The Probabilities of Conditionals Revisited

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Abstract

According to what is now commonly referred to as “the Equation” in the literature on indicative conditionals, the probability of any indicative conditional equals the probability of its consequent of the conditional given the antecedent of the conditional. Philosophers widely agree in their assessment that the triviality arguments of Lewis and others have conclusively shown the Equation to be tenable only at the expense of the view that indicative conditionals express propositions. This study challenges the correctness of that assessment by presenting data that cast doubt on an assumption underlying all triviality arguments.

Keywords: Conditionals; Probability; Triviality arguments; Semantics

1. Introduction

According to what is now commonly referred to as “the Equation” in the literature on indicative conditionals, the probability of any indicative conditional equals the probability of its consequent conditional on its antecedent, provided the latter is defined. Formally, where “if” is the indicative conditional operator,

\[ \Pr(\text{If } \varphi, \psi) = \Pr(\psi \mid \varphi), \quad \text{for any } \varphi, \psi \text{ such that } \Pr(\varphi) > 0. \] (EQ)

Stalnaker (1970) presented this as an adequacy condition to be met by any semantics for conditionals, meaning that candidate accounts of the truth conditions for conditionals should at least validate (EQ). But, while (EQ) was initially found to have considerable intuitive appeal, Lewis’s (1976) so-called triviality arguments convinced the broad
philosophical community that (EQ) can be maintained only at the expense of the view that conditionals express propositions, that is, the view that conditionals have classical truth conditions in that they are either true or false.

For a number of theorists, the triviality arguments have been a reason to reject (EQ). However, over the past decade, (EQ) has been subjected to empirical testing by various experimental psychologists, and it has been found, time and again, that people’s judged probabilities of conditionals do closely match their judgments of the corresponding conditional probabilities; see, for instance, Hadjichristidis et al. (2001), Evans, Handley, and Over (2003), Oaksford and Chater (2003), Oberauer and Wilhelm (2003), Over and Evans (2003), Evans and Over (2004), Weidenfeld, Oberauer, and Hornig (2005), Evans, Handley, Neilens, and Over (2007), Evans, Handley, Neilens, Bacon, and Over (2010), Oberauer, Geiger, Fischer, and Weidenfeld (2007), Oberauer, Weidenfeld, and Fischer (2007), Over, Hadjichristidis, Evans, Handley, and Sloman (2007), Douven and Verbrugge (2010), Pfeifer and Kleiter (2010), and Politzer, Over, and Baratgin (2010). Given these experimental results, rejecting (EQ) would amount to attributing massive error to people as far as their judgments of the probabilities of conditionals are concerned. In view of this, abandoning the assumption that conditionals express propositions has come to appear the more attractive option to many, even though—it is generally acknowledged—doing so raises some thorny issues, most notably, concerning how conditionals combine with other parts of our language and how we are to account for iterated conditionals.

The present study argues against the broadly shared view that the triviality arguments have shown (EQ) to be incompatible with the view that conditionals express propositions. Specifically, we will point out that these arguments rely on assumptions that entail a generalization of (EQ). Despite the surge of interest among psychologists in the probabilities of conditionals, the said generalized version of (EQ) has so far been ignored by experimentalists. We have, for the first time, tested this version and have obtained data refuting it. In the following, we report these data and argue that, in conjunction with the extant empirical evidence for (EQ), they suggest that (EQ) is tenable even if conditionals express propositions. We start by describing the broader theoretical background of our experimental work, in particular current approaches to the semantics of conditionals, which have raised interest in (EQ) in the first place.

2. Theoretical background

Although many proposals have been made concerning the truth conditions of conditionals, three have been clearly dominant in the literature. These are, first, the Material Conditional Account (MCA), according to which a conditional “If \( \varphi, \psi \)” is false if \( \varphi \) is true and \( \psi \) is false, and is true in all other cases; second, Stalnaker’s (1968) possible worlds semantics, which declares a conditional to be true (false) precisely if its consequent is true (false) in the closest possible world in which the conditional’s antecedent is true, where “closest” means “most similar to the actual world,” and provided there is a world in which the antecedent is true; and third, the non-propositional view, according to
which conditionals never have a truth value (the No Truth Value view, or NTV for short),
or, in a slightly less radical version, have the truth value of their consequent if their ante-
ccedent is true and otherwise are neither true nor false and so, as some like to say, have a
third “truth value” (the Three Truth Value view, or TTV).³⁴
Of these three semantics, the first was once the orthodoxy in philosophy. While that is
no longer the case, it still seems to be the “default” position, in that it is the view tacitly,
or even not so tacitly, endorsed by most philosophers not working on conditionals. By
contrast, most philosophers who presently do work on conditionals seem to favor a
version of the non-propositional view. In psychology, the situation is slightly different in
that even psychologists doing research on conditionals are divided. Those working in the
mental models tradition of Johnson–Laird typically advocate MCA, while those working
within the Bayesian approach to reasoning are more inclined toward either NTV or
TTV.⁵ Stalnaker’s semantics has never been popular as a semantics for indicative condi-
tionals, neither in philosophy nor in psychology, but at least in philosophy many believe
it to provide the best semantics of subjunctive conditionals. In linguistics, again the situa-
tion is different. According to von Fintel (2011, Sect. 5), there is hardly any support for
the non-propositional view among linguists. Instead, most linguists favor some version of
the Lewis–Kratzer view (von Fintel, 2011, Sect. 4). In this semantics, there is no such
thing as a conditional operator; rather, antecedent clauses attach to overt or covert opera-
tors, restricting the domain of quantification of those operators. This view has not
received much attention from either philosophers or psychologists.⁶
All three main semantics of conditionals are known to have their virtues. However, all
three also face severe problems. To start with MCA: It is commonly held that high probabil-
ity is at least close to being sufficient for rational credibility (that many think it is not entirely
sufficient has to do with Kyburg’s (1961) so-called lottery paradox). Given that a disjunction
is at least as probable as its most probable disjunct, it follows from MCA that a conditional
is highly probable if its antecedent is highly improbable. But consider this sentence:
  • If Manchester United finishes last in the League, they will give their manager a
raise.
Although it is exceedingly unlikely that Manchester United will end last in the League,
we find the above sentence neither highly probable nor rationally credible—quite the
contrary! This is not an isolated fact about the above conditional. Various experimental
studies carried out in the past years have shown that people’s probability judgments devi-
ate starkly from what, on the basis of MCA, one would expect these judgments to be
(see, e.g., Evans & Over, 2004; Oaksford & Chater, 2007). Besides these probabilistic
issues, it appears that ordinary people make many mistaken inferences if MCA is correct
(see, e.g., Evans, Newstead, & Byrne, 1993; Evans & Over, 1996; Oaksford, Chater, &
The possible worlds account also seems to get the probabilities of conditionals wrong
to the extent that people’s actual probability judgments of conditionals are at variance
with what the possible worlds account predicts these judgments to be (Evans & Over,
2004). Moreover, many still regard Gibbard’s (1981) argument as a decisive refutation of
the possible worlds account. This argument proceeds by presenting a story in which two protagonists each assert a conditional, where one of these conditionals is of the form “If \( \varphi, \psi \)” and the other is of the form “If \( \varphi, \neg \psi \).” The context of the story is such that both assertions appear unassailable, and even such that it would seem that if we deem one of the conditionals true, then we are reasonably obliged to deem the other true as well. But deeming both true would go against Conditional Non-Contradiction (CNC), a principle that is valid in Stalnaker’s semantics. According to CNC, pairs of conditionals with the same antecedent but contradictory consequents are themselves contraditories, provided their antecedent is consistent (which holds for the Gibbard conditionals). Of course, MCA does not validate CNC, and so it allows us to judge both Gibbard conditionals to be true at the same time. However, MCA has problems of its own, as we noted, and as Gibbard also noted, which led him to favor the NTV view.

Finally, if probabilities are operationalized as fair betting rates, then, given the NTV–TTV views and given plausible betting conditions for a conditional—a bet on a conditional is won if the conditional is true, lost if the conditional is false, and called off otherwise—the probabilities of conditionals automatically equal the corresponding conditional probabilities. Given the outcomes of the empirical work on (EQ) cited in the introduction, that is an important point in favor of the NTV–TTV views. Also, Adams (1975) has defined a notion of validity to go with these views that seems to license precisely the inferences that people deem intuitively valid. Still, these points should not blind us to the shortcomings that the NTV–TTV views also have. The most notable one stems from the fact that standard logical operators, like conjunction and disjunction, are propositional operators, meaning that they apply only to propositions, that is, things that have determinate truth conditions. If conditionals do not have full truth conditions, then they cannot occur in the scope of a logical operator. That would mean, for example, that we cannot make sense of a conjunction one of whose conjuncts is a conditional. But that seems false. While proponents of the NTV–TTV views have presented ways around this difficulty (even if these have not done much to convince the opposition), a more serious problem, one for which not even a candidate solution has been proposed so far, is that the NTV–TTV views are unable to account for so-called left-nested conditionals, that is, conditionals whose antecedent is itself a conditional. Yet there are many such sentences that qualify as bona fide conditionals, and which we have no difficulty understanding as such. Consider, for instance, the following conditionals, which are both left and right nested:

- If your mother is irritated if you come home with a B, then she will be furious if you come home with a C.
- If the watch breaks if it is dropped, then the watch will break if it is smashed against the wall.

To be able to account for sentences like these is generally regarded as one of the main challenges facing the NTV–TTV views. Perhaps this and other challenges can be met. Still, it would seem ideal if we could have a semantics that both makes conditionals come out as propositions and gets the
probabilities of conditionals right (i.e., that makes these probabilities equal the corresponding probabilities). As said, however, Lewis’s triviality arguments are generally taken to have shown for good that such a semantics is not in the offing. We now turn to these arguments.

3. Triviality

Lewis (1976) presented two triviality arguments against (EQ), the first and simplest of which goes as follows: Let Pr be a probability function for which (EQ) holds, and let \( \varphi \) and \( \psi \) be such that both \( \Pr(\varphi \land \psi) > 0 \) and \( \Pr(\varphi \land \neg \psi) > 0 \). Also, suppose that conditionals have full truth conditions. Then, from the law of total probability, it follows that

\[
\Pr(\text{If } \varphi; \psi) = \Pr(\varphi, \psi | \psi) \Pr(\psi) + \Pr(\varphi, \psi | \neg \psi) \Pr(\neg \psi).
\] (1)

Assuming (EQ), this can be rewritten as

\[
\Pr(\text{If } \varphi; \psi) = \Pr(\psi | \varphi \land \psi) \Pr(\psi) + \Pr(\psi | \varphi \land \neg \psi) \Pr(\neg \psi).
\] (2)

And this simplifies to

\[
\Pr(\text{If } \varphi; \psi) = 1 \times \Pr(\psi) + 0 \times \Pr(\neg \psi).
\]

In other words, for all \( \Pr, \varphi, \) and \( \psi \) that satisfy the aforementioned conditions, it holds that \( \Pr(\text{If } \varphi; \psi) = \Pr(\psi) \). It takes little effort to appreciate that this is not a realistic constraint for people’s belief states. For instance, one may be certain that if Carol will come to the party, then Peter will come without being certain that Peter will come. The other triviality argument to be found in the same study is a relatively minor extension of the one above. For present purposes, the important point to note is that it relies on the step from (1) to (2) as well.

The publication of these triviality arguments had a great impact in the field of philosophy. As mentioned above, these arguments were generally perceived as showing conclusively that there can be no semantics of conditionals that is descriptively adequate insofar as the probabilities of conditionals are concerned—unless the idea that conditionals express propositions is given up. For if conditionals do not express propositions, then neither of the conditional probabilities in (1) is well defined. After all, by probability theory, it should hold that \( \Pr(\text{If } \varphi; \psi | \psi) = \Pr(\text{If } \varphi, \psi | \psi) \div \Pr(\psi) \), and similarly for the other conditional probability. And, as was mentioned, conditionals can occur as conjuncts only if they are propositions. As a result, many came to regard Lewis’s triviality arguments as a major—or even the main—motivation for endorsing the non-propositional views of conditionals. However, we saw that these views have some undesirable consequences. Naturally, we might simply have to live with these consequences. But, as will be clear from the introduction, we are more hesitant than is the mainstream to believe that the triviality
arguments leave us with no other options than either to live with those consequences or to accept that our semantics of conditionals will get the probabilities of conditionals wrong.

To explain our hesitancy, we begin by noting that, just on the basis of (EQ), the move in the above argument from (1) to (2) is illicit. After all, (EQ) equates the probability of a conditional with the corresponding conditional probability, not (as in the designated move) the conditional probability of a conditional given a certain proposition with the conditional probability of the conditional’s consequent given the conjunction of the antecedent and the given other proposition. That is to say, Lewis’s triviality arguments really rely on the following generalization of (EQ):

$$\Pr(\text{If } \varphi, \psi | \chi) = \Pr(\psi | \varphi \land \chi), \text{ for any } \varphi, \psi, \chi \text{ such that } \Pr(\varphi \land \chi) > 0.$$  \hfill (GEQ)

We obtain (EQ) from (GEQ) by setting $\chi$ equivalent to an arbitrary tautology; obviously, we cannot obtain (GEQ) from (EQ). Hence, (GEQ) is a genuinely stronger premise than (EQ).

It should be remarked at once that (GEQ) is not a basic assumption of Lewis’s arguments. Rather, Lewis derives (GEQ) from (EQ) in conjunction with what is a basic assumption of those arguments, namely, that the class of probability functions for which (EQ) holds is closed under conditionalization, meaning that for any probability function $\Pr$ in the class, the probability function $\Pr_\varphi$ that results from conditionalizing $\Pr$ on $\varphi$, for any proposition $\varphi$ such that $\Pr(\varphi) \neq 0$, is in the class as well. With this extra premise, Lewis derives (GEQ) as follows: Let $\Pr$ be an arbitrary probability function in an arbitrary class $\mathcal{P}$ of probability functions such that (a) (EQ) holds for all probability functions in $\mathcal{P}$, and (b) $\mathcal{P}$ is closed under conditionalization. Then, by (b), for any $\chi$ such that $\Pr(\chi) \neq 0$, $\Pr_\chi$ is in $\mathcal{P}$. Thus, by (a), for any $\varphi$ and $\psi$ such that $\Pr_\chi(\varphi) \neq 0$, $\Pr_\chi(\text{If } \varphi, \psi) = \Pr_\chi(\psi | \varphi)$. And so, given that, by definition, $\Pr_\chi(\text{If } \varphi, \psi) = \Pr(\text{If } \varphi, \psi | \chi)$ and $\Pr_\chi(\psi | \varphi) = \Pr(\psi | \varphi \land \chi)$, it follows that (GEQ) holds for $\Pr$. Because $\Pr$ was an arbitrary element of $\mathcal{P}$, (GEQ) holds for all probability functions in $\mathcal{P}$. Finally, because $\mathcal{P}$ was chosen arbitrarily, (GEQ) holds for all probability functions in any class of such functions for which (a) and (b) hold.

Our claim is not that there is something contentious about Lewis’s closure assumption. To the contrary, we believe that this assumption is perfectly innocuous, certainly if we limit (EQ) to the class of probability functions that represent possible systems of belief, as does Lewis (1976, 82). We agree with Lewis (ibid.) that “[r]ational change of belief never can take anyone to a subjective probability function outside [this] class.” While we would object to the assumption that conditionalization is the only rational update rule (see Douven, 1999), we are unaware of any good reasons to believe that conditionalization is not a rational update rule, possibly among various other such rules. And the latter is all that Lewis needs. Indeed, the closure assumption is consistent with the supposition that rational people sometimes just give up their
probability function—throw away their priors, as some call it—and arbitrarily pick another one.

But there is another assumption underlying Lewis’s derivation of (GEQ), one that is more easily overlooked, to wit, the assumption that the conditional-forming operator has an interpretation that is independent of belief states. To see why this is necessary, suppose the interpretation of “if” were allowed to vary with a person’s belief state so that, in effect, for each Pr we might have a different, tacitly indexed, conditional-forming operator, which we might indicate by “if_i”; thus, “if_x” denotes the conditional corresponding to the probability function Pr_x that comes from Pr by conditionalizing on x. Then, from the assumption that (EQ) holds for each pair of a probability function and a corresponding conditional, we get that Pr_x(If_x φ, ψ) = Pr_x(ψ|φ). We also still have that, by the definition of conditionalization, Pr_x(If φ, ψ) = Pr(If φ, ψ|x). But, if the interpretation of conditionals depends on belief states, then there is no guarantee that Pr_x(If φ, ψ) = Pr_x(If_x φ, ψ), and so the inference of (GEQ) from (EQ) and the closure assumption is no longer valid. (See Section 6 for how conditionals might have their interpretation relative to belief states.)

We have so far considered only the triviality arguments from Lewis (1976). Since the publication of these arguments, many more triviality arguments have appeared, not all of which rely on (GEQ). But, whatever the details of these further arguments, due to a result by van Fraassen, it is known that all of them—as well as any possible future such arguments—are committed to the assumption that the conditional-forming operator has a fixed interpretation across belief states. More specifically, van Fraassen (1976) proves not only that (EQ) is cotenable with a truth-conditional semantics of conditionals, provided the interpretation of “if” may depend on a person’s belief state, but also that this remains the case if, on each interpretation, “if” is supposed to satisfy certain logical principles generally agreed to be characteristic of conditionals. Lewis was already familiar with van Fraassen’s result when writing the final version of his own 1976 article. However, while he mentions the possibility that the conditional is belief sensitive, he quickly dismisses it (more on this below).

In short, all triviality arguments have as a premise not only (EQ) but also the fixedness of the interpretation of the conditional, and these premises, when taken with the assumption that the class of probability functions that we are considering is closed under conditionalization, entail (GEQ). Although the hypothesis that people’s probability functions validate (GEQ) would seem to be testable in much the same way in which (EQ) has been tested, no empirical results concerning this thesis are yet known in the literature. To see whether people’s probability judgments are in line with a generalized version of (EQ) is not only interesting in its own right; doing so might also shed new light on the triviality results. From the perspective of those attracted to the idea that conditionals are propositions, it is already discomfiting to know that there is massive empirical support for a premise that the triviality arguments seem to reduce to absurdity. If we find support for (GEQ) as well, that would make things more discomfiting still; in that case, two theses that must be among the premises of any triviality argument jointly have a consequence that is also borne out by the data. But if, on the other hand, (GEQ) is disconfirmed, that
would supply a reason for reconsidering the fixedness thesis, and for thinking that Lewis has been too quick to deny any real importance to van Fraassen’s result. We argue for this in greater detail below. First, we turn to the question of the empirical adequacy of (GEQ).

4. Experiment 1

Whereas an impressive amount of evidence has accrued for the descriptive adequacy of a particular instance of (GEQ)—to wit, (EQ)—the goal of the present study was to investigate whether (GEQ) is descriptively correct in general. We proceeded in much the same way as previous studies on (EQ), the main difference being that we had to ask participants not to judge the probabilities of conditionals, but the conditional probabilities of conditionals, given some proposition. We realized that this might make the task presented to participants cognitively more demanding than the tasks that had been presented in the studies on (EQ). In some of these studies, a percentage of participants had shown a tendency to interpret conditionals as conjunctions.\(^{10}\) This effect was absent when real-world conditionals were used, that is, conditionals pertaining to events that have occurred or might still occur and about which participants may be expected to hold prior beliefs (Over, Hadjichristidis, Evans, Handley, & Sloman, 2007). Douven and Verbrugge (2010, Experiment 3) found a close-to-perfect match between the probabilities of conditionals and conditional probabilities in a between-subjects experiment, which is evidence that few (if any) of their participants had interpreted conditionals as conjunctions. In a control experiment to be reported in the next section, we similarly failed to find any evidence for the conjunctive interpretation. However, all these studies concern unconditional probabilities of conditionals. So, although our stimuli consisted of real-world conditionals, we still reckoned with the possibility that some participants would interpret the conditionals as conjunctions, as this might plausibly be assumed to reduce the complexity of the tasks.\(^{11}\) If this possibility materialized, then that, rather than the falsity of (GEQ), could explain whatever difference we might find between conditional probabilities of conditionals and corresponding conditional probabilities. To control for this possible confounder, we also asked a group of participants to rate the probabilities of the relevant conjunctions.

4.1. Method

4.1.1. Participants

There were 185 participants in the experiment. In return for their cooperation, they were paid a small amount of money. Participants were recruited and paid via the CrowdFlower interface (http://www.crowdflower.com), which directed them to the Qualtrics platform (http://www.qualtrics.com) on which the experiment was run. All participants were from the United Kingdom or the United States. Their native language was English.
4.1.2. Design

The question concerning the probability of the conditional, the question concerning the corresponding conditional probability, and the question concerning the probability of the conjunction were manipulated between subjects.

4.1.3. Materials and procedure

All materials were in English, the participants’ mother tongue. The participants were divided into three groups. We asked one group \( (N = 62) \) to judge probabilities of conditionals, another group \( (N = 63) \) to judge the corresponding conditional probabilities, and a third group \( (N = 60) \) to judge the probabilities of the corresponding conjunctions. Each participant had to evaluate 10 items, which were presented on screen. Items were randomized and the participants were randomly assigned to one of the three groups.

The following is an example of a question asking for the conditional probability of a conditional, which was presented to one of the three groups:\(^{12}\)

Suppose that the global economic situation stabilizes in the next year.
Then how probable is the following sentence?
If the British government is unsuccessful in creating more jobs, there will be more riots in the UK like the ones we saw a few months ago.
Highly improbable 1 2 3 4 5 6 7 highly probable\(^{13}\)

A second group received the question asking for the corresponding conditional probability:

Suppose that both of the following sentences are true:
1. The global economic situation stabilizes in the next year.
2. The British government is unsuccessful in creating more jobs.
Then how probable is the following sentence?
There will be more riots in the UK like the ones we saw a few months ago.
Highly improbable 1 2 3 4 5 6 7 highly probable

And a third group was asked to judge the corresponding conjunction of antecedent and consequent:

Suppose that the global economic situation stabilizes in the next year.
Then how probable is the following sentence?
The British government is unsuccessful in creating more jobs and there will be more riots in the UK like the ones we saw a few months ago.
Highly improbable 1 2 3 4 5 6 7 highly probable

It is worth emphasizing that in determining conditional probabilities in the above way, we have been relying on the so-called Ramsey test, according to which one determines
Pr(ψ|φ) by first adding hypothetically φ to one’s stock of beliefs and then determining how probable ψ is in that hypothetical situation. According to Bennett (2003, 53), “[t]he best definition [of conditional probability] we have is the one provided by the Ramsey test.” His view is shared by, among others, Edgington (1995, 266 f, 1997, 108 f) and Oaksford and Chater (2007, 109). Zhao, Shah, and Osherson (2009) present empirical evidence suggesting that the Ramsey test offers a way of determining conditional probabilities that is not only accurate but also more accurate than determining conditional probabilities via Kolmogorov’s ratio definition.

4.2. Results

An ANOVA analysis was conducted with question type (probability of the conditional / conditional probability / probability of the conjunction) as between-subjects variable and item as within-subjects variable. The analysis revealed a main effect of question type (F(2,182) = 40.62, p < .01; mean probability of the conditional = 4.56; mean conditional probability = 4.94; mean probability of the conjunction = 3.90). The analysis also revealed a main effect of item (F(9,1638) = 48.17, p < .01) and an interaction between question type and item (F(18,1638) = 4.33, p < .01).

Planned comparisons showed that probability of the conditional and conditional probability differed significantly (F(1,182) = 10.64, p < .01), that probability of the conditional and probability of the conjunction differed significantly (F(1,182) = 32.07, p < .01), and that conditional probability and probability of the conjunction differed significantly (F(1,182) = 79.56, p < .01).

To exclude the possibility that our findings were confounded by “spurious materials effects” (Evans, Neilens, Handley, & Over, 2008, 108), we examined the key ordinal differences within each set of materials, in the manner of Evans, Neilens, Handley, and Over (2008) and Corner, Hahn, and Oaksford (2011, Experiment 1). We found that in eight of our 10 sets of materials, mean conditional probability exceeded mean probability of conditional (p = .055, binomial test, one tailed), in 9 of the 10 sets, mean conditional probability exceeded mean probability of conjunction (p = .01, binomial test, one tailed), and also in 9 of the 10 sets, mean probability of conditional exceeded mean probability of conjunction.\(^{14}\)

4.3. Discussion

The significant difference between the conditional probability of a conditional, given a certain proposition, and the conditional probability of the consequent of the conditional, given the same proposition in conjunction with the antecedent of the conditional, presents clear evidence against (GEQ). This difference cannot be explained away by assuming that the participants interpreted the conditionals that were presented to them as conjunctions, inasmuch as we also found a significant difference between the conditional probability of a conditional given a certain proposition and the conditional probability of the corresponding conjunction given the same proposition.
5. Experiment 2

It might be objected that we were rash to take the significant difference between the conditional probabilities of conditionals and the corresponding conditional probabilities as evidence against (GEQ). Much of the experimental work on (EQ) only showed a high correlation between probabilities of conditionals and conditional probabilities, which is compatible with there being a significant difference between the two variables. As mentioned previously, given that in many of those earlier studies, some participants tended to interpret conditionals as conjunctions, a close match between probabilities of conditionals and conditional probabilities was, in effect, not to be expected. As also mentioned, however, the occurrence of the conjunctive interpretation responses may have been due to the stimuli used in the studies. No evidence, or very little, was found for that interpretation in Over, Hadjichristidis, Evans, Handley, and Sloman (2007), which asked for the probabilities or real-world conditionals. Moreover, in a control experiment in Douven and Verbrugge (2010, Experiment 3), which used as stimuli fictional vignette stories concerning ordinary matters of daily life, we did find a close match between probabilities of conditionals and conditional probabilities. That is already reason for taking the outcomes of Experiment 1 as militating against (GEQ). However, we have also conducted a control experiment specifically checking whether (EQ) holds for the conditionals that were used in Experiment 1, and not, perhaps, merely the claim that there is a high correlation between the relevant variables. Here, too, we also checked for the possibility of a conjunctive interpretation.15

5.1. Method

5.1.1. Participants

There were 188 participants in the experiment. In return for their cooperation, they received a small amount of money. Again, the participants were recruited and paid via the CrowdFlower interface, which directed them to the Qualtrics platform on which the experiment was run. All participants were from the United Kingdom or the United States. Their native language was English.

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presented on screen. Items were randomized and the participants were randomly assigned to one of the three groups.

We used exactly the same materials as in Experiment 1, except that we omitted from the items the propositions that served as suppositions relative to which the first group of participants in Experiment 1 had to judge the probabilities of the various conditionals. So, for instance, in the present experiment the following item corresponded to the item that was given earlier as an example of a question asking for the conditional probability of a conditional:

<table>
<thead>
<tr>
<th>How probable is the following sentence?</th>
</tr>
</thead>
<tbody>
<tr>
<td>If the British government is unsuccessful in creating more jobs, there will be more riots in the UK like the ones we saw a few months ago.</td>
</tr>
<tr>
<td>Highly improbable 1 2 3 4 5 6 7 highly probable</td>
</tr>
</tbody>
</table>

Similarly, the second group received this question:

<table>
<thead>
<tr>
<th>Suppose that the British government is unsuccessful in creating more jobs. Then how probable is the following sentence?</th>
</tr>
</thead>
<tbody>
<tr>
<td>There will be more riots in the UK like the ones we saw a few months ago.</td>
</tr>
<tr>
<td>Highly improbable 1 2 3 4 5 6 7 highly probable</td>
</tr>
</tbody>
</table>

And the participants in the third group were asked the following question:

<table>
<thead>
<tr>
<th>How probable is the following sentence?</th>
</tr>
</thead>
<tbody>
<tr>
<td>The British government is unsuccessful in creating more jobs and there will be more riots in the UK like the ones we saw a few months ago.</td>
</tr>
<tr>
<td>Highly improbable 1 2 3 4 5 6 7 highly probable</td>
</tr>
</tbody>
</table>

From these examples and the Appendix, it will be clear what the rest of the materials looked like.

5.2. Results

An ANOVA analysis was conducted with question type (probability of the conditional / conditional probability / probability of the conjunction) as between-subjects variable and item as within-subjects variable. The analysis revealed a main effect of question type \( (F(2,185) = 16.22, \ p < .01) \); mean probability of the conditional = 4.41; mean conditional probability = 4.50; mean probability of the conjunction = 3.92). The analysis also revealed a main effect of item \( (F(9,1665) = 53.36, \ p < .01) \) and an interaction between question type and item \( (F(18,1665) = 3.70, \ p < .01) \).

Planned comparisons showed that probability of the conditional and conditional probability did not differ significantly \( (F(1,185) = 0.71, \ p = .40) \), that probability of the
conditional and probability of the conjunction did differ significantly ($F(1,185) = 20.37$, $p < .01$), and that conditional probability and probability of the conjunction differed significantly as well ($F(1,185) = 28.14$, $p < .01$).

Here too, we compared means for each of the sets of materials separately. We found that in exactly half of the 10 sets, mean conditional probability exceeded mean probability of conditional ($p = .62$, binomial test, one tailed), in nine of the sets, mean conditional probability exceeded mean probability of conjunction ($p = .01$, binomial test, one tailed), and in seven of the sets, mean probability of conditional exceeded mean probability of conjunction ($p = .17$, binomial test, one tailed).

5.3. Discussion

The close match between probabilities of conditionals and conditional probabilities found in this experiment is clear evidence in favor of (EQ) and reconfirms the earlier results from Douven and Verbrugge (2010). Again, there was no indication that participants had interpreted conditionals as conjunctions. The simplest explanation for these results, in conjunction with the results from Experiment 1, is that people’s probability judgments accord with (EQ), but not with the logically stronger (GEQ).

6. General discussion

From a classical semantical perspective, the experimental results on (EQ) seemed perplexing. It appeared that people assign probabilities to conditionals in accordance with a thesis that—the triviality arguments seem to show—implies people’s probability functions to have all sorts of absurd features that these functions clearly do not have. As was argued in Section 3, however, it is not (EQ) by itself that gives rise to triviality, even supposing a classical truth-conditional semantics of conditionals. Rather, it is (EQ) in conjunction with the thesis that the interpretation of the conditional operator is independent of people’s belief states. We further saw that, together with the assumption that conditionalization is at least one rational way of revising one’s beliefs, the aforementioned theses entail (GEQ). It was an open question whether people’s probability judgments also conformed to (GEQ). Experiment 1 answers that question in the negative. What may we conclude from this specifically about (EQ)? Can the classical semanticist still adhere to that thesis, the triviality arguments notwithstanding?

Here, we may seem to be facing a case of what philosophers of science have dubbed “underdetermination” (cf. Douven, 2008). Jointly, three premises have a consequence that has been shown to be false, but how does that allow us to identify the bad apple (or apples) among the premises? While there is no general recipe for how to proceed in this kind of case—as Duhem, Quine, and many others have argued—in the case at hand we do seem able to differentiate between the premises involved in the derivation of the false consequence.

The crucial point to notice is that these premises were not on an equal footing to begin with. (EQ) has been tested in isolation over and over again during the past 10 years or
so, and all results obtained so far have pointed in its favor. It was further said that the premise that conditionalization is a rational update procedure is plausible on independent grounds. Can we say anything similarly positive about the remaining premise, the fixedness of the conditional?

As previously mentioned, Lewis dismissed the suggestion that this premise might fail to hold. He reasoned that if the semantics of conditionals really depended on belief states—as van Fraassen suggested—that would preclude the possibility that people can reasonably disagree about conditionals. But, even though people are rarely (if ever) in exactly the same belief state, they can have apparently reasonable disputes about particular conditionals. Were van Fraassen right, people engaged in such disputes would most likely just be talking past one another, given that they would be interpreting the conditional operator in different ways.

However, as Douven and Dietz (2011) note, Lewis’s dismissal of van Fraassen’s suggestion may well have been too quick. To give the example used by Douven and Dietz, people may meaningfully discuss the truth of

- There was a lot of snow in Amsterdam last winter.

even if they disagree about what precisely counts as a lot, or how exactly Amsterdam is to be delineated from the rest of the world, or when winter begins and when it ends, even if, that is, they interpret the above sentence in somewhat different ways. As long as they roughly agree on these issues, they may still genuinely disagree about whether there was a lot of snow in Amsterdam last winter. Disagreement does not require that people have exactly the same proposition in mind. The same may be true regarding conditionals: If the interpretation of the conditional may vary with belief states, the variation in interpretation may only be slight. Such slight variation may still be enough to explain why the data go against (GEQ) even if they support (EQ).¹⁶

For readers who believe that there is something intrinsically absurd about the idea that the interpretation of a conditional might be relative to persons’ belief states, we note that of late a number of philosophers have been propagating semantics that make the interpretation of sentences generally relative to belief states; see MacFarlane (2012) and references given therein. These philosophers vehemently deny that it follows from their views that genuine communication is impossible. In this connection, it is also worth observing that theorists who hold that similarity relations between possible worlds are involved in the truth conditions of conditionals are committed to a version of relative semantics for conditionals, given that similarity judgments are known to be sensitive to background beliefs. The same is true for adherents of Stalnaker and Jeffrey’s semantics mentioned in note 4, given that the conditional expectation of a conditional’s consequent given its antecedent depends on a person’s probability function; after all, that provides the weights for determining the expectation.¹⁷

In the philosophical literature on conditionals, van Fraassen’s tenability result has received marginal attention at most. And we are not aware of any references to this result in the psychological literature. The abundant empirical support for (EQ) might already have been reason to pay more attention to van Fraassen’s result, as it entails that the data
apparently in support of (EQ) need not be explained away, nor accepted at the expense of the view that conditionals express propositions, but can be taken at face value, as showing that people do assign probabilities to conditionals in accordance with (EQ), where these conditionals have classical truth conditions. The results reported in this article take an important step further in supporting van Fraassen’s idea that the interpretation of the conditional may be belief sensitive. After all, from the conjunction of (a) the negation of van Fraassen’s idea, (b) (EQ), a thesis for which considerable support has accrued over the years, and (c) a fairly uncontentious assumption about the class of probability functions to which (EQ) is meant to pertain (closure under conditionalization), a thesis was seen to follow—(GEQ)—that our data show to be empirically inadequate. In our view, this makes the correctness of van Fraassen’s idea the best explanation of the data.

Finally, it is worth mentioning that the empirical results reported in this article should be of interest independently of the debate about the semantics of conditionals or the status of the triviality arguments. People are able not only to evaluate the unconditional probabilities of conditionals but also the probabilities of conditionals, conditional on a given proposition. However, (EQ) does not make any predictions regarding these conditional probabilities of conditionals, nor can we conclude anything with respect to them from the empirical work done on that thesis. If nothing else, this article reports the first experiments on the conditional probabilities of conditionals and it shows that they are not correctly predicted by the most straightforward generalization of (EQ), to wit, (GEQ). That leaves wide open the question of which principle (if any) describes those probabilities, which we take to present an important avenue for future research for all working within the Bayesian approach to conditionals.

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Appendix

This appendix contains the nine items that, together with the item given as an example in Section 4, constitute the materials used in Experiment 1. We present only the items that asked for the probability of a conditional, given a specific proposition. With these correspond items that asked for the probability of the given conditional’s consequent conditional on its antecedent and the said further proposition, as well as items that asked for the probability of the conjunction of the antecedent and the consequent conditional on the same further proposition, as explained in Section 4. The items
without the first sentences also served as materials for Experiment 2, in the way explained in Section 5.

Suppose that oil prices will keep going up for some time to come. Then how likely is the following sentence? If the frequency of traffic jams increases over the next years, then more people will start to consider travelling by public transport.

Suppose that the economic crisis in Europe gets worse. Then how likely is the following sentence? If European governments take further austerity measures, the unemployment in Europe will reach unprecedented levels in the following years.

Suppose that there is an increase in emigration from Arab countries to the US. Then how likely is the following sentence? If there will be another terrorist attack in the United States, this will lead to strong tensions between Arab immigrants and the rest of the American population.

Suppose that primary schools start to give more attention to musical education. Then how likely is the following sentence? If musical instruments become cheaper, more children will want to learn to play an instrument.

Suppose that Harvard University receives fewer and smaller gifts from alumni. Then how likely is the following sentence? If Harvard University does not raise tuition fees, they will have to start paying their professors lower salaries.

Suppose that the emission of greenhouse gases from India will double over the next 20 years. Then how likely is the following sentence? If Europe and the United States do not succeed in reducing their emission of greenhouse gases, sea levels will rise dramatically.

Suppose that the prices of tobacco products go down. Then how likely is the following sentence? If more people become aware of the risks of tobacco use, smokers will come to be a minority relatively soon.

Suppose that baseball will get less popular in the United States over the next 5 years. Then how likely is the following sentence? If the US national team wins the 2014 World Championship in Football, then soccer will become just as popular as baseball in the United States.

Suppose that American universities are going to raise their tuition fees. Then how likely is the following sentence? If the next president of the United States—whoever that will be—is going to raise taxes, a college education will become unaffordable for many Americans.

Notes

1. This thesis also goes by the names “Stalnaker’s hypothesis” (e.g., Douven & Verbrugge, 2010) and “the conditional probability hypothesis” (e.g., Gilio & Over, 2012).
2. Unless specified otherwise, we refer to indicative conditionals as “conditionals” from here on.

3. Most advocates of TTV take the third value to stand for something like “indeterminate” or “void,” following suggestions made by Ramsey (1926/1990, 1929/1990) and de Finetti (1937/1964); see on this Politzer, Over, and Baratgin (2010) and Baratgin, Over, and Politzer (unpublished data).

4. Stalnaker and Jeffrey (1994) in effect develop an infinitely valued version of the non-propositional view. In their account, a conditional has the semantic value of its consequent if its antecedent is true and has as its semantic value the mathematical expectation of its consequent conditional on the antecedent if the antecedent is false (which can be any value in the interval [0,1]).

5. Evans and Over (2004), Pfeifer and Kleiter (2010), and Politzer, Over, and Baratgin (2010) all advocate TTV. In support of their view, these authors adduce evidence to the effect that people tend to regard possibilities in which a conditional’s antecedent is false as being irrelevant to the evaluation of the conditional as a whole.

6. Although see Égré and Cozic (2011) for a recent discussion of the Lewis–Kratzer view from a philosophical perspective.

7. See, for example, Dietz and Douven (2010). Proponents of the NTV–TTV views appeal to the so-called Import–Export principle (cf. note 8) to account for right-nested conditionals; see McGee (1985, 1989) and Adams (1998), Appendix 4.

8. However, many of these arguments do rely on (GEQ), or even on the Import–Export principle (e.g., the triviality arguments offered in Blackburn, 1986, Jeffrey, 2004, 15 f, and Douven, 2011, 394), according to which “If $\varphi$, then if $\psi$, then $\chi$” is logically equivalent to “If $\varphi$ and $\psi$, then $\chi$,” which in the face of (EQ) is an even stronger assumption than (GEQ). After all, from Import–Export and the fact that probability theory respects logic, it follows that

$$\Pr(\text{If } \varphi, \text{then if } \psi, \chi) = \Pr(\text{If } \varphi \text{ and } \psi, \text{then } \chi).$$

Applying (EQ) to both sides of the above equation yields

$$\Pr(\text{If } \psi, \chi|\varphi) = \Pr(\chi|\varphi \land \psi),$$

which is (GEQ).

9. That is to say, they are committed to this assumption supposing—as van Fraassen does—that a person’s belief states are represented by probability assignments to all models of her language. Some have contested this supposition, but see Douven, in press, for a defense.

10. See Evans and Over (2004, 136 ff) and Oberauer, Geiger, Fischer, and Weidenfeld (2007). Barrouillet and Lecas (1999) and Lecas and Barrouillet (1999) found that the conjunctive interpretation is the predominant one in young children. See also Evans, Handley, Neilens, and Over (2007) and Evans, Handley, Neilens, Bacon, and Over (2010).

11. We thank David Over for drawing our attention to this possibility.
12. See the Appendix for the rest of the materials.
13. The numbers 2–6 were also labeled, in the obvious way (2 = Improbable, 3 = Somewhat improbable, etc.).
14. We thank Mike Oaksford for suggesting that we perform this additional analysis.
15. We thank Jonathan Evans for suggesting that we conduct this control experiment.
16. To make precise the notion of one proposition being roughly the same as another, one may draw on the formal machinery developed in the context of the debate on truth approximation in the philosophy of science. Various authors have offered detailed accounts of what it is for one theory to be close to another theory, where these theories are propositions in the standard sense, that is, sets of models or possible worlds. See Oddie (2007) for an overview.
17. We owe the latter observation to David Over.
18. We owe the main point of this paragraph to David Over.

References


