Abstract—Statistical shape models which represent the shape variations within a population are used in a variety of medical image applications, such as statistical shape analysis, model-guided segmentation and registration. This paper presents our Statistical Surface Wavelets Model that has three highly desirable properties of a shape model: compact representation, multi-scale shape description, and spatial-localization of the shape variation. To the best of our knowledge, there is no known prior work that achieves all these properties simultaneously. We have implemented our model to perform shape analysis. Preliminary results achieved from the cerebral lateral ventricle and caudate nucleus are encouraging.

Keywords—statistical shape analysis, wavelet, spatial localization, multiscale

I. INTRODUCTION

Given a population, there are generally pronounced anatomical variations among the subjects. Statistical shape analysis of medical images aims to study the various statistical quantities of such variations. For example, it estimates the mean shape and shape variation mode in a population. Collectively, these statistical quantities define a statistical shape model that captures the essence of the shapes in population.

A good statistical shape model should have the following three properties: compact representation, multi-scale shape description, and spatial-localization of the shape variation. A representation is considered compact if the shape is described with a given accuracy by a small number of parameters or features (that incorporate the mentioned statistical quantities). This property is good in reducing the dimensionality of the feature space of the shape. As shape variations have quite different scales, a multi-scale property allows a model to focus the analysis on the variation within the specified scale of interest. With the property of spatial-localization, the variation of one specific parameter in the model results only in a corresponding local variation of shape without propagating any shape variation to other parts of the model. This property is useful, for example, when a statistical shape model is used to guide the segmentation of medical images as it can lead to a more efficient optimization algorithm by fitting locally the model with the image.

Before presenting our proposed shape model, we first discussed the related works on statistical shape models based on active shape and parametric modal decomposition models. In the Active Shape Model (ASM) [4], a shape is represented by a set of boundary sample points and a series of connection relations among these points (edges and facets). In a population, after normalization to size, orientation and position, a mean shape is formed from the mean points’ coordinates over all members in the population at each sample point. Through a standard way of principal component analysis on the population, eigenvectors which represent the eigen shape variation modes are computed and new shapes are modeled by a linear combination of these eigenvectors. As the population is usually small in size, relative to the dimensionality of the shape space, the possible ways to deform a shape are limited to a linear subspace of the complete shape space. Moreover, the representation of shape here is composed of discrete points, thus, we know the geometry of a shape only at a finite set of points. When a high degree of precision is required on the shape, a correspondingly dense sampling is needed. This shape model can be verbose and not quite efficient.

On the other hand, parametric modal decomposition models provide continuous and concise representation of a shape. The decomposition basis is usually a set of different frequency harmonics. The decomposition results in a small number of coefficients which capture the overall shape. The Fourier [14][15] and spherical harmonics [9] are the most commonly used bases. However, the basis functions of the Fourier and spherical harmonics are periodic and globally supported (rather than localized in space), so a small perturbation of a parameter can affect the entire outline of a shape. These are thus not efficient and effective in describing shapes with only local deformation.

In contrast to the Fourier and spherical harmonics, wavelet basis functions are compactly supported and have the localization property both in frequency and space. Statistical shape models based on wavelets for 2D shape were presented by [3][5]. Their works explicitly parameterize the shape boundaries and then decompose the parameterized boundary with the so-called first-generation wavelets that work with manifolds defined on regular grids. The rigorous requirements in this explicit surface parameterization are the main obstacles to extend this wavelet basis to 3D surface. We need, firstly, a surface parameterization that provides the correspondences between objects in the population. Secondly, the first-generation wavelets requires the surface to be parameterized by regular grids. Furthermore, since
the anatomical structures are usually of sphere topology, to parameterize such a surface with sphere topology needs 2 parameters, longitude and latitude. However, distortions will inevitably occur at the south and north poles.

In our proposed model, we adopt a newly developed surface wavelet scheme in [1] with the wavelet basis defined directly on a surface mesh. This so-called second-generation wavelet scheme can work with manifolds defined on non-regular grids, more specifically, on the Catmull-Clark subdivision mesh. With it, our model is a concise shape description to represent shape variations in a population.

The remaining part of the paper is organized as follows. Section II addresses the solution to the correspondence problem on anatomical data, and Section III discusses our proposed Statistical Surface Wavelets Model (SSWM) and compares it with other shape models. Then, Section IV presents our preliminary results by using SSWM to analyze cerebral lateral ventrical and 18 samples of the caudate nucleus obtained from the Internet Brain Segmentation Repository. Lastly, Section V summarizes the work done so far and presents possible future research directions.

II. The Correspondence Finding

Our work uses anatomical objects obtained from MRI images. After binary volumetric data of the interested structure is obtained through a segmentation process, the outmost facets of the voxels are extracted as the surface mesh of the anatomical object. Figure 1(a) gives an example of the extracted surface mesh of a brain ventricle.

In order to measure shape variation at a specific part of objects in a population, we first need to match up the corresponding part of the objects. We address this problem with a spherical harmonics (SPHARM) normalization. By its name, the SPHARM description [2] is a parametric surface description using spherical harmonics as basis functions. This method finds the correspondences in the spectral space through a normalization process by using the rotation-invariant property. Note that this method maps the object to a unit sphere, and, as a result, it becomes easy to construct the Catmull-Clark subdivision mesh connectivity needed by our wavelet scheme, since a re-meshing can be performed directly on the unit sphere domain.

In the following, we describe the three steps of the correspondence method: (1) unit sphere mapping, (2) SPHARM expansion, and (3) SPHARM normalization.

Unit sphere mapping aims to create a continuous and uniform mapping from the object surface to the surface of a unit sphere. This mapping is formulated as a constrained optimization problem with the goals of topology and area preservation and distortion minimization[2]. The result is a bijective mapping (Figure 1(b)) between each point \( V(x, y, z) \) on a surface and two spherical coordinates \( \theta \) and \( \phi \) on a unit sphere:

\[
V(x, y, z) = \begin{pmatrix} x(\theta, \phi) \\ y(\theta, \phi) \\ z(\theta, \phi) \end{pmatrix}
\]

where \( x, y \) and \( z \) denote the Cartesian object space coordinates and \( \theta \) and \( \phi \) are the two spherical coordinates. When the free variables \( \theta \) and \( \phi \) run over the whole sphere, \( V(x, y, z) \) runs over the whole surface of the input object.

SPHARM expansion expands the object surface into a set of SPHARM basis functions \( Y_l^m \), where \( Y_l^m \) denotes the spherical harmonic of degree \( l \) and order \( m \). The spherical harmonics are defined as

\[
Y_l^m(\theta, \phi) = \sqrt{\frac{2l + 1}{4\pi} \frac{(l - m)!}{(l + m)!}} P_l^m(\cos \theta)e^{im\phi}
\]

where \( P_l^m(\cos \theta) \) are associated Legendre polynomials (with argument \( \cos \theta \)), and \( l \) and \( m \) are integers with \(-l \leq m \leq l\). After the unit sphere mapping, the Cartesian object space coordinates \( x, y \) and \( z \) of the points on the surface are functions of variables \( \theta \) and \( \phi \) defined on the unit sphere. Thus, each of them can be decomposed with the SPHARM basis functions:

\[
\begin{pmatrix} x(\theta, \phi) \\ y(\theta, \phi) \\ z(\theta, \phi) \end{pmatrix} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_{xl}^m Y_l^m(\theta, \phi)
\]

Let \( c_{xl}^m = (c_{xl}^m, c_{yl}^m, c_{zl}^m)^T \). Then, the surface SPHARM expansion is:

\[
V(x(\theta, \phi), y(\theta, \phi), z(\theta, \phi)) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_{xl}^m Y_l^m(\theta, \phi)
\]
Given these coefficients $c_m^n$, by resampling on the unit sphere to specify the $\theta$ and $\phi$ in the right side of equation 4, inversely, we can reconstruct the surface points. And using more coefficients leads to a more detailed reconstruction. See Figure 1(c) for an example.

**SPHARM normalization** removes translation, rotation, and scaling in objects to generate a normalized set of SPHARM coefficients for objects, which are comparable across objects. In brief, rotation invariance is achieved by aligning the degree 1 reconstruction, which is always an ellipsoid, scaling invariance by dividing all the coefficients by a scaling factor, and translation invariance by ignoring degree 0 coefficient. Figure 1 shows the normalized reconstruction without translation, rotation and scaling factors. After normalization, the correspondence between object surfaces can be defined using parameter $\theta$ and $\phi$. Points on two surfaces with the same spherical coordinates $\theta$ and $\phi$ are corresponding points.

III. **THE STATISTICAL SURFACE WAVELETS MODEL**

Having solved the correspondence issue and re-meshing to obtain the required subdivision mesh, we can now build our Statistical Surface Wavelets Model. But before that, we first provide some background of the adopted subdivision-surface wavelets.

**A. Subdivision Surface Wavelets**

Though wavelets have been applied in many domains since 1980s, there was no surface wavelet scheme reported until much later. Wavelets representing surfaces of arbitrary topology were originally explored by Lounsbery et al.[11][12]. Subsequently, other subdivision-surface wavelet constructions for functions defined on triangulated spherical domains were introduced by Schroder and Sweldens [13]. However, these approaches are used for constructing functions on given domains rather than representing the underlying domain geometries. Recently, Bertram et al. [1] introduced a new biorthogonal wavelet decomposition and synthesis scheme based on generalized B-subdivision surfaces. This method also uses Lifting Scheme [16], but is capable of representing two-manifold geometries as well as functions defined on the two-manifold. More importantly, both its decomposition and synthesis are linear to the number of points on the mesh.

The subdivision-surface wavelet we used [1] is a second-generation wavelets. It is based on the hierarchical mesh connectivity defined by Catmull-Clark subdivision. Figure 2 shows an example. Here, we call the initial mesh sub-mesh (right) and the mesh resulting from subdivision or reconstruction super-mesh (left), since it contains more vertices. A mesh is refined (from sub-mesh to super-mesh) by inserting a new vertex inside every face and on every edge and by connecting these vertices to quadrilaterals. Vertices in a super-mesh correspond to a face (polygon), an edge, or a vertex in the sub-mesh and are denoted by f, e, and v, respectively. A decomposition step can be considered as an operation applied to a super-mesh that computes $v'$ vertex positions for an approximating sub-mesh and replaces the remaining $e$ and $f$ vertices by difference vectors $(e', f')$ representing details that are missing in the sub-mesh. The $v'$ vertices represent coefficients for scaling functions, $e'$ vertices represent wavelet coefficients corresponding to edges, and $f'$ vertices represent wavelet coefficients corresponding to faces. The reconstruction (synthesis) is then a step of reconstructing the super-mesh from the sub-mesh by adding the detail shape information contained in the wavelet coefficients.

We can denote the set of all vertices contained on the mesh after $j$ subdivisions as $V(j)$. When we get a finer resolution mesh $V(j+1)$ through subdivision, we denote the $e'$ vertices and $f'$ vertices added in the subdivision as a vertex set $W(j)$. The complete set of vertices in the finer resolution $j + 1$ is $V(j+1) = V(j) \cup W(j)$. Let $S$ be a surface and $x \in \mathbb{R}^3$ is a vertex on $S$. Using wavelet transform, surface $S$ is represented by a set of basis consisting of dilated and translated versions of a wavelet $\psi$ and a scaling function $\phi$ on the surface of $S$.

$$S(x) = \sum_{j \geq 0} \sum_{m \in W(j)} w^j_m \psi^j_m(x) + \sum_{n \in V(0)} v^0_n \phi^0_n(x)$$

where, $\phi^0_n$ is the scaling function of the coarsest scale at vertex $n$ and $\psi^j_m$ is the wavelet function of scale $j$ at vertex $m$. $v^0_n$ and $w^j_m$ are the corresponding coefficients respectively. Here, in 3D, every coefficient is actually a vector in which each element represents one of the coordinates $x$, $y$, $z$. Figure 3 shows scaling and wavelet functions examples on a sphere.

**B. Building Statistical Surface Wavelets Model**

With the method described in Section II, each shape can be decomposed and represented by a vector consisting of the
wavelets coefficients as follows:

\[ p = \{ v^0_{m,x}, v^0_{m,y}, v^0_{m,z}, w^j_{m,x}, w^j_{m,y}, w^j_{m,z} | j = 1, 2, \ldots \}; \]

\[ m \in W(j); n \in V(0) \]  

(5)

For a population with \( N \) objects (or shapes), let \( p_i \) denotes the \( i^{th} \) shape in the population. A matrix defined as \( P = [p_1, p_2, \cdots, p_N] \) is now a representation of the whole population, in which each column represents one shape. With this, elements in one row of \( P \) are the measures of some part of the objects across the population. We can thus analyze this part across the population to obtain a statistical shape model. Thus, we call \( P \) a Statistical Surface Wavelets Model (SSWM). As mentioned before, this model possesses the three most desirable properties of a statistical shape model. We next illustrate these properties through examples.

We use the proposed wavelet transform to analyze a cerebral lateral ventricle as shown in Figure 4. Figure 4(a) is the original shape of the brain ventricle before decomposition. By reconstructing the shape from the wavelet coefficients up to different scale of level \( j \) in Equation 5, we get the multi-scale representation in different scale levels as shown in Figure 4(b)-(f). Figure 4(e) is a reconstruction up to scale level 3 using 384 wavelet coefficients. This reconstructed mesh with 24,578 vertices approximates the original shape very well. This is a demonstration of the conciseness of the SSWM. Figure 5 shows the spatial-localized shape representation property by comparing SSWM with SPHARM descriptor. Due to the fact that the basis function of the SPHARM descriptor is periodic and global supported, a small change of one coefficient can affect the entire outline of a shape (Figure 5(c)). However, the SSWM uses the wavelet basis functions with local support (examples are shown in Figure 3). Each coefficient only defines local shape in certain scale. Therefore, it provides the ability to focus the shape analysis at any specific part of the object.

IV. STATISTICAL SHAPE ANALYSIS USING SSWM

We have completed an experiment of using our SSWM model to analyze 18 samples of caudate nucleus available from The Internet Brain Segmentation Repository (IBSR) [8]. IBSR provides the caudate nucleus segmented from MRI images at a resolution of \( 256 \times 256 \times 128 \). Figure 6 shows the 18 samples of right caudate nucleus. Note that the shape has been normalized by the method shown in Section II.

We compute the 18 shapes in the vector form \( p_i (i = 1, \cdots, 18) \) as defined by Equation 5. With this, we can compute the mean shape of a population by averaging the corresponding coefficients: \( \overline{p} = \frac{1}{N} \sum_{i=1}^{N} p_i \).

The mean shape \( \overline{p} \) is shown in Figure 7(a). Also, the spatial distribution of the shape deviations away from the mean shape can be computed. This is shown in Figure 7(b) with different colors representing the different extent of deviation. From this figure, we can see that the tail of the caudate nucleus is the most variable part, and the middle body part is relatively stable in shape.

Besides the mean shape, we can perform statistical shape analysis at different scales since the shape model is multi-scale. By performing a principal component analysis on each specified scale level, we can understand the most significant variation of the shape in that scale level as shown in Figure 8. Note that the variation scale changes from coarse to detail in Figure 8(a) to Figure 8(d). The ASM model [4], because of its discrete shape representation using PDM (points distribution model), can’t provide this kind of multi-scale shape analysis capability. Models based on the Fourier [15] or spherical harmonics [9] basis can produce similar results as shown in Figure 8. However, due to their global supported basis function, they don’t have the ability to focus the shape analysis only on parts of the surfaces. In contrast, because of the spatial-localization property of SSWM, we now can perform shape analysis not only at a specified scale level, but also at a specified location on the surface. To do so, using the SSWM, the method is quite straightforward. Instead of doing a principal component analysis on all the wavelet coefficients...
corresponding to the specified scale level, we only perform the analysis on the wavelet coefficients of the specified scale level corresponding to some region of interest on the surface. The results are shown in Figure 9. In the figure, the region of interest is denoted by the dashed red line. From Figure 9(a)-(d), the shape variation modes are constrained locally with different scales, while the other part of the shape remains unchanged. Such capability of SSWM compares favorably to other models mentioned previously which usually only support global variations (as shown in Figure 8). In medical shape analysis, such localized shape analysis can be very useful as diseases, such as cancer, may only affect a small portion of an organ.

V. CONCLUDING REMARKS AND FUTURE WORKS

We propose in this paper the Statistical Surface Wavelet Model (SSWM). This is the first application of the subdivision-surface wavelets in the shape analysis domain. In contrast to other shape models, our model is compact in its shape representation. Moreover, it supports shape analysis at multi-scales and multiple locations from wavelet coefficients at different scales and locations. Experiment using this model has shown shape variations of cerebral lateral caudate nucleus at the specified scale level, as well as, at the specified location on the surface. The proposed model is potentially useful in supporting model-guided segmentation. Its multiscale and spatial-localized representation can lead to a multiscale and spatial-localized optimization algorithm to perform model-fitting during segmentation. This is good news as the optimization can possibly be divided into much smaller steps at different scales and locations and therefore, be conducted more efficiently and effective.

Another possible application of the proposed SSWM model is the classification of 3D neuroanatomic structures. This classification aims to discriminate between healthy and pathological neuroanatomic structures based on their surface...
to select a subset of the most useful elements from the shape descriptors which are focused on the most significant surface region at the most significant scales. Although PDM is spatial-localized, it is not multi-scale. Therefore, it is only selective to the locations on the surface. SPHARM is multi-scale, but because of its global supported basis function, it is only selective to the scale. In contrast, the SSWM provides selectivity both in scales and spatial locations. So, when used as the shape descriptor, SSWM can potentially improve the classification accuracy.

ACKNOWLEDGMENT

The authors wish to thank Martin A. Syner of the Department of Computer Science, University of North Carolina at Chapel Hill for providing the spherical mapping and SPHARM normalization routines discussed in [2].

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