Analysis of Intersymbol Interference due to Overlap in DM-BPSK

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SUMMARY The relationship between the degree of overlap in direct modulation chirp spread spectrum systems with binary phase shift keying and intersymbol interference (ISI) is analyzed. It is observed that the ISI due to overlap fluctuates or monotonically increases as the number of overlaps changes and that, in some cases, the overlap does not incur ISI at all.

key words: CSS, DM, BER, chirp, overlap

1. Introduction

The chirp spread spectrum (CSS) technique, developed and deployed originally for military radar systems, spreads the data signal over a frequency bandwidth wider than the minimum bandwidth necessary to send the data signal via chirp signals for transmission. Recently, CSS has attracted much attention in the field of wireless communications due to its various advantages such as high processing gain and time resolution, low power consumption, and anti-jamming and anti-multipath capabilities [1]. It has also been adopted as a physical layer implementation of IEEE 802.15.4, the standard for low-rate wireless personal area networks [2].

One of the techniques for increasing the bit rate in CSS is time overlapping, a technique in which more than one symbol are transmitted with an interval shorter than the symbol duration. More overlaps can clearly offer higher data throughput. However, overlaps may cause more intersymbol interference (ISI), eventually resulting in serious degradation of the bit error rate (BER) [3]. Thus, in order to utilize the overlap technique suitably in CSS, the ISI due to overlap should be analyzed. Assuming that the ISI due to overlap is caused by only two overlapped symbols immediately adjacent to the symbol of interest, the influence of the overlap on the direct modulation CSS scheme with binary phase shift keying (DM-BPSK) was investigated preliminarily in [4]: without providing a clear relation between the degree of overlap and ISI, it was simply mentioned that more overlaps do not always cause more ISI. In [5], an approximate closed-form BER expression was derived for DM-BPSK with overlap: yet, the relation between the overlap and ISI was not analyzed thoroughly.

In this letter, we derive an explicit expression of the ISI due to overlap in DM-BPSK and provide a detailed discussion on the relation between the ISI and overlap, allowing an efficient use of the overlap technique in DM-BPSK.

2. Analysis of the ISI due to Overlap

The complex baseband equivalent \( c(t) \) of a chirp waveform can be expressed as

\[
c(t) = \sqrt{\frac{1}{T_c}} \exp(j\pi \mu t^2), \quad |t| < \frac{T_c}{2},
\]

where \( T_c \) denotes the chirp duration and the non-zero parameter \( \mu \) denotes the chirp rate defined as the rate of instantaneous frequency change of the chirp signal. When \( \mu \) is positive (negative), the chirp signal is called the up-chirp (down-chirp) signal and the instantaneous frequency increases (decreases).

Figure 1 shows the complex baseband model of the DM-BPSK system with overlap. Input data is first encoded every \( \tau = T_c / L \) seconds by a polar nonreturn-to-zero (NRZ) level encoder, where \( L \in \{1, 2, \cdots\} \) and \( \tau \) are called the

![Fig. 1 The complex baseband model of the DM-BPSK system with overlap.](image-url)
and two-sided power spectral density

\[ N \]

ted data taking a value in

additive white Gaussian noise (AWGN)

DM-BPSK symbol is contaminated during transmission by

\[ L \]

number of overlaps and overlap interval, respectively. It
should be noted that the DM-BPSK overlapped with \( L = 1 \)
is the normal DM-BPSK without any overlap: some examples of DM-BPSK symbols overlapped are shown in Fig. 2
for \( L = 1, 2, \) and 3.

Subsequently, the encoded data is multiplied by an up-chirp signal \( e(t - k\tau) \). The DM-BPSK symbol \( s(t) \) can thus be expressed as

\[ s(t) = \sqrt{E_b} \sum_{k=-\infty}^{\infty} b_k e(t - k\tau), \]

where \( E_b \) is the bit energy and \( b_k \) denotes the \( k \)-th transmitted data taking a value in \( \{1, -1\} \) with equal probability. The DM-BPSK symbol is contaminated during transmission by additive white Gaussian noise (AWGN) \( w(t) \) with mean zero and two-sided power spectral density \( N_0/2 \).

At the receiver, the contaminated DM-BPSK symbol is correlated with a down-chirp signal \( e^*(t - i\tau) \), producing the \( i \)-th correlator output \( g_i \). Clearly, the real part

\[ h_i = \int_{i\tau - \Delta\tau/2}^{i\tau + \Delta\tau/2} \text{Re} \left[ \{ s(t) + w(t) \} e^*(t - i\tau) \right] dt = \sqrt{E_b} b_i + \sqrt{E_b} z_i + n_i \]

of the correlator output \( g_i \) is sufficient as the decision statistic [6], where the noise

\[ n_i = \int_{i\tau - \Delta\tau/2}^{i\tau + \Delta\tau/2} \text{Re} \left[ w(t) e^*(t - i\tau) \right] dt \]

is a Gaussian random variable with mean zero and variance \( N_0/2 \) and

\[ z_i = \sum_{k=1}^{i+1} b_k p_{i,k} \]

represents the normalized ISI due to overlap with

\[ p_{i,k} = \int_{i\tau - \Delta\tau/2}^{i\tau + \Delta\tau/2} \text{Re} \left[ e^*(t - k\tau) e^*(t - i\tau) \right] dt \]

\[ = \int_{\min\left(\frac{\Delta\tau}{2}, \frac{\Delta\tau}{2} + (k-i)\tau \right)}^{\max\left(\frac{\Delta\tau}{2}, \frac{\Delta\tau}{2} + (k-i)\tau \right)} T_c^{-1} \times \text{Re} \left[ \exp\left( j\pi\mu (y - (k-i)\tau)^2 - y^2 \right) \right] dy \]

\[ = \frac{\sin\left( (k-i)\pi S_1 \frac{(1 - |k-i|^2)}{(k-i)\pi S_1} \right)}{(k-i)\pi S_1}. \]

In (6), \( S = BT_c/L = B\tau \) is the processing gain per overlap with \( B = |\mu|T_c \) the CSS bandwidth. Finally, the \( i \)-th output data \( \hat{b}_i \) is determined by comparing the decision variable \( h_i \) with a threshold of zero.

3. Discussion

It is interesting to see from (6) that, if \( L^2 \) is a divisor of the processing gain \( BT_c \), we have \( p_{i,k} = 0 \) for any non-zero integer \( k - i \) in \( \{ -L + 1, -L + 2, \ldots, L - 1 \} \) since \( S \left( 1 - \frac{|k-i|^2}{L^2} \right) = (L - |k-i|)\frac{BT_c}{L} \) is an integer in such a case: an important implication of the fact \( p_{i,k} = 0 \) for all \( k \neq i \) is that the normalized ISI \( z_i \) shown in (5) becomes zero, and consequently, the overlap does not incur any ISI. For instance, overlaps incur no ISI for \( L \in \{ 1, 5 \}, \{ 1, 2, 5, 10 \}, \) and \( \{ 1, 2, 3, 4, 6, 12 \} \) when \( BT_c = 75, 100, \) and 144, respectively. This is clearly observed in Fig. 3(a), which shows the expected amplitude \( E[|z_i|] \) of the ISI when \( BT_c = 75, 100, \) and 144. From Fig. 3(a), we can additionally observe that, with the processing gain \( BT_c \) of the CSS system fixed, the ISI due to overlap is a fluctuating and a monotonically increasing function of the number \( L \) of overlaps when \( L < BT_c \) and \( L \geq BT_c \), respectively. This can be explained as follows. First, among the set \( \{ p_{i,k} \} \) of semi-sinc functions, \( p_{i,i+1} \) most strongly influences the amplitude of ISI \( z_i \). As shown in Fig. 3(b), \( p_{i,i+1} \) fluctuates as the number \( L \) of overlaps varies when \( L < BT_c \) with the zero-crossings located approximately at

\[ L = \left[ \frac{BT_c}{q} - 1 \right] \]

for \( q \) the first few natural numbers when \( BT_c \gg 1 \), where
some cases when $L < BT_c$ whereas any increase of $L$ would inevitably result in an increase of the ISI when $L \geq BT_c$.

Figure 4 shows the BER curves of the DM-BPSK for various values of $L$ when $BT_c = 100$, where the chirp duration ($T_c$), rate ($\mu$), and bandwidth ($B$) are set to $0.5\mu$s, 400 MHz/$\mu$s, and 200 MHz, respectively. As expected from the discussions above, it is clearly observed that no degradation of the BER due to the ISI is incurred when $L \in (2, 5, 10)$: note that the BER curve for $L = 1$ is the ideal one without overlap. It is also observed that an increase of $L$ does not necessarily imply a degradation of the BER.

### 4. Conclusion

We have addressed the ISI due to overlap in the DM-BPSK system. Specifically, it is found that the ISI due to overlap changes from a fluctuating function to a monotonically increasing function when $L$, the number of overlaps, exceeds $BT_c$. In addition, it is observed that overlaps do not incur any ISI if the processing gain $BT_c$ of the system and the number $L$ of overlaps satisfy certain conditions. The results derived in this letter should be quite useful for the implementation of the DM-BPSK system with overlap.

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