint of leg 1. Finally, \( \theta \) is the angle between the x and x’ axes, measured about the z axis (not shown).

Furthermore, \( \rho_1 \) is the active-joint variable associated with leg 1, and is defined as the distance between the y axis and the axis of the revolute joint of leg 1. The active-joint variable \( \rho_2 \) is defined as the distance between the x axis and the axis of the revolute joint of leg 2 (i.e. O is chosen in such a way that it lies on the axis of the revolute joint of leg 2, when \( \rho_2 = 0 \)). Similarly, \( \rho_3 \) is the active-joint variable associated with leg 3, and is defined as the distance between the x axis and the axis of the revolute joint of leg 3. Finally, \( s \) is the distance between the planes of motion of the axes of the revolute joints of legs 2 and 3.

Given the active-joint variables, we are able to define the position and orientation of the mobile platform (i.e. of the mobile reference frame). The orientation angle is easily obtained, as

\[
\theta = \tan^{-1}\left(\frac{\rho_3 - \rho_2}{s}\right),
\]

while the position of the mobile platform is given by

\[
x = \rho_1, \\
y = \rho_2 + \rho_1\left(\frac{\rho_3 - \rho_2}{s}\right).
\]

As can be observed, the direct kinematic equations of the PreXYT are relatively simple, and the platform’s x coordinate is directly defined by actuator 1, which is why our parallel robot is partially decoupled (in theory).

The inverse kinematics problem is also simple. Given the position and orientation of the platform, the active-joint variables are

\[
\rho_1 = x, \\
\rho_2 = y - x\tan \theta, \\
\rho_3 = y + (s - x)\tan \theta.
\]

It is evident that the PreXYT has no singularities.
Prototype

A prototype of the PreXYT, shown in Figure 2, has been constructed at the École de technologie supérieure. It comprises three screw-driven linear guides from LinTech: two from the 130 series, and one from the 100 series. Both the 130 series linear guides have a pivoting block attached to the carriage through a deep-groove single-row bearing (revolute joints 2 and 3). A steel shaft is rigidly attached to one of the two pivoting blocks and, through a simple linear ball bearing, to the other pivoting block. The mobile platform slides along the rod through a pair of the same linear ball bearings. The carriage of a roller monorail guide is attached to the carriage of the LinTech 100 series linear guide, so that the two guides are perpendicular. The monorail guide is fixed to the tapered block, which holds a large deep-groove double-row ball bearing (revolute joint 1) that is attached to the mobile platform.

![Prototype Diagram](image)

Figure 2. Experimental setup for calibrating the PreXYT with a CMM.

The mobile platform can rotate up to ±30° and translate inside a rectangle 170 mm × 300 mm, when at 0°. Or, referring to the more practical workspace, the set of positions attainable with any orientation between −17° and 17° is at least a disk of diameter 170 mm. It is the absolute accuracy throughout this entire workspace area that we improve in this work.

The three linear guides used in this robot are driven by brushless servo motors without gear boxes. Since the linear guides use Acme precision ball screws with a 5.080 mm (0.2 in) lead, a specified maximum lead error of 50 µm per 300 mm, and bidirectional repeatability of 5 µm. Finally, note that the lead errors are not considered in the nominal model described in the previous section.

Calibration model

The nominal kinematic model previously presented assumes that the machining and assembly of all robot components is perfect, which is clearly not the case. Various errors should therefore be taken into account, and a more realistic, so-called calibration model should be developed. The purpose of the calibration process will then be to identify these errors, so that the calibration model will more closely represent the actual robot.

In this work, as well as the distance s, several additional parameters are considered. Referring to Figure 3, let \( H_0 \) be a point in the \( Oxy \) plane such that \( C \) coincides with \( H_0 \) when the first actuator is homed (i.e. when \( r_1 = 0 \)) and actuators 2 and 3 are at their mid-travel lengths. With this definition in mind, the additional parameters are:
- \(d_1\) (offset of actuator 1): distance between \(H_0\) and the \(y\) axis;
- \(d_3\) (offset of actuator 3): distance between the \(x\) axis and the axis of the revolute joint of leg 3 when actuator 3 is homed (i.e. when \(r_3 = 0\));
- \(h\): distance between \(H_0\) and the \(x\) axis;
- \(\alpha\): angle between the \(x\) axis and the positive direction of actuator 1;
- \(\beta\): angle between the \(y\) axis and the positive direction of actuator 3;
- \(\gamma\): angle between the \(y\) axis and the direction of the passive joint of leg 1.

![Figure 3. Schematic diagram of the calibration model.](image)

As already mentioned, the lead errors are neglected in the nominal model, yet these errors degrade the accuracy of the robot. These errors should be added to the kinematic model of the robot, in order to be estimated and compensated for by the calibration process. Although we can directly measure these errors, it is better to include them in the calibration model and identify them at the same time as all the other parameters. We assume that the lead errors are a linear function of the actuator motion. Therefore, the actuator displacements are given by

\[ \rho_{i,R} = \delta_i \rho_i, \quad (7) \]

where \(\rho_{i,R}\) and \(\rho_i\) are the real and commanded displacement respectively of actuator \(i = 1, 2, 3\), and \(\delta_i\) is the correction coefficient for actuator \(i\).

Since we use the so-called direct kinematic calibration approach, our calibration model will be based on the direct kinematic equations. Given the active-joint variables, \(\rho_i\), and skipping the calculation details, the orientation \(\theta\) and position \((x, y)\) of the mobile platform are given by

\[ \theta = \tan^{-1}\left( \frac{\delta_i \rho_1 \cos \beta + d_1 - \delta_2 \rho_2}{s - \delta_3 \rho_3 \sin \beta - d_3 \tan \beta} \right), \quad (8) \]

\[ x = \frac{\delta_3 \rho_3 \cos \alpha (1 - \tan \alpha \tan \gamma) + d_1 - \tan \gamma (h - \delta_2 \rho_2)}{1 - \tan \gamma \tan \theta}, \quad (9) \]

\[ y = \delta_3 \rho_2 + x \tan \theta. \quad (10) \]

By analyzing the above equations, we observe that, if all our measurements are made on the platform, then parameters \(\delta_1\) and \(\alpha\) are interdependent (i.e. they cannot be identified separately), as are \(h\) and \(d_1\), which, of course, is quite logical. Therefore, the expressions \(\delta_1 \cos \alpha (1 - \tan \alpha \tan \gamma)\) and \(d_1 - h \tan \gamma\) are
replaced by two new parameters, denoted $\delta$ and $d$ respectively. These two new parameters have a relatively complex geometric meaning, which need not be described here. Thus, equation (9) can be rewritten as

$$x = \frac{\rho_1 \delta + d + \delta_2 \rho_2 \tan \gamma}{1 - \tan \gamma \tan \theta}. \quad (11)$$

As can be observed, the platform’s $x$ coordinate is no longer defined by actuator 1 only. Therefore, in reality, our parallel robot is not partially decoupled.

Finally, given the position and orientation of the platform, the active-joint variables are defined by:

$$\rho_2 = \frac{y - x \tan \theta}{\delta_2}, \quad (12)$$

$$\rho_3 = \frac{\tan \theta (s - d_3 \tan \beta - x) + y - d_3}{\delta_3 (\cos \beta + \sin \beta \tan \theta)}, \quad (13)$$

$$\rho = \frac{x - d - y \tan \gamma}{\delta}, \quad (14)$$

Once the nine parameters ($s$, $\delta$, $\delta_2$, $\delta_3$, $d$, $d_3$, $\beta$, and $\gamma$) are identified, the above inverse and direct kinematic equations are introduced into the robot controller.

**World reference frame**

The reference frames used in this work are presented in Figure 4. We start by describing our $WF$, which is fixed in the base of the robot. The axes of this frame are denoted by $x_w$ and $y_w$. The frame is determined only once, at the beginning of the calibration process, all further measurements being relative to it, and is then transferred to become relative to the $BF$.

First, three points on the rail of linear guides 2 and 3 are measured and used to define the $x_w y_w$ plane of our robot. All further measurements will be projected in this plane. Next, the PreXYT is sent to its home position (i.e. all three actuators are homed). Then, we measure the positions of a precision ball, denoted by $R$ and fixed to the carriage of guide 2, at the home position of actuator 2 and at its upper limit. The line that passes through the two positions defines the direction of the $y_w$ axis. The center of ball $R$, at the home position of actuator 2, projected on the $x_w y_w$ plane, gives us the origin $R_0$ of the $WF$. Finally, the $x_w$ axis is orthogonal to the $y_w$ axis, in the $x_w y_w$ plane and pointing towards actuator 3.
Figure 4. PreXYT’s reference frames.

**Base and mobile reference frames**

The $x'y'$ plane of the mobile reference frame coincides with the $x_0y_0$ plane. Assuming that the three revolute axes are parallel and coplanar, $x'$ is defined so as to intersect all of them. The origin $C$ lies on the axis of the revolute joint of leg 1.

The $xy$ plane of the $BF$ coincides with the $x_0y_0$ plane and the $BF$ has the same orientation as the $WF$. The origin $O$ is chosen in such a way that it lies on the axis of the revolute joint of leg 2, when the actuator is homed (i.e. when $\rho_2 = 0$). Therefore, only a translation vector is used to obtain the end-effector’s measurement positions relative to the $BF$:

$$p_C^{BF} = p_C^{WF} - p_O^{WF},$$

where $p_C^{BF} = [x, y]^T$ and $p_C^{WF}$ represent the platform position relative to the $BF$ and to the $WF$ respectively, and $p_O^{WF} = [x_0, y_0]^T$ is the position of the $BF$’s origin with respect to the $WF$. The offset coordinates $x_0$ and $y_0$ are obtained by the calibration process, because we cannot measure them directly.

Similarly, $p_C^{WF}$ cannot be obtained by direct measurement because there is no measurement target (e.g. a tooling ball) placed exactly at (or above) the platform’s center $C$. Thus, two tooling balls are attached to the platform, referred to as $K$ and $L$, as shown in Figure 4. These balls are used to define a reference frame denoted by $KL_F$ with its origin at the projection of $K$ in the $x_0y_0$ plane and its $x''$ axis passing through the projection of $L$ in the $x_0y_0$ plane. Note that the position of $C$ with respect to $KL_F$ is a constant vector $p_C^{KL_F} = [x_c, y_c]^T$. Then, to measure the pose of the mobile platform, we measure the centers of balls $K$ and $L$ with respect to the $WF$, namely the coordinates $x_K$, $y_K$, $x_L$, and $y_L$.

Finally, we obtain the platform position with respect to the $BF$ using the equation:

$$p_C^{BF} = p_C^{WF} - p_O^{WF} = p_C^{WF} + R_{KL_F}^{WF} [x_c, y_c]^T - p_O^{WF},$$

where $R_{KL_F}^{WF}$ is the rotation matrix representing the orientation of $KL_F$ with respect to the $WF$. Knowing that the orientation angle of $KL_F$ with respect to the $WF$ is $\lambda$, equation (16) can be rewritten as follows:

$$p_C^{BF} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_c + x_c \cos \lambda - y_c \sin \lambda - x_0 \\ y_c + x_c \sin \lambda + y_c \cos \lambda - y_0 \end{bmatrix},$$

where

$$\lambda = \text{atan} 2(y_L - y_K, x_L - x_K).$$

**Orientation measurement error**

Ideally, the orientation $\lambda$ of $KL_F$ with respect to the $BF$ should be equal to the measured platform orientation, $\theta_{\text{meas}}$. However, the $x''$ axis is not perfectly parallel to the $x'$ axis, and there is an offset between $\theta_{\text{meas}}$ and $\lambda$. This offset, $\varepsilon_\theta$, is evaluated by the calibration process. To do so, the platform orientation error is introduced at every position measured:

$$\theta_{\text{meas}} = \lambda - \varepsilon_\theta.$$ 

To summarize, in our calibration process, we have to identify a total of thirteen independent parameters: $x_0$ and $y_0$ (the $BF$ translation with respect to the $WF$), $x_c$ and $y_c$ (the mobile frame translation with respect to $KL_F$), $s$, $d$, $d_3$, $\delta$, $\beta$, $\gamma$, the correction coefficient for the lead errors $\delta_2$ and $\delta_3$, and, finally,
the orientation measurement offset, \( \varepsilon_0 \). Reference our assumption that the axes of the three revolute joints are parallel, coplanar, and normal to the directions of all the prismatic joints.

**Calibration method**

As mentioned previously, the aim of our calibration process is to reduce the end-effector’s position and orientation absolute errors by accurately identifying the geometric parameters, the lead error parameters, and the frame locations. In contrast to a previous work (Joubair et al., 2011), nearly all possible kinematic errors in the PreXYT are considered. In the current work, the calibration process is based on the *direct kinematic method*, which compares the end-effector’s estimated and measured poses. We do not use the *inverse kinematic method* because (a) the direct kinematic model of the PreXYT has a unique trivial solution (unlike most parallel) and (b) we have complete pose measurement devices at hand. There is hardly any doubt that direct kinematic calibration methods based on complete pose measurement yield best results.

For calibration pose \( j \), the measured position \( x_{\text{meas},j}, y_{\text{meas},j} \) and orientation \( \theta_{\text{meas},j} \) are based on measurements of the \( K \) and \( L \) precision balls, while the estimated poses \( x_{\text{est},j}, y_{\text{est},j}, \theta_{\text{est},j} \) are obtained from the direct kinematic equations (8), (10), and (11), which can also be represented as

\[
x_{\text{est},j} = f_x(\rho_{h,j}, \rho_{2,j}, \rho_{3,j}, s, \delta_2, \delta_3, d, \beta, \gamma),
\]

\[
y_{\text{est},j} = f_y(\rho_{h,j}, \rho_{2,j}, \rho_{3,j}, s, \delta_2, \delta_3, d, \beta, \gamma),
\]

\[
\theta_{\text{est},j} = f_0(\rho_{h,j}, \rho_{2,j}, \rho_{3,j}, s, \delta_2, \delta_3, d, \beta, \gamma),
\]

where \( \rho_{h,j}, \rho_{2,j}, \) and \( \rho_{3,j} \) are the actuator values commanded at the \( j \)th calibration pose.

The equations of the measured position and orientation can be rewritten based on equations (17) and (19), as

\[
x_{\text{meas},j} = \hat{f}_x(x_{O}, y_{O}, x_{C}, x_{K,j}, y_{K,j}, x_{L,j}, y_{L,j}),
\]

\[
y_{\text{meas},j} = \hat{f}_y(x_{O}, y_{O}, x_{C}, x_{K,j}, y_{K,j}, x_{L,j}, y_{L,j}),
\]

\[
\theta_{\text{meas},j} = \hat{f}_0(x_{O}, y_{O}, x_{C}, x_{K,j}, y_{K,j}, x_{L,j}, y_{L,j}, \varepsilon_0),
\]

where \( x_{K,j}, y_{K,j}, x_{L,j}, \) and \( y_{L,j} \) represent the measured coordinate for precision balls \( K \) and \( L \) with respect to the WF, at the \( j \)th calibration pose.

All the parameters are identified using the least squares minimization method. The cost function allows the identifications of parameters that minimize the square residuals between the commanded poses and the measured poses:

\[
\text{minimize} \sum_{j=1}^{n} \left( (x_{\text{est},j} - x_{\text{meas},j})^2 + (y_{\text{est},j} - y_{\text{meas},j})^2 + (\theta_{\text{est},j} - \theta_{\text{meas},j})^2 \right),
\]

where \( j \) is the reference number of the calibration pose and \( n \) is the total number of calibration poses, the position coordinates are in millimetres and the orientation angles in radians. This non linear model is solved using the Matlab optimization toolbox. Therefore, the function without constraints \texttt{fminunc} is used with large-scale (trust-region Newton algorithm). The minimization is achieved by providing the gradient of the objective function, and the nominal values of all parameters are used as a first guess.

**Calibration poses**

Our optimization problem needs at least as many independent constraints as there are parameters. Knowing that we have thirteen parameters to identify and that every pose provides three independent
equations (i.e., equations of $x$, $y$, and $\theta$), theoretically, only five poses are necessary. However, we chose to overconstrain the model by increasing the number of calibration poses, if necessary, and our goal is to find the smallest set of calibration poses for which the simulated position accuracy is equal or lower than 2.7 $\mu$m, which is the uncertainty of the measurement instrument used for validation (CMM). To choose these calibration poses, a series of simulations were performed. The results show that the farther the calibration positions from the workspace center and the bigger the orientation angle ($\theta$), the better the accuracy improvement in the target workspace. As mentioned earlier, our target workspace is limited to orientations within the range $\pm$17°. Therefore, poses outside the target workspace, with $\theta = \theta_{\text{max}} = 30^\circ$ and $\theta = \theta_{\text{min}} = -30^\circ$, are chosen in addition to other poses. These other poses are uniformly distributed within the position workspace for smaller orientations of the mobile platform. The retained calibration poses, referred to as set 1 are shown in Figure 5. Note that, using calibration poses uniformly distributed within only the target workspace requires a very large number of poses to reach our limit criteria. To illustrate that, Figure 5 presents a calibration set (set 3) composed of sixteen such poses, but even sixteen of them do not yield to as good results as those for set 1.

Before performing the actual calibration process, a sensitivity analysis is performed. The purpose of this simulation is to analyse the sensitivity of parameter identification to the measurement noise. Calibration poses are illustrated in Figure 5, while the robot’s nominal and actual parameters used in this simulation are given in Table 1. The actual geometric parameter errors are randomly generated between $\pm$2.000°, $\pm$5.500 mm, and $\pm$0.050 for the angle, distance, and coefficient errors respectively. Furthermore, random errors of 0.015 mm are introduced for the simulated measurements and the impacts of those errors are investigated using several sets of calibration. Five sets are presented in this paper (set 1, set 1, set 2, set

![Figure 5. Sets of calibration poses.](image)

**Simulated calibration**

Before performing the actual calibration process, a sensitivity analysis is performed. The purpose of this simulation is to analyse the sensitivity of parameter identification to the measurement noise. Calibration poses are illustrated in Figure 5, while the robot’s nominal and actual parameters used in this simulation are given in Table 1. The actual geometric parameter errors are randomly generated between $\pm$2.000°, $\pm$5.500 mm, and $\pm$0.050 for the angle, distance, and coefficient errors respectively. Furthermore, random errors of 0.015 mm are introduced for the simulated measurements and the impacts of those errors are investigated using several sets of calibration. Five sets are presented in this paper (set 1, set 1, set 2, set
2 and set 3), but many more were tested. Sets 1 and 1 correspond to the same positions, but the first set has larger orientations, and similarly for sets 2 and 2. In addition, the workspace delimited by the positions of set 1 is larger than that delimited by sets 2 and 3 (Figure 5).

Table 1. Results of simulations for parameter identification.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nominal</th>
<th>Actual</th>
<th>Identified</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Set 1</td>
<td>Set 1</td>
<td>Set 2</td>
</tr>
<tr>
<td>( \beta ) (deg)</td>
<td>0.000</td>
<td>1.204</td>
<td>1.203</td>
</tr>
<tr>
<td>( \gamma ) (deg)</td>
<td>0.000</td>
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<td>-1.523</td>
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<td>( \varepsilon_0 ) (deg)</td>
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<td>-0.954</td>
</tr>
<tr>
<td>( s ) (mm)</td>
<td>394.000</td>
<td>393.564</td>
<td>393.565</td>
</tr>
<tr>
<td>( d_1 ) (mm)</td>
<td>0.000</td>
<td>0.648</td>
<td>0.650</td>
</tr>
<tr>
<td>( d ) (mm)</td>
<td>115.000</td>
<td>120.473</td>
<td>120.473</td>
</tr>
<tr>
<td>( x_C ) (mm)</td>
<td>73.000</td>
<td>73.513</td>
<td>73.511</td>
</tr>
<tr>
<td>( y_C ) (mm)</td>
<td>-73.000</td>
<td>-72.822</td>
<td>-72.822</td>
</tr>
<tr>
<td>( x_O ) (mm)</td>
<td>23.500</td>
<td>23.246</td>
<td>23.244</td>
</tr>
<tr>
<td>( y_O ) (mm)</td>
<td>-30.500</td>
<td>-30.053</td>
<td>-30.053</td>
</tr>
<tr>
<td>( \delta_1 )</td>
<td>1.000</td>
<td>0.986</td>
<td>0.986</td>
</tr>
<tr>
<td>( \delta_2 )</td>
<td>1.000</td>
<td>0.994</td>
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</tr>
<tr>
<td>( \delta_3 )</td>
<td>1.000</td>
<td>0.976</td>
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</tr>
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</table>

After carrying out the simulated calibration, errors on the identified parameters (i.e., the differences between the identified and actual values) are calculated, and are illustrated in Figure 6. This diagram shows that the errors using set 1 are smaller than those obtained using the other sets. The identification errors for the coefficients \( \delta_1, \delta_2, \) and \( \delta_3 \) are not presented, because they are nearly zero for all five sets.

Figure 6. Results from simulations for parameter identification errors.

The position and orientation accuracy was evaluated using the parameters identified for every set. This simulation analysis is performed on our target workspace (Figure 5) using thousand positions uniformly distributed within the target workspace. Every group of positions is simulated with three
orientations: 0, −15°, and 15°. The results are presented in Table 2 and show that set 1 gives the best accuracy.

Table 2. Position and orientation error after simulated calibration.

<table>
<thead>
<tr>
<th>Errors</th>
<th>Set 1</th>
<th>Set 11</th>
<th>Set 2</th>
<th>Set 22</th>
<th>Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$xy_{\text{Max}}$ [μm]</td>
<td>2.2</td>
<td>13.3</td>
<td>125.5</td>
<td>369.2</td>
<td>83.3</td>
</tr>
<tr>
<td>$xy_{\text{RMS}}$ [μm]</td>
<td>1.3</td>
<td>12.4</td>
<td>123.2</td>
<td>362.4</td>
<td>79.0</td>
</tr>
<tr>
<td>$x_{\text{Max}}$ [μm]</td>
<td>0.1</td>
<td>12.2</td>
<td>121.8</td>
<td>357.3</td>
<td>79.5</td>
</tr>
<tr>
<td>$x_{\text{RMS}}$ [μm]</td>
<td>0.0</td>
<td>12.0</td>
<td>120.7</td>
<td>354.5</td>
<td>75.9</td>
</tr>
<tr>
<td>$y_{\text{Max}}$ [μm]</td>
<td>2.2</td>
<td>5.3</td>
<td>31.0</td>
<td>93.6</td>
<td>24.8</td>
</tr>
<tr>
<td>$y_{\text{RMS}}$ [μm]</td>
<td>1.3</td>
<td>3.0</td>
<td>24.4</td>
<td>74.9</td>
<td>21.9</td>
</tr>
<tr>
<td>$\theta_{\text{Max}}$ [deg]</td>
<td>0.0004</td>
<td>0.0004</td>
<td>0.0008</td>
<td>0.0010</td>
<td>0.0022</td>
</tr>
<tr>
<td>$\theta_{\text{RMS}}$ [deg]</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0006</td>
<td>0.0007</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

Referring to Tables 1 and 2, we can conclude that the larger the calibration workspace, the better the parameter identification (Table 1) and accuracy improvement (Table 2). Thus, set 1, which gives an accurate identification of the parameters and better robustness to the measurement noise, will be used to perform the actual robot calibration. Note that the eight poses of set 1 correspond to the eight combinations of min/max actuator positions, which is not surprising since PreXYT is essentially a decoupled robot.

**Actual calibration and validation**

Two coordinate measurement machines were used in our experimental work. The first is a FaroArm Platinum measurement arm (Figure 7) with an uncertainty evaluated at ±7.5 μm (for each coordinate) inside PreXYT’s workspace, as per the ISO/IEC Guide 98-3 (2008), while the second is a Mitutoyo BRIGHT-STRATO 7106 CMM (Figure 2) with a volumetric accuracy of ±2.7 μm, according to its latest calibration certificate. The coordinates of precision balls $K$ and $L$ for the eight poses of calibration set 1 were measured using both machines separately.
The calibration process is performed using the eight poses of set 1, as follows. First, the \( WF \) was defined, as explained previously. Second, using the nominal kinematic model, the robot mobile platform is sent consecutively to each pose of set 1 and the corresponding positions of ball \( K \) and \( L \) are measured with respect to the \( WF \). Thereafter, the measurement results: \( x_{K,j} \), \( y_{K,j} \), \( x_{L,j} \), and \( y_{L,j} \), along with the corresponding actuator variables \( \rho_{1,j} \), \( \rho_{2,j} \), and \( \rho_{3,j} \) (where \( j = 1, 2, \ldots, 8 \) represent the calibration pose of set 1), are used in the optimization model, equation (26), to identify the robot’s parameters. The calibration process is repeated five times with each measurement instrument, and the results of this process are shown in Table 3 (mean values ± 3 × standard deviation).

Table 3. Experimental results for parameter identification using the FaroArm and the CMM.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nominal</th>
<th>Identified with FaroArm</th>
<th>Identified with CMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ) [deg]</td>
<td>0.000</td>
<td>−0.0297±0.0023</td>
<td>−0.0142±0.0085</td>
</tr>
<tr>
<td>( \gamma ) [deg]</td>
<td>0.000</td>
<td>0.1067±0.0020</td>
<td>0.0828±0.0008</td>
</tr>
<tr>
<td>( \epsilon ) [deg]</td>
<td>0.000</td>
<td>−0.0219±0.0046</td>
<td>−0.0217±0.0046</td>
</tr>
<tr>
<td>( s ) [mm]</td>
<td>394.000</td>
<td>393.8972±0.0783</td>
<td>393.9288±0.0194</td>
</tr>
<tr>
<td>( d_1 ) [mm]</td>
<td>0.000</td>
<td>0.7324±0.1131</td>
<td>0.8528±0.0469</td>
</tr>
<tr>
<td>( d ) [mm]</td>
<td>115.000</td>
<td>116.0683±0.0193</td>
<td>116.2344±0.0198</td>
</tr>
<tr>
<td>( x_C ) [mm]</td>
<td>73.000</td>
<td>73.0165±0.0604</td>
<td>73.088±0.0229</td>
</tr>
</tbody>
</table>
The mean values for the identified parameters are then inserted into the calibration kinematic model of the robot, equations (12-14), and these new kinematic equations are, in turn, introduced into the robot controller, replacing the nominal model. Then, the accuracy of the robot is evaluated using the CMM only. Fifty-one poses in our target work space are tested: 17 positions uniformly distributed on two concentric circles, with three orientations for each position (0°, −15°, and 15°). The outside circle has a diameter of 170 mm, while the inside one has a diameter of 85 mm.

The results before calibration (i.e. using the nominal model) are illustrated in Figure 8. The results for the same 51 poses, after calibration (i.e. using the calibrated model), are shown in Figures 9 and 10. Moreover, the maximal and RMS values of position and orientation accuracy are shown in Table 4.

Referring to Table 3, the parameters $x_0$ and $y_0$ of the BF had large errors (0.17 mm and 0.19 mm, respectively). This means that these parameters had a considerable effect on the inaccuracy of our robot before calibration. Therefore, if the calibration was achieved without identifying these parameters, the positions relative to the BF would not be known accurately, and only the relative accuracy would have been improved.

Table 4. Position and orientation errors before and after calibration, as measured on a CMM in 51 poses uniformly distributed within the target workspace.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Before calibration</th>
<th>After calibration with FaroArm</th>
<th>After calibration with CMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_C$ [mm]</td>
<td>−73.000</td>
<td>−73.0121±0.0119</td>
<td>−73.0214±0.0008</td>
</tr>
<tr>
<td>$x_O$ [mm]</td>
<td>23.500</td>
<td>23.3658±0.0438</td>
<td>23.3405±0.0044</td>
</tr>
<tr>
<td>$y_O$ [mm]</td>
<td>−30.500</td>
<td>−30.6902±0.0784</td>
<td>−30.7055±0.0383</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1.000</td>
<td>0.9992±0.0001</td>
<td>0.9992±0.0001</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>1.000</td>
<td>0.9996±0.0003</td>
<td>0.9996±0.0001</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>1.000</td>
<td>0.9999±0.0000</td>
<td>0.9998±0.0000</td>
</tr>
</tbody>
</table>
Figure 8. Position accuracy before calibration

Figure 9. Position accuracy after calibration using a FaroArm Platinum
Conclusions

A calibration model for the kinematic identification of a 3-DOF planar parallel robot was presented. The calibration approach is based on the direct kinematics, and consists of minimizing the sum of the square residuals between the measured and estimated poses. A total of thirteen parameters were considered, of which three are related to the screw lead errors of the linear actuators, two define the base frame, and three define the mobile frame. Only eight calibration poses need to be measured. Simulations were conducted to confirm that our model is not sensitive to measurement noise. In addition, experimentation showed that the absolute accuracy improved, from maximum position and orientation errors of 1.432 mm and 0.107°, respectively, before calibration, to 0.094 mm and 0.039°, after calibration with a FaroArm, and 0.044 mm and 0.009°, after calibration with a CMM. These results were obtained by measuring, using a CMM, the position and orientation of the mobile platform in 51 poses equally distributed in the target workspace of our robot.

Results show that identification using the CMM leads to better robot accuracy, because the parameter uncertainty (±3σ) is smaller than that obtained by the FaroArm. Nevertheless, the results obtained using the FaroArm are very satisfactory, and demonstrate that a small range measurement arm can be used to calibrate a parallel robot and bring its positioning accuracy to 0.094 mm or better.

The accuracy achieved in this work can hardly be better, considering the fact that the multidirectional repeatability of the robot was measured to be about 30 μm (Joubair et al., 2011).

References


