Benefits of Transmit Antenna Selection in Ad Hoc Networks

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Abstract — In this paper, we investigate the benefits of providing limited-feedback and using transmit antenna selection (TAS) in ad hoc networks. We find that the TAS scheme can provide throughput gains of up to 50% compared to the non-feedback scheme. We also find that the performance gains of the TAS scheme are more significant for low path loss exponents and single-antenna transmission. Our results are obtained by deriving new closed-form expressions for the network throughput and performance measures to be obtained. Moreover, we also propose design guidelines for TAS to determine the optimal number of antennas used for transmission.

I. INTRODUCTION

Multiple-antenna technologies are well known to offer significant performance gains in wireless communication systems, providing substantial data rates through the use of spatial multiplexing techniques. These gains can be further enhanced if the transmitter designs its transmission strategy based on the transmitter-receiver channel. Such strategies require the feedback of channel-dependent data from the receiver to the transmitter, typically via a low-rate feedback link. A practical limited-feedback scheme is for the receiver to select a set of transmit antennas to be used for transmission based on the corresponding channel strengths, and then inform the transmitter of this selection by feeding back antenna selection bits. This transmit antenna selection (TAS) approach has been thoroughly studied in the context of point-to-point multiple-antenna systems (see e.g., [1]), however, it has seen far less attention in the context of ad hoc networks. For such networks, TAS presents an extremely attractive transmission option since it provides a low complexity and low feedback method which can achieve performance improvements through selection diversity, whilst minimizing the aggregate interference.

In this paper, our aim is to investigate the performance of TAS in an ad hoc network setting, and to characterize the benefits which it can achieve over non-feedback schemes. We consider spatial multiplexing TAS systems, where the receiver selects a subset of one or more transmit antennas to employ based on the measured channel state information (CSI). The number of transmit antennas is a key design parameter, which we aim to characterize. As in [1], we assume that the receiver employs zero-forcing (ZF) filters to perform detection. The transmitting nodes are spatially distributed according to a homogeneous Poisson point process (PPP) on a 2-D plane. Besides corresponding to realistic network scenarios, modeling the nodes according to a PPP has the benefit of allowing network performance measures to be obtained.

Various prior contributions have considered both non-feedback and feedback schemes in ad hoc networks. For non-feedback schemes, [2] considered diversity-based approaches, namely single-antenna transmission with maximal ratio combining receivers and orthogonal space-time block codes, whereas [3] considered spatial multiplexing schemes with ZF receivers, with the transmit antennas chosen at random. Feedback schemes were also considered in [2, 4], where complex beamforming weights were sent from each receiver to their corresponding transmitting node. These schemes, however, require a relatively high-rate feedback1. They also assume that all transmit antennas are active, which can be a significant limiting factor since it yields a large amount of network interference. TAS, in comparison, yields the minimum network interference with only very low feedback requirements, whilst still providing boosted channel gains through selection diversity.

A primary objective of our work is to establish how much gain can be achieved by TAS by exploiting feedback, and under what network conditions are the maximum gains achieved. We point out that the single-antenna transmission TAS scheme was considered in [2], but it was only briefly discussed and the answer still remains unclear. The key finding of this paper is that limited-feedback can provide significant performance gains, however, these gains depend heavily on the system parameters. We show that feedback is most beneficial for low path loss exponents and single-antenna transmission, but this benefit reduces substantially if the number of receive antennas is large. We establish these results by deriving new closed-form expressions for the network throughput and the transmission capacity. Based on our transmission capacity expression, we also derive a very simple formula which gives design guidelines for determining the number of transmit antennas to employ with TAS to maximize performance.

II. SYSTEM MODEL

We consider an ad hoc network comprising of transmitter-receiver pairs, where each transmitter communicates to its corresponding receiver in a point-to-point manner, treating all other transmissions as interference. The transmitting nodes are distributed spatially according to a homogeneous PPP of intensity $\lambda$ in $\mathbb{R}^2$. Each transmitting node transmits with

1Strictly speaking, they require an infinite number of feedback bits, which in practice would be approximated by sending quantized estimates.
probability $p$ according to a slotted ALOHA medium access protocol, and communicates with its corresponding receiving node located at a distance $r_{tr}$.

We aim to establish network performance measures. To obtain such measures, it is sufficient to focus on a typical transmitter-receiver pair, denoted by index 0, with the typical receiver located at the origin. This is due to the Slivnyak’s theorem of a PPP [5]. We consider a network where each node is equipped with $N$ antennas, and the transmitting nodes, with the exception of the typical transmitter, constitute a marked PPP [5]. This is denoted by $\Phi = \{ (D_{\ell}, H_{\ell 0}), \ell \in \mathbb{N} \}$, where $D_{\ell}$ and $H_{\ell 0} \sim \mathcal{CN}_{N,N} (0_{N \times N}, I_N)$ model the location and channel matrix respectively of the $\ell$th transmitting node with respect to (w.r.t.) the typical receiver. Each transmitting node sends the same transmission power $P$, and the transmitted signals are attenuated by a factor $1/r^\alpha$ with distance $r$ where $\alpha > 2$ is the path loss exponent.

At the physical layer, we consider spatial multiplexing transmission where the $\ell$th transmitting node sends $M \leq N$ data streams separately from $M$ selected transmit antennas, with each receiving node utilizing a ZF receiver to cancel the interference from its corresponding transmitting node. The received $M \times 1$ data estimate vector at the typical receiver is given by

$$x_0 = \frac{1}{r_{tr}^{\alpha}} R_0 H_{00} T_0 x_0 + \sum_{\ell \in \Phi} \frac{1}{|D_{\ell}|^\alpha} R_\ell H_{00} T_\ell x_\ell + R_0 n_0$$

(1)

where $x_\ell$ is the $M \times 1$ symbol vector sent from the $\ell$th transmitting node satisfying $E[x_\ell x_\ell^H] = I_M P_{\ell}$ and $n_0 \sim \mathcal{CN}_{M,N}(0_{N \times 1}, N_0 I_N)$ is the complex additive white Gaussian noise vector. The $N \times M$ precoding and $M \times N$ receiver combining matrix for the $\ell$th transmitting and receiving node are given respectively by

$$T_\ell = S_{\ell,q_\ell} \quad \text{and} \quad R_\ell = (H_{\ell q_\ell} H_{\ell,q_\ell}^H)^{-1} H_{\ell q_\ell}$$

(2)

where $H_{\ell q_\ell} \sim \mathcal{CN}_{N,N} (0_{N \times N}, I_N)$ is the channel matrix corresponding to the $\ell$th transmitting-receiver pair, $H_{\ell q_\ell} = H_{\ell q_\ell} S_{\ell,k_\ell}$ and $S_{\ell,q_\ell}$ is a $N \times M$ transmit antenna selection matrix, formed by choosing $M$ columns from $I_N$ as described in [1]. We see in (2) that the TAS scheme involves each receiving node sending $\log_2 \left( \binom{N}{M} \right)$ bits back to its corresponding transmitting node, representing the selected transmit antennas. Substituting (2) into (1), the received SINR for the $k$th stream at the typical receiver can be shown to be given by (3) at the top of the next page, where $\rho = \frac{P}{N_0}$ and $[X]_{k,l}$ denotes the $(k,l)$th element of $X$.

Although the antennas selected for transmission should ideally be chosen to maximize the SINR in (3), this will require that each receiving node has knowledge of all the interferers’ CSI, which is usually difficult to obtain. We consider the more practical scenario where each receiving node has knowledge of only the channel to its corresponding transmitting node. We thus choose $q_0$ as

$$q_{0,\max} = \arg \max_{q \in Q} \min_{w \in \{1, \ldots, M\}} \frac{1}{\left( H_{00, q_0} H_{00,q_0}^H \right)^{-1}}$$

(4)

where $Q$ is the set of all possible $\binom{N}{M}$ transmit antenna subsets.

To illustrate the benefits of employing feedback for antenna selection, we also consider a spatial multiplexing scheme without feedback. This scheme, which we denote as N-TAS, operates in the same way as the TAS scheme except that the $M$ antennas used for transmission are selected randomly.

III. PERFORMANCE ANALYSIS

In this section, we consider three important network performance measures: outage probability, network throughput and transmission capacity.

A. Outage Probability

The outage probability is defined as the probability that the mutual information of any data stream falls below a spectral efficiency of $R$ bits/sec/Hz, and is given by

$$F(\beta) = \Pr \left( \bigcup_{k=1}^{M} \left\{ \log_2 \left( 1 + \text{SINR}_{0,k,q_{0,\max}} \right) \leq R \right\} \right)$$

$$= \Pr \left( \log_2 \left( 1 + \min_{k=1,\ldots,M} \text{SINR}_{0,k,q_{0,\max}} \right) \leq R \right) \approx \Pr \left( \log_2 \left( 1 + \text{SINR}_{0,w_{\min},q_{0,\max}} \right) \leq R \right)$$

$$= \Pr \left( \text{SINR}_{0,w_{\min},q_{0,\max}} \leq \beta \right)$$

(5)

(6)

where $\beta = 2^R - 1$ is the SINR threshold and $w_{\min} = \min_{w \in \{1, \ldots, M\}} \left( \left( H_{00, q_0, w, q_0, \max} \right)^{-1} \right)$. As we will show in Fig. 1, the approximation in (6) is very accurate. This definition is used for pure spatial multiplexing systems with independent encoding and decoding of data streams [6], and implies that if $F(\beta) = \epsilon$, then the $M$ streams can support a spectral efficiency of $R$ bits/sec/Hz with probability $1-\epsilon$.

The outage probability approximation for the TAS scheme is given by the following lemma:

Lemma 1: The outage probability of the TAS scheme with ZF receivers can be approximated by (7) at the top of the next page, where $\eta(M, \lambda, \alpha, \beta) = - \frac{\rho M \left( M + \beta \right) \Gamma(1-\frac{\beta}{M})}{M} \zeta(1-\frac{\beta}{M})$, $s(\cdot, \cdot)$ is the Stirling number of the first kind [7], $S(\cdot, \cdot)$ is the Stirling number of the second kind [7], $C = \binom{N}{M}$ and $b(i, \alpha)$ is the coefficient of $x^i$ in the expansion of

$$\left( \sum_{k=0}^{N-1} \frac{x^k}{k!} \right)^{\alpha+1} M^{-1}$$

(7)

Proof: We present a brief summary of the proof due to space limitations. To derive the outage probability of the TAS scheme, we are first required to obtain a distribution of the equivalent channel gain for the desired data stream, given by the numerator in (3). This is difficult due to (i) the dependence between the different channel subsets chosen by the selection procedure in (4), and (ii) the correlation between the different received data streams for any given channel subset. To address this problem, we assume that the channel gains for each data stream are mutually independent, which allows us to use
\[ \text{SINR}_{0,k,q_0} = \frac{\rho}{M} \left[ (H_{00,q_0} H_{00,q_0})^{-1} \right]_k \left( H_{00,q_0} H_{00,q_0}^{-1} \right) \left( \sum_{D_j \in \Phi} \frac{H_{D_j,q_j} H_{D_j,q_j}^*}{|D_j|^\alpha} \right) \left( H_{00,q_0} H_{00,q_0}^{-1} \right)_k + 1 \]  

\[ \tilde{F}_{\text{TAS}}(\beta) = 1 - \frac{CM}{(N - M + 1)} \sum_{a=0}^{C-1} \frac{(C - 1)}{\alpha} e^{-(a+1)M} \frac{M!}{\rho \alpha} \sum_{i=0}^{N - M + i} b(i, a) \sum_{q=0}^{N - M + i} b(i, a) (N - M + i) \]

\[ \left( \frac{M \beta^\alpha}{\rho} \right)^{i-q} (-1)^{i+q} \sum_{h=0}^{q} \frac{2^h}{h} s(q + 1, h + 1) \sum_{l=0}^{h} \eta(M, \lambda, \alpha, \beta) ((a + 1)M)^{-(q+1-\frac{2l}{\alpha})} S(h, l) e^{\eta(M, \lambda, \alpha, \beta)((a + 1)M)^{\frac{2l}{\alpha}}} \]

\[ T = RM \rho \lambda (1 - F(\beta)) \]  

results from order statistics [8] to obtain the distribution of the equivalent channel gain. To proceed, we follow a similar method to that described in [3].

For the N-TAS scheme, the outage probability expression can be derived under a similar procedure to [3] and using the same independence assumption in Lemma 1. The accuracy of (7) is confirmed in Fig. 1. The ‘TAS - Analytical’ curves are based on our outage probability approximation, and are shown to closely match the ‘TAS - Monte Carlo’ simulated curves which are based on the exact outage probability definition in (5). Both curves show a substantial decrease in outage probability compared to the N-TAS outage probability curves, which are generated by Monte Carlo simulation based on (5). Note that when \( M = 1 \), the outage probability expression in (7) is exact.

**B. Network Throughput**

The network throughput is defined as the total number of successful transmitted bits/sec/Hz/m², and is given by

\[ T = RM \rho \lambda (1 - F(\beta)) \]  

We approximate the throughput by substituting the outage probability expressions of TAS and N-TAS (not explicitly shown due to space limitations) respectively into (8).

Fig. 2 shows the throughput vs. spectral efficiency for the TAS and N-TAS schemes, and for different numbers of data streams. We see that the TAS performs better than the N-TAS scheme as expected. The throughput gains of the TAS over the N-TAS scheme can be up to 50%, and these gains are more dominant for spectral efficiency regimes corresponding to the maximum throughput, but become negligible for small and large spectral efficiency regimes. This is because for small and large spectral efficiency regimes, the outage probability converges to zero and one respectively, and thus the throughput of both schemes converge as well. We also see that the throughput gains of the TAS scheme over the N-TAS scheme are more dominant when only one antenna is used for transmission, i.e., the difference in throughput between the TAS and N-TAS schemes is larger, compared to when two antennas are used for transmission. Moreover, Fig. 2 shows that for all but very small spectral efficiency regimes, using one transmit antenna is optimal in terms of achieving the highest throughput.
C. Transmission Capacity

Although throughput is an important performance measure, it may be obtained at the expense of unacceptably high outage levels, which is undesirable for some applications. This has motivated the introduction of the transmission capacity [9], defined as the maximum throughput subject to an outage constraint \( \epsilon \), where the maximization is performed over all intensities \( p \lambda \). The transmission capacity is thus given by

\[ c(\epsilon) = R M \lambda(\epsilon)(1 - \epsilon) \tag{9} \]

where \( \lambda(\epsilon) \) is the optimal contention density, defined as the inverse of \( \epsilon = F(\beta; p \lambda) \) taken w.r.t. \( p \lambda \). Note that here we have made explicit the dependence of the outage probability on \( p \lambda \). We focus on deriving an expression for the transmission capacity in the high SNR and large antenna regime, subject to a low outage constraint. This is given in the following lemma:

Lemma 2: In the high SNR regime, the transmission capacity of the TAS scheme with ZF receivers, subject to a low outage constraint, can be approximated for large \( N \) and fixed \( M \) by (10) at the top of the next page\(^3\), with scaling\(^4\)

\[ \tilde{c}_{\text{TAS}} = \Theta \left( M^{1 - \frac{2}{\alpha}}(N - M + 1)^{\frac{2}{\alpha}} \right) \tag{11} \]

where \( \text{erf}^{-1}(\cdot) \) is the inverse error function.

Proof: See the Appendix.

Taking the derivative of (11) w.r.t. \( M \), we see that for large \( N \), the transmission capacity scaling increases with \( M \) if

\[ M < \left( 1 - \frac{2}{\alpha} \right)(N + 1). \tag{12} \]

From this result, we can make some interesting observations. First, we see that for small path loss exponents (e.g., \( \alpha \approx 2 \)), it is always best to select only a single transmit antenna. As \( \alpha \) increases, however, a higher transmission capacity is obtained by selecting multiple transmit antennas. As a rough guideline, (12) implies that for medium path loss exponents (e.g., \( \alpha \approx 4 \)), one should select approximately half of the antennas for transmission, whereas for high path loss exponents (e.g., \( \alpha \approx 6 \)) approximately two-thirds of antennas should be selected. It is also interesting to note that at no point should all \( M = N \) antennas be employed for transmission. Although these results are derived for large \( N \), we see from Fig. 3 that the same trends hold for small \( N \) also. In particular, Fig. 3 plots the transmission capacity vs. path loss exponent for different numbers of data streams \( M \), for \( N = 3 \) antennas. The TAS curves are generated by numerically taking the inverse of \( F(\beta; p \lambda) \) w.r.t. \( p \lambda \), and substituting the results into (9). We see that for path loss exponents \( \alpha \) below 5, the best performance is achieved by transmitting only a single data stream, whereas for \( \alpha \) above 5, selecting the best 2 out of 3 antennas gives better performance, which are in agreement with the guidelines provided by (12). Interestingly, we see that the transmission capacity is a decreasing function of the path loss exponent \( \alpha \) for \( M = 1 \), however, it is an increasing function for \( M = 2 \). Moreover, we also see that there is a substantial performance loss for all \( \alpha \) values if all \( M = 3 \) antennas are used for transmission. Clearly, this case should be avoided in practice.

To compare the spatial multiplexing schemes with and without feedback for large \( N \), we rewrite (10) as

\[ \tilde{c}_{\text{TAS}}(M, N) \sim \frac{R N^{\frac{2}{\alpha}} \Gamma(M + 1) \epsilon}{\pi \Gamma(M + \frac{2}{\alpha}) \beta^{\frac{2}{\alpha}} r_{tr}^2} \left( 1 - \frac{M}{N} + \frac{1}{N} \right) \tag{13} \]

\[ + \left[ 2 \log \left( \frac{N^{\frac{2}{\alpha}}}{\sqrt{\pi M}} \right) + o \left( \frac{\log \left( \frac{N^{\frac{2}{\alpha}}}{\sqrt{\pi M}} \right)}{N} \right) \right] \frac{2}{\alpha} \Gamma(M + 1) \epsilon \]

\[ = \frac{R N^{\frac{2}{\alpha}} \left( 1 + \sqrt{\frac{2}{N}} + o \left( \frac{1}{\sqrt{N}} \right) \right)^{\frac{2}{\alpha}} \Gamma(M + 1) \epsilon}{\pi \Gamma(M + \frac{2}{\alpha}) \beta^{\frac{2}{\alpha}} r_{tr}^2} \]

where the first line is obtained by applying [10, Eq. (2)] in (10) followed by algebraic manipulation, and the last line is obtained using L’Hopital’s rule.

For the N-TAS scheme with no feedback, the transmission capacity when \( M = 1 \) is given by [2]

\[ c_{\text{N-TAS}(1,N)} \sim \frac{R \Gamma(N) \epsilon}{\Gamma \left( N - \frac{2}{\alpha} \right) \pi \Gamma \left( 1 + \frac{2}{\alpha} \right) \beta^{\frac{2}{\alpha}} r_{tr}^2}. \tag{14} \]

Thus, focusing on the case \( M = 1 \), from (13) and (14) we have

\[ \frac{c_{\text{TAS}(1,N)}}{c_{\text{N-TAS}(1,N)}} \sim \left( 1 + \sqrt{\frac{2}{N}} + o \left( \frac{1}{\sqrt{N}} \right) \right)^{\frac{2}{\alpha}} \geq 1 \tag{15} \]

As expected, we see that the transmission capacity of TAS is always greater than that of the N-TAS scheme. However, the benefit is reduced if the number of receive antennas is large, which is explained by noting that for large \( N \) the large receive array gains for both the TAS and N-TAS schemes dominate any additional gains obtained by the transmit antenna selection process. We also observe from (15) that the additional transmission capacity benefit of TAS over the N-TAS scheme is less for higher path loss exponents. This can be explained by first noting that the distributions of the interference component of both the TAS and N-TAS schemes are equal. The key difference lies in the effective channel gain of the desired signal, which is higher for the TAS scheme than for the N-TAS scheme. However, as the path loss exponent is increased, the disparity between the effective channel gains for the TAS scheme and the N-TAS scheme becomes less, and therefore so does the difference in performance. This behavior is confirmed in Fig. 3, where the TAS and N-TAS curves become closer together as the path loss exponent is increased. In this figure, the N-TAS curves are generated based on outage probability expressions (not explicitly shown due to space limitations), which are derived using a similar procedure to that proposed in [3] and using the same independence assumption as in Lemma 1. The figure also confirms that the performance benefit of TAS over the N-TAS approach can be very significant, particularly if single-steam transmission is
\[ \hat{c}_{\text{TAS}}(\epsilon) \sim \frac{R(N - M + 1)^{\frac{1}{2}}}{\pi \Gamma(M + \frac{2}{\alpha}) \beta^2 r^2} \epsilon^2 \]  

where \( V_M \) is the maximum of \( \binom{N}{M} \) independent random variables \( V_1 \), where \( V_1 \) is the minimum of \( M \) chi-squared random variables \( X \) with \( 2(N - M + 1) \) degrees of freedom.

To prove this case, we begin by noting that the random variable \( V_M^{-\frac{1}{2}} \) is equivalent to the minimum of \( \binom{N}{M} \) independent random variables \( V_2 \), where \( V_2 \) is the maximum of \( M \) random variables \( Y = X^{-\frac{1}{2}} \). The distribution of \( X^{-\frac{1}{2}} \) can thus be written as

\[ F_Y(y) = 1 - F_X\left(y^{-\frac{1}{2}}\right) \]

and the distribution of \( V_2 \) by

\[ F_{V_2}(v) = \left(1 - F_X\left(v^{-\frac{1}{2}}\right)\right)^M. \]

By applying a general result from order statistics [8, Eq. (4.5.1)] for large \( N \) to (18) and after some algebraic manipulation, we have

\[ E_{V_M}[v^{-\frac{1}{2}}] \approx \left(F_X^{-1}\left(\frac{1}{\left(\frac{N}{M}\right)^{M+1}}\right)^{-\frac{1}{2}}\right)^{-\frac{1}{2}} \]

For large \( N \), \( X \) approaches a Gaussian distribution with mean \( N - M + 1 \) and variance \( N - M + 1 \). Thus

\[ F_X^{-1}\left(1 - \frac{M}{N}\right) = N - M + 1 + \sqrt{2(N - M + 1)} \times \text{erf}^{-1}\left(1 - 2\frac{M}{N}\right). \]

Substituting (20) into (19), and the resultant expression into (16) we obtain (10), (11) follows by taking \( N \) large in (10).

**REFERENCES**


