A novel effective model for solving 3D forward scattering problems in a homogeneous background

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A new model for solution of 3-D forward scattering problems particularly suitable for lossy scenarios is presented. The proposed model is a suitable extension to the vectorial case of the Contrast Source – Extended Born method recently proposed in literature with reference to the 2-D scalar case. Based on the proposed model a new series is introduced, which represents a convenient way to solve the forward problem in many cases. Moreover, analytical tools are provided to foresee the actual behavior of the series which are also of interest for the corresponding inverse problem [1],[2].

Introduction

One of the main drawbacks which limits practical application of electromagnetic diagnostic techniques arises from the large computational efforts required when facing “real-world” situations, such as for instance 3D vectorial problems. This issue claims for methods capable of dealing with 3D problems effectively, i.e. reducing as much as possible the computational costs as well as storage requirements. Of course, this is not only just a matter of optimizing existing tools but stimulates development of suitable models driven by physics-based considerations.

In this respect, with reference the “canonical” 2D scalar case, the recently proposed Contrast Source – Extended Born model (CS-EB) [1] has shown that improved performances in the solution of both forward and inverse scattering problems can be achieved by means of a “simple” rewriting of the pertaining scalar integral equation in which the peculiar behaviour of the Green’s function in lossy media is exploited. The remarkable results obtained [1] suggest the extension of this model to more complex 3D vectorial case. Note that application of the CS-EB proves to be particularly fruitful (but not limited to) the case in which losses are present in the investigated scenario. According to this, in the following we will consider the 3D scattering problem in which (possibly lossy) scatterers are embedded within a (possibly lossy) homogeneous medium. In particular, this communication is focused on extending the CS-EB to the 3D forward problem, while the extension of CS-EB in the 3D inverse problem can be found in [2].
Mathematical formulation

Let us consider a system of scatterers of equivalent permittivity \( \varepsilon_x(r) \), enclosed in a volume \( V \) and embedded in a homogeneous medium of equivalent permittivity \( \varepsilon_b \). The total electric field \( E \) presents in \( V \) when a known incident field \( E_{\text{inc}} \) is impinging is described by the so-called Electric Field Integral Equation (EFIE):

\[
E(r) = E_{\text{inc}}(r) + k_b^2 \int_V G(r, r') \chi(r') E_{\text{inc}}(r') dr' = E_{\text{inc}} + A_i[\chi E],
\]

wherein \( \chi = \varepsilon_x/\varepsilon_b - 1 \) defines the contrast function and the \( G \) denotes the dyadic Green’s function [3] in a homogeneous medium, \( A_i \) denotes the dyadic integral radiation operator, while \( k_b \) is the background (complex) wave-number.

In order to take advantage of the singular behaviour of \( G \) for \( r' = r \), it proves convenient to separate the Green’s function into two contributions, i.e. \( G = G_D + G_{ND} \), wherein \( G_D \) denotes the diagonal terms of \( G \) and \( G_{ND} \) the other ones.

Note that due to the behaviour of the Green’s function, the second and third integral are more and more negligible for increasing losses in the background.

Now, by multiplying Eq.(1) times the contrast and then adding and subtracting the so-called ‘contrast-source’ \( J(r) = \chi(r)E(r) \) inside the integral with kernel \( G_D \), Eq.(1) becomes:

\[
\chi(r)E_{\text{inc}}(r) + \chi(r) k_b^2 \int_V G_D(r, r') dr'J(r) +
\]

\[
+ \chi(r) k_b^2 \int_V G_D(r, r')J(r)dr' + \chi(r) k_b^2 \int_V G_{ND}(r, r')J(r')dr' = E_{\text{inc}}.
\]

Note that due to the behaviour of the Green’s function, the second and third integral are more and more negligible for increasing losses in the background.

Let us now introduce two auxiliary dyads, defined as:

\[
F_v(r) = k_b^2 \int_V G_D(r, r') dr', \quad F_v \]

\[
P(r) = \chi(r)[I - \chi(r)F_v](r)^{-1}.
\]

By taking into account the above, Eq.(2) can be synthetically recast as:

\[
J(r) = PE_{\text{inc}}(r) + PA_{MOD}[J],
\]

wherein the dyadic integral operator \( A_{MOD}[J] \) is given by \( A_{MOD}[J] = A_i[J] - F_vJ \), i.e. by the second and third term at the right hand side of Eq.(2).

As in Eq.(4) the dyadic operators \( A_{MOD}[J] \) and \( P \) play the same role of the (scalar) operator defined in [1] to introduce the CS-EB model, Eq.(4) defines what we call the dyadic Contrast Source – Extended Born (CS-EB-d) model. In the following we adopt this model to solve the 3D forward scattering problem.

Note that, as the contrast function is now embedded into the auxiliary dyad \( P \), the adoption of Eq.(4) is not convenient in the inverse scattering problem since a scalar unknown would indeed require the determination of a dyadic quantity. A suitable reformulation of the CS-EB to overcome this problem is given in [2].
The CS-EB-d series: an effective tool to solve the forward problem

In order to study the applicability of the CS-EB-d model to the forward problem, let us consider the formal inversion of Eq.(4), i.e.:

$$J(r) = (I - PA_{MOD})^{-1} P E_{inc}(r).$$  \hspace{1cm} (5)

In particular, as long as \( |PA_{MOD}| < 1 \), the inverse dyadic operator appearing in (5) can be expanded into a Neumann series around the zero as:

$$J = \sum_{n=0}^{\infty} (PA_{MOD})^n [PE_{inc}].$$  \hspace{1cm} (6)

which defines the CS-EB-d series. Note the series can be computed in a very easy fashion by iterations of the kind:

$$J_0 = PE_{inc}, \quad J_n = PE_{inc} + PA_{MOD} [J_{n-1}], \quad n \geq 1$$  \hspace{1cm} (7)

where \( J_n \) is the partial sum (up to the \( n \)-th term) of the series (6).

Let us remark that the norm \( |PA_{MOD}| \) plays a key role in the actual possibility of solving Eq.(4) by means of a series expansion. Therefore, it proves convenient to provide some estimate of this quantity. Of course, as \( P \) embeds the information related to the scatterers, \( |PA_{MOD}| \) changes from case to case. However, one can take advantage of the fact that

$$|A_{MOD}| \leq |P|, |A_{MOD}|,$$

so that it is possible to achieve a sufficient criteria to establish applicability of the series expansion (6). As matter of fact, one can separately study the two norms exploiting the circumstance that, while \( |P| \), whose determination is anyway straightforward, varies with the experiment (i.e. depends by the scatterers), \( |A_{MOD}| \) only depends on the background characteristics and the shape of the investigated region. Accordingly, one can separate the effect of the objects from that of the scenario. In particular, one can achieve some “universal” plots which express \( |A_{MOD}| \) as function of the dimension \( d \) of the region under test (as normalized to the wavelength) and the background’s tangent loss. The “universal” plot is shown in Fig.1. Note that the evaluation of the this norm would require to build and store a very large matrix, which results in an unfeasible task as discretization increases. In order to overcome this problem we have iteratively computed the norm by means of the power method [4], which allow to exploit the convolutional nature of dyadic operator \( A_{MOD} \).

Numerical examples

In order to show the effectiveness of the introduced CS-EB-d series and usefulness of the tools described in the previous Section, let us consider two objects characterized by permittivity \( \varepsilon_1 = 12, \varepsilon_2 = 7 \) and conductivity \( \sigma_1 = 30 \text{mS/m}, \sigma_2 = 10 \text{mS/m} \), respectively, embedded in a lossy homogeneous background having permittivity \( \varepsilon_0 = 5 \) and conductivity \( \sigma_0 = 20 \text{mS/m} \). The permittivity profile is depicted in Fig2. The scatterers are supposed to be enclosed in a cube (discretized into 34x34x34 cells) whose side is \( 1.8\lambda_b \), \( \lambda_b \) being the background’s wave-length.
at the frequency \( f = 600 \text{MHz} \). In this case the background’s tangent loss is 0.12, therefore \( |A_{\text{MOD}}| \approx 2 \) while \( |P| = 0.94 \).

In order to illustrate the behaviour of the CS-EB-d series in Fig.3 we have plotted the normalized relative error given by \( \|J_n - J_{n+1}\|/\|J_1\| \), where \( J_n \) is the contrast source estimated by considering \( n \) terms of the series. For the sake of comparison, we have also considered the behaviour of the Born series (dashed line) and that of the series arising when considering the Contrast Source model (dotted line) [5]. Fig.3 shows that in this case only the CS-EB-d series converges, as foreseen through the estimated bound.

Fig.1: Universal plot of \( |A_{\text{MOD}}| \)

Fig.2: Permittivity profile

Fig.3: Behaviour of CS-EB-d (solid line), Born (dashed line) and CS (dotted line) series

References:


