Outage Probability and SER of Cooperative Selection Diversity in Nonregenerative MIMO Relaying

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Abstract—Cooperative diversity is a promising solution in wireless distributed networks where integrating multiple antennas onto small mobile devices is practically impossible due to size and cost constraints. As such, we consider a cooperative diversity network where the source and the destination user-pair are equipped with single antennas while the relay is a wireless access point equipped with \( N \) antennas. For such networks, we focus on cooperative selection diversity (CSD) to select a single link with the highest instantaneous received signal-to-noise ratio (SNR) between the direct link and the multiple-input multiple-output (MIMO) relay link. We present new closed-form expressions for the exact outage probability and the exact symbol error rate (SER) based on the cumulative distribution function (cdf) of the instantaneous received SNR. Our expressions are valid for arbitrary \( N \) antennas and apply to general operating scenarios with distinct average received SNRs in each link. Furthermore, we present a high SNR analysis of the outage probability and SER to explicitly characterize the diversity order and array gain. We show that the diversity order increases with the number of antennas according to \( N + 1 \).

I. INTRODUCTION

Transmission strategies that offer superior capacity and reliability in wireless distributed networks such as ad-hoc and sensor networks is vital for realizing future generation ubiquitous communications. Employing multiple-input multiple-output (MIMO) techniques can boost the transmission capacity and the link reliability without incurring power or bandwidth penalties [1–3]. However, integrating multiple antennas onto small mobile devices would face the practical limitation of size and cost constraints. Against this background, cooperative diversity [4–6] has emerged as a prominent strategy which allows distributed terminals to create a virtual antenna array. In this paper, we consider a cooperative diversity network where the source and the destination user-pair are equipped with single antennas while the wireless access point operates as a MIMO relay.

MIMO relaying has been considered from an information theoretic perspective (e.g., see [7–9] and the citations therein). While these works provide insights into the ergodic capacity of MIMO relaying, other metrics such as outage probability and symbol error rate (SER) are also important in network design and optimization. Existing studies on the outage and error performance of multiple antenna cooperative networks [10–12] have focused on the cell extension scenario involving multiple antennas at the source and/or destination with only a single antenna at the relay. Among them, [10] considered multiple antennas at the source with maximal-ratio transmission (MRT) and the outage probability was analyzed. In [11], the exact SER was derived when multiple antennas were used at both the source and the destination. In both [10] and [11], the direct link from the source to the destination was not included. In [12], the direct link was included with maximal-ratio combining (MRC) and the authors considered the impact of multiple antennas at the destination on the SER. To assist the tractability in [12], a sufficiently large signal-to-noise ratio (SNR) in the source-to-relay link was assumed.

In this work, we propose cooperative selection diversity (CSD) in MIMO relaying as a practical cost-conscious strategy for wireless ad-hoc networks. We focus on nonregenerative MIMO relaying where the relay does not need to decode the source signal. When considering both the direct and the relay links, CSD simply selects the link with the strongest instantaneous received SNR. In such networks, we derive new exact closed-form expressions for the outage probability and SER which are valid for arbitrary \( N \) antennas at the relay. Our analysis applies to general operating scenarios with distinct average received SNRs in each link. To attain further insights, we present a high SNR analysis of the network which explicitly characterizes two important design parameters of the array gain and the diversity order. Specifically, our observations are summarized as follows: First, we prove that the diversity order increases with the number of antennas according to \( N + 1 \). Second, under the same diversity order, we show that the array gain increases with the per-hop average received SNRs.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a cooperative relaying network employing \( N \) antennas at the relay, and single antennas at the source and the destination user-pair. We adopt a time multiplexing approach in which the communication between the source and the destination occurs over two time slots: In the first time slot, the source transmits to the relay and the destination. In the second time slot, the source remains silent.
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Fig. 1. Illustration of cooperative selection diversity in MIMO relaying where the source and the destination are communicating either directly or indirectly via a nonregenerative relay with $N$ transmit and receive antennas.

and the relay forwards a scaled version of its received signal to the destination.

Let $\lambda_S$ be a scalar symbol transmitted from the source with zero mean and unit variance. In the first time slot, the received signal from the source to the destination is

$$y_{SD} = \sqrt{\mathbb{E}_S} h_0 \lambda_S + \eta_D, \tag{1}$$

where $\mathbb{E}_S$ is the average transmit energy available at the source, $h_0$ is the channel coefficient of the direct link, and $\eta_D$ is the additive white Gaussian noise (AWGN) term satisfying $\mathbb{E}[\eta_D^2] = N_0$, with $\mathbb{E}[\cdot]$ denoting the expectation. From (1), the instantaneous received SNR of the direct link is

$$\gamma_0 = \frac{\mathbb{E}_S}{N_0} |h_0|^2. \tag{2}$$

Given that the relay is equipped with $N$ antennas, the received signal vector from the source to the relay can be written as

$$y_{SR} = \sqrt{\mathbb{E}_S} \mathbf{h}_1 \lambda_S + \mathbf{\eta}_R, \tag{3}$$

where $\mathbf{h}_1$ is the $N \times 1$ channel vector of the first hop and $\mathbf{\eta}_R$ is the $N \times 1$ AWGN vector satisfying $\mathbb{E}[\mathbf{\eta}_R^H \mathbf{\eta}_R] = \mathbf{I}_N N_0$, with $\mathbf{I}_N$ as the $N \times N$ identity matrix and $(\cdot)^H$ denoting the conjugate transpose operation.

In the second time slot, the relay coherently combines $y_{SR}$ as per the rules of MRC to obtain the scalar symbol of

$$\lambda_R = \sqrt{\mathbb{E}_S} \mathbf{h}_1^H \mathbf{h}_1 \lambda + \mathbf{h}_1^H \mathbf{\eta}_R. \tag{4}$$

A scaling gain $\alpha$ is then applied to (4) and the resulting signal is forwarded to the destination with MRT [3]. As such, the received signal from the relay to the destination is given by

$$y_{RD} = \sqrt{\mathbb{E}_R} \mathbf{w}_2 \mathbf{h}_2 \alpha \left( \sqrt{\mathbb{E}_S} \mathbf{h}_1 \lambda + \mathbf{\eta}_R \right) + \eta_D, \tag{5}$$

where $\mathbb{E}_R$ is the average transmit energy available at the relay, $\mathbf{h}_2$ is the $N \times 1$ channel vector of the second hop and $\mathbf{w}_2 = \mathbf{h}_2^H/(\mathbf{h}_2^H \mathbf{h}_2)$ is the $N \times 1$ transmit weight vector, with $(\cdot)^T$ denoting the transpose operation.

From (5), we can express the instantaneous end-to-end SNR of the relay link as

$$\gamma_R = \frac{\mathbb{E}_R}{N_0} \frac{1}{\alpha} |\mathbf{h}_1||F| + \frac{\mathbb{E}_R}{N_0} |\mathbf{h}_2||F|^2, \tag{6}$$

in which the relay scaling gain, $\alpha$, is set as

$$\alpha = \frac{1}{\sqrt{\mathbb{E}_S} |\mathbf{h}_1||F|^2}, \tag{7}$$

such that the relay transmits within its available power constraint. We use $|\cdot|$ $\in F$ to denote the Frobenius norm i.e., $|\mathbf{h}_1||F| = \sqrt{\mathbf{h}_1^H \mathbf{h}_1}$ and $|\mathbf{h}_2||F| = \sqrt{\mathbf{h}_2^H \mathbf{h}_2}$.

The destination receiver selects the strongest link between the direct and the MIMO relay links. As such, the instantaneous received SNR at the destination is given by

$$\gamma_{CSD} = \max(\gamma_0, \gamma_R), \tag{8}$$

where $\gamma_0$ and $\gamma_R$ are given in (2) and (6), respectively.

III. EXACT OUTAGE PROBABILITY

The outage probability, $P_{out}$, is defined as the probability that the SNR measured at the destination is lower than a specified threshold value, $\gamma_{th}$. Mathematically, $P_{out}$ can be written as

$$P_{out} = \text{Pr}[\gamma_{CSD} \leq \gamma_{th}] = F_{\gamma_{CSD}}(\gamma_{th}), \tag{9}$$

where $F_{\gamma_{CSD}}(\cdot)$ is the cdf of $\gamma_{CSD}$ defined in (8).

As such, we now proceed to evaluate the exact closed-form expression for $F_{\gamma_{CSD}}(\gamma_{th})$ in (9). In doing so, we first present a new result for the cdf of $\gamma_R$ defined in (6), as given in the following lemma.

**Lemma 1:** We consider Rayleigh fading in each hop of the MIMO relay link, and hence the per-hop instantaneous received SNRs follow a gamma distribution. As such, we derive the cdf of $\gamma_R$ based on the statistics of the per-hop instantaneous received SNRs as

$$F_{\gamma_R}(\gamma) = 1 - 2e^{-\gamma/(\Gamma(N+1) + \frac{1}{\gamma})} \sum_{n=0}^{N-1} \sum_{k=0}^{N-n-1} \left(\begin{array}{c} N+n-1 \\ k \end{array}\right) \frac{\gamma^{n+N}}{\pi_{12}} K_{N-k} \left(\frac{2 \gamma}{\sqrt{\gamma_{12}}}\right), \tag{10}$$

where $\Gamma(\cdot)$ is the Gamma function [13, eq. 6.1.1] and $K_{\nu}(\cdot)$ is the $\nu$-th order modified Bessel function of the second kind [13, eq. 9.6.2]. In (10), we denote $\pi_{12} = \mathbb{E}_N \left[ |\mathbf{h}_n|^2 \right]$, $\forall \ n \in \{1, \ldots, N\}$, as the average received SNR at the relay per receive antenna. In the second hop, the available energy at the relay per transmit antenna is $\pi_{12}$. As such, we denote $\pi_{21} = \mathbb{E}_N \left[ |\mathbf{h}_{2n}|^2 \right]$, $\forall \ n \in \{1, \ldots, N\}$ as the average received SNR at the destination per transmit antenna.

**Proof:** See Appendix A.

We also consider Rayleigh fading between the source and the destination. As such, the instantaneous received SNR, $\gamma_0$, follows an exponential distribution with cdf given by

$$F_{\gamma_0}(\gamma) = 1 - e^{-\frac{\gamma}{\gamma_0}}, \tag{11}$$

where $\gamma_0 = \frac{\mathbb{E}_S}{N_0} \mathbb{E}_N \left[ |h_0|^2 \right]$ is the average received SNR of the direct link.
Since the fading is independent in all the links, the cdf of $\gamma_{\text{CSD}}$ can be expressed as

$$F_{\gamma_{\text{CSD}}} (\gamma) = F_{\gamma_0} (\gamma) F_{\gamma_R} (\gamma),$$  \hspace{1cm} (12)

where $F_{\gamma_0} (\gamma)$ and $F_{\gamma_R} (\gamma)$ are the cdfs of $\gamma_0$ and $\gamma_R$, respectively.

This results in a new closed-form expression for the outage probability, found by substituting (10) and (11) into (12) together with (9), which yields

$$P_{\text{out}} = 1 - e^{-\gamma_{\text{th}}/\gamma_0} - \frac{2e^{-\gamma_{\text{th}}(\frac{1}{1+\beta})}}{\Gamma(N)\pi_{1/2}^N} \sum_{n=0}^{N-1} \sum_{k=0}^{N-n-1} \left( \frac{N+n-1}{1+n} \right) \frac{\gamma_{\text{th}}^{n+N}}{\pi_1^n n!} \times \left( \frac{\gamma_{\text{th}}}{\pi_1} \right)^{k} K_{N-k} \left( \frac{2\gamma_{\text{th}}}{\sqrt{\pi_1} \tau_2} \right)$$

$$+ \frac{2e^{-\gamma_{\text{th}}(\frac{1}{1+\beta})}}{\Gamma(N)\pi_{1/2}^N} \sum_{n=0}^{N-1} \sum_{k=0}^{N-n-1} \left( \frac{N+n-1}{1+n} \right) \frac{\gamma_{\text{th}}^{n+N}}{\pi_1^n n!} \times \left( \frac{\gamma_{\text{th}}}{\pi_1} \right)^{k} K_{N-k} \left( \frac{2\gamma_{\text{th}}}{\sqrt{\pi_1} \tau_2} \right).$$  \hspace{1cm} (13)

Our result in (13) is valid for arbitrary $N$ number of antennas at the relay. It also includes the general case of unbalanced hops where the relay link may not necessarily exhibit the same average received SNRs in each hop (i.e., $\pi_1 \neq \gamma_0$). We note that (13) permits a quick and straightforward evaluation of the outage probability since it consists of simple summations of standard functions.

### IV. Exact Symbol Error Rate

In this section, we derive a closed-form expression for the SER based on our cdf of $\gamma_{\text{CSD}}$ which is valid for arbitrary $N$ antennas at the relay and unbalanced hops. The exact SER is derived by utilizing the following cdf-based approach given by [14]

$$P_s = \frac{1}{\sqrt{2\pi}} \int_0^\infty F_{\gamma_{\text{CSD}}} (t^2/\beta) e^{-t^2/2} dt.$$  \hspace{1cm} (14)

The expression in (14) applies to all general modulations that have an average SER of the form

$$P_s = E \left[ Q \left( \sqrt{\beta \gamma} \right) \right],$$  \hspace{1cm} (15)

where $Q(x) = \frac{1}{\sqrt{\pi}} \int_x^\infty e^{-y^2/2} dy$ is the Gaussian Q-function and $\beta$ is a modulation specific constant. This includes important modulation formats such as binary phase-shift keying (BPSK) when $\beta = 2$, quadrature phase-shift keying (QPSK) when $\beta = 1$, and binary frequency shift keying (BFSK) with minimum correlation when $\beta = 1 + \frac{2}{\pi \gamma}$. [15].

We proceed by substituting (12) into (14) which yields

$$P_s = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-t^2/2} - e^{-t^2/2} \left( \frac{t}{t^2/2} + \frac{\pi}{t^2/2} \right) dt$$

$$- \sum_{n=0}^{N-1} \sum_{k=0}^{N-n-1} \frac{2^k (N+n-1)!}{\Gamma(N)\pi_{1/2}^N} \left( \frac{\gamma_{\text{th}}}{\pi_1} \right)^{N-k}$$

$$\times \int_0^\infty \frac{t^{2n+2N} e^{-t^2/2} \left( \frac{t}{t^2/2} + \frac{\pi}{t^2/2} \right) K_{N-k} \left( \frac{2t^2}{\beta \sqrt{\pi_1} \tau_2} \right)}{dt} dt$$

$$+ \sum_{n=0}^{N-1} \sum_{k=0}^{N-n-1} \frac{2^k (N+n-1)!}{\Gamma(N)\pi_{1/2}^N} \left( \frac{\gamma_{\text{th}}}{\pi_1} \right)^{N-k}$$

$$\times \int_0^\infty \frac{t^{2n+2N} e^{-t^2/2} \left( \frac{t}{t^2/2} + \frac{\pi}{t^2/2} \right) K_{N-k} \left( \frac{2t^2}{\beta \sqrt{\pi_1} \tau_2} \right)}{dt} dt,$$  \hspace{1cm} (16)

where $I_1$ is easily solved as

$$I_1 = \sqrt{\frac{\pi}{2}} - \sqrt{\frac{\pi}{2}} \left( 1 + \frac{2}{\beta \gamma_0} \right)^{-\frac{1}{2}}.$$  \hspace{1cm} (17)

Next, we solve $I_2$ and $I_3$ by performing a change of variables to apply the identity [16, eq. 2.16.6.3]

$$\int_0^\infty x^{\mu-1} e^{-px} K_v (cx) dx = \frac{\Gamma(\mu - \nu) \Gamma(\mu + \nu)}{2^{2\mu}} \times 2F_1 \left( \frac{\mu - \nu}{2}, \frac{\mu + \nu + 1}{2}; \mu + 1; 1 - \frac{t^2}{\beta^2} \right),$$  \hspace{1cm} (18)

where $2F_1 (a; b; c; z)$ is the Gauss hypergeometric function [13, eq. 15.1.1].

The result is a new closed-form expression for the exact SER given by (19). Our analytical expression can be utilized to easily evaluate the exact SER since it contains finite summations of standard functions that are readily available in numerical software such as Matlab and Mathematica.

### V. High SNR Analysis of Outage Probability and SER

In this section, we present a high SNR analysis of the outage probability and SER to investigate the diversity order and array gain of the network. We adopt a similar approach to [17] which states that if the outage probability can be approximated by a single polynomial term for $\gamma \rightarrow \infty$ as

$$P_{\text{out}} \approx \frac{a}{t+1} \left( \frac{\gamma_{\text{th}}}{\gamma} \right)^{t+1} + o(\gamma^{-(t+1)}),$$  \hspace{1cm} (20)

then at high SNRs the average SER is given by

$$P_s = \frac{2^\alpha \Gamma (t + \frac{2}{\pi \gamma})}{\sqrt{\pi} (t + 1)} (\beta \gamma)^{-(t+1)} + o(\gamma^{-(t+1)}).$$  \hspace{1cm} (21)
For the special case where \( t \) is a positive integer, we can further simplify (21) as

\[
P_s^\infty = \frac{a(2t+1)!}{2^{t+1}(t+1)!} (\beta_\gamma)^{-t+1} + o(\gamma^{-t+1}),
\]

by expanding the Gamma function according to its explicit formula for half-integer values: \( \Gamma(n + 1/2) = \sqrt{\pi}(2n)!/2^n n! \), where \( n! \) denotes the double factorial.

We present a new result for the asymptotic outage probability in the following proposition, which is derived from our exact outage probability result in (13).

**Proposition 1:** For sufficiently high SNRs, the outage probability is given by

\[
P_{out}^\infty = \left( \frac{1}{N_\gamma_1} + \frac{1}{N_\gamma_2} \right)^{N+1} + o(\gamma_0^{-N+1}).
\]

In (23), we define \( \kappa_1 = \gamma_1/\gamma_0 \) and \( \kappa_2 = N\gamma_2/\gamma_0 \) as the ratios of the total per-hop average received SNRs relative to the average receive SNR of the direct link.

**Proof:** See Appendix B.

Next, we consider the asymptotic error performance by comparing our outage probability result in (23) with the identity in (20). Substituting the corresponding values of \( a \) and \( t \) from (23) into (22) results in a high SNR approximation of the SER given in (24) at the top of the following page.

Our result in (24) allows us to easily find two important performance parameters of the array gain and the diversity order. According to [17], we re-express (24) as

\[
P_c^\infty = (S_a\gamma_0)^{-S_d},
\]

and find that the SER diversity order is

\[
S_d = N + 1,
\]

and the SER array gain is

\[
S_a = \beta \left[ \frac{(2N+1)!}{2^{N+1}(N!)^2} \left( \frac{1}{\kappa_1 N} + \frac{1}{\kappa_2 N} \right)^N \right]^{\kappa_0\gamma_0^{-\kappa_0}}
\]

From (25), it is clear that the diversity order, \( S_d \), increases with increasing number of antennas at the relay. In addition, (26) indicates that for a given number of antennas \( N \), the array gain, \( S_a \), increases with increasing per-hop average received SNRs represented by \( \kappa_1 \) and \( \kappa_2 \).

**VI. NUMERICAL RESULTS**

We now present numerical plots to demonstrate the impacts of the number of antennas and the average received SNRs on the network performance. Due to space limitations, we present results for the SER. However, extensive simulations have also been carried out to verify the accuracy of our exact and high SNR outage probability results in (13) and (23), respectively. In each of the figures, the solid curves represent the exact analytical SER from (19) and the dotted curves represent the high SNR analytical SER from (24). We use the symbol ‘◦’ to mark points generated by Monte Carlo simulations.

Fig. 2 plots the SER with BPSK modulation and balanced hops, i.e., \( \gamma_1 = \gamma_2 = \gamma_0 \). We define \( \gamma_2 = N\gamma_2 \) as the total average received SNR in the second hop. The balanced hops condition describes a wireless network where the relay is equidistant from the source and the destination. We see in the figure that the simulation points validate the analytical curves in all cases. Furthermore, we see that our high SNR curves accurately describe the asymptotic behavior of the exact SER. For example, we see in the figure that the diversity order is correctly predicted to be \( S_d = 2 \) when \( N = 1 \). Clearly, the diversity order increases with increasing \( N \) according to (25).

Fig. 3 plots the SER with QPSK modulation and unbalanced hops. Here, the unbalanced hops condition describes a wireless network where the relay is not equidistant from the source and the destination. More specifically, we consider the following two cases based on the values of \( \gamma_1 \) and \( \gamma_2 \) relative to \( \gamma_0 \):

- **Case 1:** Relay close to the source where we set \( \gamma_1 = 2\gamma_0 \) and \( \gamma_2 = \gamma_0 \) with \( \gamma_2 = N\gamma_2 \) (i.e., \( \kappa_1 = 2 \) and \( \kappa_2 = 1 \)).
- **Case 2:** Relay close to the destination where we set \( \gamma_1 = \gamma_0 \) and \( \gamma_2 = 2\gamma_0 \) (i.e., \( \kappa_1 = 1 \) and \( \kappa_2 = 2 \)).

Fig. 3 compares the SER of Case 1 and Case 2 for \( N = 2, 4 \), and 6. We see that for the same number of antennas, Case 1 and Case 2 exhibit the same diversity order of \( N + 1 \), indicated by the parallel high SNR curves. In addition, we see that Case 2 attains an SNR advantage over Case 1 which...
\[ P_s^\infty = \frac{(2N + 1)!}{2^{N+1}(N!)^2} \left( \frac{1}{\kappa_1} + \frac{1}{(\kappa_2/N)^N} \right) \left( \beta^{\frac{1}{\alpha}} \right)^{(N+1)} + o(\beta^{\frac{1}{\alpha}}). \] (24)

Fig. 2. Exact and asymptotic SER versus \( \gamma_0 \) with BPSK modulation for the case of balanced hops where we set \( \tau_1 = \Gamma_2 = \gamma_0 \) with \( \Gamma_2 = N\gamma_2 \).

is characterized by the difference in their array gains. This SNR advantage can be easily evaluated using our result in (26). Fig. 3 demonstrates that when the relay is equipped with multiple antennas, a lower SER is achieved when the relay is placed closer to the destination compared to the source.

VII. CONCLUSION

We derived new exact closed-form expressions for the outage probability and SER of cooperative selection diversity in nonregenerative MIMO relaying. Our analytical results are valid for arbitrary number of antennas at the relay and arbitrary average received SNRs in each link. Furthermore, we presented a high SNR analysis to explicitly characterize two important design parameters of the diversity order and the array gain. Based on these, we made the following two observations: First, the diversity order increases with the number of antennas according to \( N + 1 \). Second, under the same diversity order, the array gain increases with the per-hop average received SNRs. Finally, we applied our results to identify the impact of the relay location on the SER. We showed that a lower SER is achieved by placing the MIMO relay closer to the destination relative to the source.

APPENDIX A

By substituting (7) into (6), the instantaneous end-to-end SNR of the relay link can be rewritten as

\[ \gamma_R = \frac{XY}{X+Y} \] (27)

where \( X = \frac{e^2}{X_0} ||h_1||^2 \) and \( Y = \frac{e^2}{X_0} ||h_2||^2 \) are the per-hop instantaneous received SNRs of the source-to-relay and relay-to-destination link, respectively. The cdf of \( \gamma_R \) can then be written in terms of the cdf of \( X \) and the pdf of \( Y \) as

\[ F_{\gamma_R}(\gamma) = 1 - \int_0^\infty F_X \left( \gamma + \frac{\gamma^2}{\omega} \right) f_Y(\gamma + \omega)d\omega, \] (28)

where \( F_X(\cdot) = 1 - F_X(\cdot) \).

Since the per-hop signals experience Rayleigh fading, it follows that the per-hop instantaneous received SNRs with multiple antennas at the relay follow a gamma distribution. As such, the cdf of \( X \) and the pdf of \( Y \) is given by

\[ F_X(x) = 1 - e^{-x/\tau_1} \sum_{n=0}^{N-1} \frac{(x/\tau_1)^n}{n!}, \] (29)

and

\[ f_Y(y) = \frac{y^{N-1}e^{-y/\tau_2}}{\Gamma(N)\tau_2^N}, \] (30)
respectively, where $\tau_1 = \frac{\xi_1}{N_0} E \left[ |h_{1n}|^2 \right]$, $\forall n \in \{1, \ldots, N\}$, is the average received SNR at the relay per receive antenna and $\tau_2 = \frac{\xi_2}{N_0} E \left[ |h_{2n}|^2 \right]$, $\forall n \in \{1, \ldots, N\}$ is the average received SNR at the destination per transmit antenna.

Substituting (29) and (30) into (28) and applying the binomial expansion results in

$$
F_{\tau_1}(\gamma) = 1 - \sum_{n=0}^{N-1} \sum_{k=0}^{N+n-1} \frac{\gamma^{n+k}}{n! \Gamma(N)\tau_2^{N-n}} \times \int_0^\infty \omega^{N-k-1} e^{-\left(\frac{\gamma}{\tau_1} + \frac{\omega}{\tau_2}\right)} d\omega.
$$

Finally, to solve the integral in (31), we utilize the identity

$$
\int_0^\infty x^{\nu-1} e^{-\frac{x}{\lambda}} dx = 2 \left(\frac{\beta}{\lambda}\right)^{\frac{\nu}{2}} K_\nu(2\sqrt{\beta\lambda}),
$$

which gives the desired closed-form result in (10).

**APPENDIX B**

We begin by defining $\kappa_1 = \frac{1}{\tau_1}/\tau_0$ and $\kappa_2 = \frac{N\tau_2}{\tau_0}$ as the ratios of the per-hop average received SNRs to the average received SNR in the direct link. As such, our expression for the cdf of $\gamma_{\text{CSD}}$ in (13) can be re-written as

$$
F_{\gamma_{\text{CSD}}} (\gamma) = 1 - e^{-\gamma/\tau_0} \times 
\sum_{n=0}^{N-1} \sum_{k=0}^{N+n-1} \frac{\gamma^{n+k}}{\kappa_1^k n!} \frac{\kappa_2^k}{\kappa_1^k} \frac{1}{(N+n+1)} \frac{1}{(N+n+1)}
\times K_{N-n-k} \left( \frac{2\gamma/\tau_0}{\sqrt{\kappa_1 \kappa_2}} \right) \quad (33)
$$

where $\kappa_1$ and $\kappa_2$ are real-valued positive constants.

The first order Taylor series expansion of (33) as $\tau_0 \to \infty$ is derived as

$$
F_{\gamma_{\text{CSD}}} (\gamma) = \left(\frac{\gamma}{\tau_0} + o(\tau_0^{-1})\right) \times 
\sum_{n=0}^{N} \sum_{k=0}^{n} (-1)^k \gamma^{n+k} \frac{n!}{\kappa_1^k} \frac{1}{\kappa_2^k} \frac{1}{(N+n+1)} \frac{1}{(N+n+1)}
\times K_{N-n-k} \left( \frac{2\gamma/\tau_0}{\sqrt{\kappa_1 \kappa_2}} \right) \quad (34)
$$

where we make use of the generalized power series representations for the exponential function given by [18, eq. 1.211/1]

$$
e^{-x} = \sum_{k=0}^{\infty} \frac{(-x)^k}{k!},
$$

and for $K_\nu(x)$ given by [18, eq. 8.446]

$$
K_\nu(x) = \frac{1}{2} \sum_{i=0}^{\nu-1} \frac{(-1)^i \nu - i - 1}{i!} \frac{(\frac{x}{2})^{2i-\nu}}{\nu!(i+\nu)!} + \frac{1}{2} \sum_{i=0}^{\infty} \frac{(-1)^i \nu - 1}{i!} \frac{(\frac{x}{2})^{2i+\nu}}{\nu!(i+\nu)!}.
$$

Finally, the outage probability for sufficiently high SNRs is obtained by substituting $F_{\gamma_{\text{CSD}}} (\cdot)$ from (34) into (9), thus completing the proof.

**REFERENCES**


