Abstract — A new coupling mechanism is used to synchronize two Van der Pol oscillators. This coupling uses the second harmonic appearing in common mode current of each oscillator. The common mode current is measured by a current mirror, and is amplified by a current amplifier. The amplifier introduces negative feedback, so that the current in the current mirror measuring diode of the first oscillator is nearly equal to the common mode current of the second oscillator, hence the coupling is established. It is shown that the system of two oscillators is described by two differential equations where the coefficients in one equation have the perturbations defined by the second oscillator, and vice versa. The coupling amplifier gain is defined. The developed concepts are demonstrated on a 5 GHz CMOS LC oscillator with quadrature outputs. The oscillator phase noise is lower than -116 dBc/Hz at 1-MHz offset.

Index term — Van der Pol oscillators, quadrature LC Oscillators.

I. INTRODUCTION

Quadrature oscillators are required in design of modern RF transceivers. Sinusoidal signals in quadrature can be obtained by using [1]: two nonlinear systems, each exhibiting self-sustained periodic oscillations, which interact through a coupling circuit. It is important to select the technique, type, and strength of coupling in order to generate accurate quadrature signals. Several coupling techniques have been presented in the literature [2,3,4], using either passive [2] or active [3] networks. A VCO followed by RC networks was used in [2], for quadrature generation. Transistors were used as coupling elements in [3] to force two oscillators to operate in quadrature. The former technique suffers from the need to include buffers between the VCO and the RC network [4]. This technique has the drawback of increased power consumption, and there is a trade off between phase noise and quadrature accuracy. The different coupling techniques were compared in [4].

Coupling modes can be established using odd or even harmonics. The amplitude stabilization results in currents with rich harmonic content. The differential current component is close to sinusoidal, with the frequency equal to the oscillator natural frequency. The common mode current component is also close to sinusoidal, but with double the oscillation frequency.

For synchronization, the differential current component provided by the synchronization circuit should be in phase with the current supplied to the parallel LC-circuit by the amplitude stabilization circuit. The synchronizing circuit will partially substitute the amplitude stabilization circuit. The common mode component may also be used for synchronization, using the second harmonic in the common mode output.

Most of the coupling circuits in the literature use the first or odd harmonics. The second harmonic common mode coupling was presented in [4], using passive narrowband networks. This has the drawback of the extra die area to implement the passive devices. The use of first and second harmonics simultaneously was demonstrated in [5], by designing a low power, low phase noise 5 GHz LC oscillator with quadrature outputs. The benefit of using simultaneous odd and even harmonic coupling is discussed in [5].

For quadrature generation, the self-oscillating nonlinear systems considered in the literature include RC [6] or Robinson type oscillators. Both types of oscillators were compared in [6].

This paper presents a new second harmonic coupling technique to enforce quadrature relation between two Van der Pol (VDP) oscillators.

In Part II, the circuit implementation and the conditions to establish a strong second harmonic component in the common mode current are derived. Part III presents the simulation results, and highlights the benefits and drawbacks of the new coupling technique. Finally, in Part IV we draw some conclusions.

![Van der Pol LC-oscillator](image_url)
II. SYNCHRONIZATION THEORY

One consider that each individual oscillator (Fig. 1) has a nonlinear block realized by transistors $M_1$ and $M_2$. This block is described by the input-output characteristic

$$i_d = -2B_N (V_{DD} - V_{TN}) v_d \left(1 - \frac{v_d^2}{4V_{DD}^2}\right)$$

where $v_d = v_1 - v_2$, $i_d = i_1 - i_2$, $B_N = (\mu N C_{ox} / 2)(W / L)$, and $V_{TN}$ is the threshold voltage of n-channel transistors. It is possible to show that the common mode current is:

$$i_c \approx (B_N / 2)v_d^2$$

where $i_c = i_1 + i_2$. Equation (2) does not include the dc component $2B_N (V_{DD} - V_{TN})^2$ and does not describe the falling part of the dependence $i_c = f_c(v_d)$ (this falling part is important for evaluation the stability of coupling using second harmonic, and this problem is not considered in this paper).

![Figure 2. VDP oscillator equivalent circuit](image)

For the standard [7] equivalent circuit of VDP oscillator (Fig. 2) one can write the equation

$$\frac{1}{C} \int i_{cd} dt = L \frac{di_{ld}}{dt} + R_S i_{ld}$$

Using the approximation $L \frac{di_{ld}}{dt} \approx v_d$ and introducing the new variable $\dot{v} = \alpha_0 v_d$, one can transform (3) into

$$\ddot{v} + \alpha_0^2 v = -\alpha_0^2 R_S C \dot{v} - \alpha_0^3 L i_d$$

Substituting (1) in (4) one obtain

$$\ddot{v} + \alpha_0^2 v = -\alpha_0^2 R_S C \dot{v} + \alpha_0^3 \left[2B_N (V_{DD} - V_{TN}) \left(1 - \frac{v_d^2}{4V_{DD}^2}\right)\right]$$

Two VDP oscillators synchronized by the second harmonic are shown in Fig. 3. The synchronization circuit consists of the sensing current mirrors between the power supply and the coil tap in each oscillator and the coupling differential current amplifier with gain $A_i$. The synchronization circuit introduces a small variation of the supply voltage in each oscillator done to the coupling current $i_{cplL}$ (or, correspondingly, by $i_{cplR}$) provided from the second oscillator. This small supply voltage variation modifies the coefficients of the differential equation (5), and this mechanism results in the synchronization of the oscillators. In this paper we only demonstrate that this coupling exists, i.e. that one can find the relationships between the oscillator parameters (basically we show that one can find an amplifier current gain) such that two differential equations with variable parameters are satisfied in steady-state operation. The formal derivation of the solution, and verification of its stability will be demonstrated during conference presentation.

![Figure 3. Synchronization of two VDP oscillator](image)

We will describe the left-hand side oscillator by the variable $v_d = v_1 - v_2$ and the right-hand side oscillator by $u_d = u_1 - u_2$. First, we notice that the coupling currents and the common mode currents of the oscillators are described by the equations

$$\begin{align*}
(i_{cplL} + i_{cplR}) A_i &= i_{cplR} \\
(i_{cplR} + i_{cplL}) A_i &= i_{cplL}
\end{align*}$$

From (6) one can find, for example, that with reasonably high $A_i$ the coupling current $i_{cplL} \approx -i_{cplR} (-i_{cplL} / A_i)$, and the current of the sensing diode is $i_c \approx -i_{cplR} / A_i$. Thus, the amplifier introduces the feedback that cancels the common mode current from the oscillator where this measuring diode is connected, yet allows flowing in this diode the current of another oscillator. This is exactly what is required for coupling.

To obtain the differential equation of the left-hand side oscillator in (5), we replace $V_{DD}$, by the expression

$$V_{DD} - |V_{TP}| - \frac{i_{cR}}{g_d A_i} = V_{DD} - |V_{TP}| - \frac{B_N u_d^2}{2 g_d A_i}$$

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where \(1/g_d\) is the resistance of the sensing diode-connected transistor (equation (7) has a small error, it is better to use \(V_{SG}\) voltage and not \(|V_{TP}|\) but this will only complicate the derivation).

With steady-state oscillation \(u_d = U_d \omega_0 \cos \omega_0 t\), which gives us the differential equation

\[
\ddot{v} + \omega_0^2 v = -\omega_0^2 R_S C \dot{v} + \frac{2B_N \left( V_{DD} - V_{TN} - |V_{TP}| \right) + \frac{B_N U_d^2 \cos 2\omega_0 t}{4g_d A_i}}{1 - \frac{\dot{v}^2}{4\omega_0^2 (V_{DD} - |V_{TP}|)^2 (1 \pm b U_d^2 \cos 2\omega_0 t)^2}}
\]

(8)

Introducing the notations

\[
G = 2B_N (V_{DD} - V_{TN} - |V_{TP}|)
\]

\[
a = B_N / (4g_d A_i (V_{DD} - V_{TN} - |V_{TP}|))
\]

\[
b = B_N / (4g_d A_i (V_{DD} - |V_{TP}|))
\]

one can rewrite (8) in the more convenient form

\[
\ddot{v} + \omega_0^2 v = -\omega_0^2 R_S C \dot{v} + \omega_0^2 G \dot{v} (1 \pm a U_d^2 \cos 2\omega_0 t)
\]

\[
\left[ 1 - \frac{\dot{v}^2}{4\omega_0^2 (V_{DD} - |V_{TP}|)^2 (1 \pm b U_d^2 \cos 2\omega_0 t)^2} \right]
\]

(9)

Considering that \(a U_d^2 \cos 2\omega_0 t << 1\) and \(b U_d^2 \sin 2\omega_0 t << 1\), one rewrites (9) as

\[
\ddot{v} + \omega_0^2 v = -\omega_0^2 R_S C \dot{v} + \omega_0^2 G \dot{v} (1 \pm a U_d^2 \cos 2\omega_0 t)
\]

\[
\left[ 1 - \frac{\dot{v}^2}{4\omega_0^2 (V_{DD} - |V_{TP}|)^2 (1 \pm b U_d^2 \cos 2\omega_0 t)^2} \right]
\]

(10)

or, finally, as

\[
\ddot{v} + \omega_0^2 v = -\omega_0^2 R_S C \dot{v} + \omega_0^2 G \dot{v} \left[ 1 \pm a U_d^2 \cos 2\omega_0 t \right]
\]

\[
\left[ \frac{\dot{v}^2}{4\omega_0^2 (V_{DD} - |V_{TP}|)^2 (1 \pm b U_d^2 \cos 2\omega_0 t)^2} \right]
\]

(11)

Similar transformations and approximations can be done for the second oscillator. With coupling, the second oscillator will be described by

\[
\ddot{u} + \omega_0^2 u = -\omega_0^2 R_S C \dot{u} + \omega_0^2 G \dot{u} \left[ 1 \pm a U_d^2 \cos 2\omega_0 t \right]
\]

\[
\left[ \frac{\dot{u}^2}{4\omega_0^2 (V_{DD} - |V_{TP}|)^2 (1 \pm b U_d^2 \cos 2\omega_0 t)^2} \right]
\]

(12)

The choice of signs before the coefficients \(a\) and \(2b\) is defined, by the coupling connections.

Assume the steady state quadrature solution

\[
\begin{align*}
\dot{v} &= V_d \omega_0 \sin \omega_0 t \\
u &= U_d \omega_0 \cos \omega_0 t
\end{align*}
\]

(13)

The existence of this solution requires, first, that

\[
G = \frac{R_S C}{L}
\]

(14)

i.e., that the negative resistance compensate the losses in the coil. Then, using (11) one finds that it is necessary to satisfy the condition (and similar condition is obtained from (12))

\[
\pm a U_d^2 \cos 2\omega_0 t - \frac{V_d^2}{8(V_{DD} - |V_{TP}|)^2} \left[ 1 \pm \frac{1}{2} (\pm 2b \mp a) \right] = 0
\]

(15)

One finds the coefficient of the second harmonic in (15) and discards the terms including small dc component (this can always be taken care of by proper modification of \(G\)). We also neglect the forth harmonic, since its frequency is to high for it to exist in a practical oscillator. Then one obtains that it is necessary to satisfy the condition

\[
\pm a U_d^2 - \frac{V_d^2}{8(V_{DD} - |V_{TP}|)^2} \left[ 1 \pm \frac{1}{2} (\pm 2b \mp a) \right] = 0
\]

(16)

After a similar treatment of (12), discarding the small terms within square brackets, and choosing the negative sign in the first term, one arrives at

\[
\begin{align*}
- a U_d^2 + \frac{V_d^2}{8(V_{DD} - |V_{TP}|)^2} &= 0 \\
- a V_d^2 + \frac{U_d^2}{8(V_{DD} - |V_{TP}|)^2} &= 0
\end{align*}
\]

(17)

These two conditions require that

\[
8a(V_{DD} - |V_{TP}|)^2 = 1
\]

(18)

which is satisfied by the choice of the coupling amplifier gain

\[
A_i = \frac{2B_N (V_{DD} - |V_{TP}|)^2}{g_d (V_{DD} - V_{TN} - |V_{TP}|)}
\]

(19)
For typical values of the parameters, (19) results in very moderate values of the gain.

III. CIRCUIT IMPLEMENTATION

The previous theoretical analysis indicates that the second harmonic can be used to synchronize two LC oscillators. To demonstrate the theory we simulated the oscillator in Fig. 3. The conventional first harmonic coupling circuit (not shown in this figure) was used to start the synchronization. Then, this coupling circuit was disconnected, leaving the full synchronization load carried by the second harmonic coupling circuit.

The circuit was designed with the 0.18 μm CMOS technology, for an oscillation frequency of 5 GHz. The second harmonic synchronization transistors have \((W/L) = 10 \mu m / 0.18 \mu m\). The current sensing transistors have the dimensions \((W/L) = 4 \mu m / 0.18 \mu m\). The core transistors forming the active nonlinear circuit have the dimensions of \((W/L) = 100 \mu m / 0.18 \mu m\). The resonant LC circuit is realized using an inductor equal to 2nH and a capacitor of 150 fF. The second harmonic coupling transistors are biased by a tail current of 100 μA. The value of \(A_i\) is 2.25. The supply voltage is 1.8V and the supply current is 1.2 mA.

IV. SIMULATION RESULTS

The oscillator phase noise at 1 MHz offset is lower than -116 dBc/Hz as shown in Fig. 4. This phase noise is 5 dB higher than the value obtained in [5], but this is compensated by the benefit that the die area is spared by not using passive elements in the coupling circuit. A plot of the quadrature outputs at 5 GHz is shown in Fig. 5.

![Figure 4. Oscillator phase noise.](image)

![Figure 5. Quadrature Outputs.](image)

V. CONCLUSION

Synchronization of LC oscillators usually requires strong coupling, entailing the use of high bias currents in the coupling circuits. Excluding the synchronizing circuit with high bias current results in the minimal shift in the oscillation frequency compared to the oscillation frequency of the stand-alone oscillator. Hence the degradation in oscillator phase noise can be minimized. Coupling using the second harmonic does not create any additional load on each oscillator. An advantage of the proposed coupling mechanism is that passive components (large area) are not necessary. The oscillator of Fig. 3 requires a start-up circuit to establish the synchronism. This can be any conventional first harmonic coupling, which may be disconnected, once the second harmonic coupling is established.

A new type of coupling circuit using second harmonic is presented, which is realized using active elements. A theory was developed to obtain the necessary conditions for stable common mode second harmonic coupling. The conclusions are confirmed by the design of a 5 GHz low power quadrature oscillator, which has a simulated phase noise of -116 dBc/Hz at 1 MHz offset.

REFERENCES