Costly bilingualism model in a population with one zealot

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We consider a costly bilingualism model in which one can take two strategies in parallel. We investigate how a single zealot triggers the cascading behavior and how the compatibility of the two strategies affects when interacting patterns change. First, the role of the interaction range on the cascading is studied by increasing the range from local to global. We find that people sometimes do not favor to take the superior strategy even though its payoff is higher than that of the inferior one. This is found to be caused by the local interactions rather than the global ones. Applying this model to social networks, we find that the location of the zealot is also important for larger cascading in heterogeneous networks.

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I. INTRODUCTION

One of the fundamental issues in social science is to understand how new strategies, technologies, and ideologies spread and diffuse through population [1, 2]. One possible mechanism that may explain this phenomenon is bilingualism, where people can adopt two traits – such as languages, technologies, and ideas – in parallel. Many of the related researches are performed also in physics as well as economics and mathematical sociology [2-4].

In the recently studied bilingualism models by Kleinberg et al., a population with two early adopters has been considered and how the compatibility of bilingualism influences the cascading behavior has been examined [3, 4]. Motivated by these studies, we investigate the population with single zealot since we realized even one zealot can trigger off the cascades. In particular, we focus on how the interaction range influences the cascading behavior, and explore the system by systematically changing the range from local to global. Moreover, in order to see how the compatibility affect competition in the real world, we also apply the model to the real social networks, and investigate its cascading behavior.

This paper is organized as follows: a costly bilingualism model is introduced in Sec. II then we reports the analytic and numerical results on one dimensional ring with an increasing interaction range in Sec. III A and those of the globally coupled case in Sec. III B. In Sec. IV an application to social networks are displayed. Finally we summarize and discuss our results in Sec. V.

II. COSTLY BILINGUALISM MODEL

Let us consider a population where individuals can choose one of the three strategies $A$, $B$, and $AB$. $A$ and $B$ can be regarded as two languages, and $AB$ represents bilingualism. Learning multiple languages or technologies usually requires lots of time and resources and thus we assume that the bilingual strategy is costly. The costly bilingualism model is played as follows [3, 4]: If two individuals with the same strategy $A$ (or $B$) interact, they get the same payoff $a$ ($b$). If two with different strategies – one with $A$ and the other with $B$ – interact, no one gets any payoff. When a bilingual ($AB$) interacts with a monolingual, each gets the monolingual’s payoff. The bilingual individual pay the cost $c$ of adopting bilingualism. When two bilingual individuals interact with each other they get the payoff of $\max(a, b)$ while paying the same cost $c$. The payoff matrix is shown in Table I.

At each time step, each individual updates its strategy choosing one among three strategies to maximize its payoff with probability 1 with no transition cost. While updating one’s strategy, we assume there is no change on his/her neighborhood. In this model, a direct update between ‘monolingual’ strategies such as change from $A$ to $B$ are allowed, differently from other model [2] where all shifts between monolingualism have to pass through the ‘bilingual’ strategy $AB$. This cascading update continues until no one wants to change anymore. Note that we do not concern the details of how one can change his/her strategy and its transition cost. It can be an imitation or adopting an already known-technology freely, to maximize players’ utility function.

We introduce a zealot, who does not change his/her

<table>
<thead>
<tr>
<th>neighbors’ strategy</th>
<th>$A$</th>
<th>$B$</th>
<th>$AB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$a$</td>
<td>0</td>
<td>$a$</td>
</tr>
<tr>
<td>$B$</td>
<td>0</td>
<td>$b$</td>
<td>$b$</td>
</tr>
<tr>
<td>$AB$</td>
<td>$a - c$</td>
<td>$b - c$</td>
<td>$\max(a, b) - c$</td>
</tr>
</tbody>
</table>

TABLE I. Payoff matrix of the costly bilingualism model. Here $\max(a, b)$ represents the larger of $a$ and $b$, and $-c$ in the last row denotes the cost of bilingualism.
strategy from the strategy $A$ at all, in the population of $B$. Then we study the $A$'s cascading behavior for various values of $a$ and $c$, under a fixed value of $b$, where $b$ is set to 1, and $a$ is assumed to be larger than $b$. When the number of interacting neighbors is more than one, we rescale the value of $a$ and $b$ by its number of neighbors to keep a balance with the cost $c$. For example, when one interacts with three neighbors using $A$, the individual gets the payoff $a/3$ per neighbor.

III. RESULTS

A. Local Interactions

We first consider one dimensional ring with an interaction range $\ell$, where each individual interacts with $2\ell$ neighbors. $\ell = 1$ means the conventional one-dimensional lattice that each individual has two nearest neighbors, and $\ell = 2$ represents the system where each one interacts with four neighbors (nearest neighbors and the next nearest neighbors). We study how the cascading behavior changes as we vary the interaction range from the local ($\ell = 1$) to the global ($2\ell = N - 1$).

Figure 1(a) and (b) show how the individuals updates their strategy for $\ell = 1$, and for $\ell = 2$ with the payoff $a = 2$, $b = 1$, and the cost $c = 0.5$ [5]. For $\ell = 1$, two nearest neighbors with $B$ next to the $A$ zealot (colored in red at $t = 0$ in Fig. 1(a)) first adopt $AB$ at $t = 1$. The next nearest neighbors then adopt $AB$, and the former two $AB$s change into $A$. At $t = 6$, all takes $A$ and the cascade completes. The strategy $AB$ naturally emerges even though we start with no $AB$. The person who choose the strategy $B$ usually adopts $AB$ first, and then changes into $A$, but directly adopt $A$ when the cost $c$ is too high. For $\ell = 2$, on the other hand, the four nearest neighbors next to the zealot first adopt $AB$ at $t = 1$. At $t = 2$, the other four next nearest neighbors adopt $AB$, and the former two $AB$s change into $A$ at $t = 3$. The cascade completes at $t = 4$, as shown in Fig. 1(b). The duration of cascades decreases as the interaction range $\ell$ increases.

We investigate the cascading behavior at various values of the interaction range, payoff, and cost. Figure 2 shows the phase diagram in the plane of $a$ and $c$ for various interaction ranges $\ell$ from $\ell = 1$ to $\ell = 3$ [Fig. 2(a)-(c)], and the globally-coupled version [Fig. 2(d)], respectively. In particular, we note that the phase diagram shown in Fig. 2(a)-(c) is divided into three regions: Two phases labeled (in bold) $A$ and $B$, where $A$ represents the phase that all individuals in the population choose the strategy $A$ in the long time, and $B$ denotes the phase that all (except the one $A$-zealot) takes the strategy $B$. The other one is the phase $R$, where the strategy $AB$ survives at the boundary and blocks the $A$’s spread through $B$, making the cascade of $A$ impossible. This $R$ phase appears in the carved region of the phase diagram in Fig. 2(a)-(c). Interestingly, people in this phase does not favor to take the strategy $A$ even though the payoff $a$ from taking $A$ is larger than the payoff $b$ from the strategy $B$. We note that the phase $R$ consists of $2\ell$-wide $AB$s located at the next to the $A$ zealot and all $B$s for the others, so the number of $AB$s increases as the interaction range $\ell$ increases. In the $R$ phase, the $AB$s shield the $A$-zealot, which makes a $AB$-buffering zone, prohibiting the cascade.

It is noteworthy to point out that reentrant transition occurs in the phase $R$, e.g., for $\ell = 1$ the population shows the $A$-cascade for $a = 1.5$ and $c = 0.1$, but if we increase the cost up to $c = 0.3$, the $AB$s next to the zealot shield the $A$, blocking the $A$’s spread, which impedes the $A$’s cascade. However, if we further increase the value of $c$ up to $c = 0.6$ the population reaches the cascade again [see Fig. 2(a)]. We also find that the area of this reentrant-transition zone shrinks as $\ell^2$, as shown in Fig. 2(a)-(c) [see later], and the zone eventually disappears in the globally-coupled system as shown in Fig. 2(d). This implies that the phase $R$ is caused by the local interaction not by the long-range (global) one. We note that the occurrence of the phase $R$ has also been
reported in Ref. [3], where the authors studied for $\ell = 1$
case with the two early adapters (zealots). Our study,
on the other hand, exhibits that even only one zealot can
tigger off the cascading, furthermore we find that the
phase $R$ strongly depends on the interaction range $\ell$.

B. Globally Coupled Case

We now consider the globally-coupled version of the
model, i.e., the complete graph with the population size
$N$, where each individual interacts with all the others,
and analyze the behaviors of the system. The phase
boundary between the phases $A$ and $B$ in Fig. 2(d)
can be obtained by analyzing the break-even point of the
payoff. The payoff of the $i$th individual, from choosing
the strategy $A/B/AB$, is given by

\[
p^i_A = \hat{a}(N^i_A + N^i_{AB}), \quad p^i_B = \hat{b}(N^i_B + N^i_{AB}),
\]

for $i = 1, \ldots, N$, where $N^i_A$, $N^i_B$, and $N^i_{AB}$ represent
the number of the interacting neighbors of the $i$th individual,
who plays $A$, $B$, and $AB$, such that $N^i_A + N^i_B + N^i_{AB} = k^i$,
where $k^i$ is the number of neighbors (degree) of the $i$-
that individual. And $\hat{a} = a/k^i$ and $\hat{b} = b/k^i$ denote the
rescaled payoff for the balance with the cost $c$. For
the complete graph, all the neighbors have the same
number of neighbors ($k^i = N - 1$) with no difference,
which allows us to suppress the site index $i$ in the Eq. (1),
which reads $p_A = a \left(1 - \frac{N_A}{N-1}\right)$, $p_B = b \left(1 - \frac{N_B}{N-1}\right)$, and
$p_{AB} = a - c - (a-b) \frac{N_{AB}}{N-1}$.

We now start with the initial condition that there is
only one $A$-zealot ($N_A = 1$) and the others are $B$'s ($N_B =
N - 1$). To choose the strategy $A$, the payoff $p_A$ should
be larger than the other ones $p_B$ and $p_{AB}$. Similarly, to
take the strategy $B$ the payoff $p_B$ should be larger than
$p_A$ and $p_{AB}$, and to take the strategy $AB$ the payoff $p_{AB}$
should be larger than the others. From these conditions,
we find that the strategy $A$ is chosen for

\[
a > b(N - 2) \quad \text{and} \quad c > \frac{N - 2}{N - 1},
\]

the $B$ is chosen for

\[
a < b(N - 2) \quad \text{and} \quad c > \frac{a}{N - 1},
\]

and the $AB$ is chosen for

\[
c < b \frac{N - 2}{N - 1} \quad \text{and} \quad c < \frac{a}{N - 1}.
\]

We find that Eq. (3) with the condition $a > b$ (initially
assumed) determines the phase boundary of the phase $B$,
as shown in Fig. 2(d). On the other hand, Eq. (2) and
(4) decide the boundary of the parameter region where
the strategies $A$ and $AB$ are chosen, respectively. Let us
suppose that we are now in the parameter region where
$AB$ is chosen. For a given value of $a$ and $c$ in this region,
the individuals first choose $AB$, the system then consists
of one $A$-zealot and $AB$s for the remains, i.e., $N_A = 1,$
$N_B = 0,$ and $N_{AB} = N - 1$, since all $B$s except the $A$-
zealot turn into $AB$ due to the “all-to-all” coupling in the
complete graph. We find that all $AB$s take the strategy $A$
ext time since the payoff obtained from taking $A$ is the
largest one, which makes all $AB$s in this region turn into
$A$s and they remain ever since, which yields the phase
boundary shown in Fig. 2(d).

The phase boundary for the system with local interaction
in Fig. 2(a)-(c) can also be obtained from the
analysis of break-even point of the payoffs of $A$, $B$, and
$AB$, similarly to the globally-coupled case. However, the
density-level description of the globally-coupled system is
impossible, instead the node(site)-level one is available,
we should decide the strategy of each node one by one,
considering all available situation. Substituting $k^i = 2\ell$
and the rescaled payoff $\hat{a} = \frac{a}{2\ell}$ and $\hat{b} = \frac{b}{2\ell}$ into Eq. (1),
the payoff of each individual is obtained, and the same
analysis about the break-even point of the payoffs leads
us to have the phase boundary as shown in Fig. 2(a)-(c),
where the boundary has been also confirmed numerically.
We find that the three points which consist of the trian-
gular region of the phase $R$ located at the carved zone are
given by the three points $(b, 0)$, $(b, \frac{b}{2\ell})$, and $(b + \frac{a}{2\ell}, \frac{b}{2\ell})$.
Accordingly, the size of the phase $R$-region is given by
$\frac{a^2}{2\ell}$, and it vanishes in the globally-coupled system. And
the kinked corner point on the site of large $a$ and $c$ for
$\ell = 2$ and $3$ is found to be given by $(b(2\ell - 1), b - \frac{b}{2\ell})$.

IV. APPLICATION TO REAL SOCIAL NETWORKS

A natural question we can ask now is this: does the reentrant transition occur in a real social system, too?
How does the compatibility of the strategies influence the
cascading behavior in the real social networks? To
address these questions, we now analyze real social networks
and explore how the compatibility of the two strategies
affect the cascading behavior. We consider the Zachary’s
karate club network [6]. The network is known as the
social network of friendships between 34 members of a
karate club at a US university in the 1970s. The network
size is just 34, which allows us to do complete investiga-
tion of the cascading behavior depending on the location
of the zealot. In particular, we examine how the network
properties affect the cascading behavior.

We perform the numerical simulations on the karate
club network, where individuals sequentially update their
strategies [3]. The sequence of updates is determined
according to the expected payoff change; the node that
can achieve the largest change in their payoff get the
priority since the large potential payoff change can be
considered as a high social pressure from the neighbors.
Again we find that even a single $A$-zealot can trigger the
cascades in the karate club network. In addition, we find
that the location of the zealot is crucial. We consider all 34 possible locations of the zealot, and explore how the phase diagram changes by the location of the zealot. We obtain 34 phase diagrams for the different sites of the zealot and find that the phase diagrams can be classified into the three representative ones as shown in Fig. 3.

We find that, in addition to the phases A and B, we have more phases named $S_1$, $S_2$, $S_3$, and $S_4$, where the phase $S_1$ ($S_2$) represents the mixed state of the strategy $A$ and $B$ with no $AB$, while $A$ ($B$) is superior to $B$ ($A$); the phase $S_3$ ($S_4$) is the state where $A$, $B$, and $AB$ all coexist, where the $A$ ($B$) is the superior one. We note that the phases $S_3$ and $S_4$ include the $AB$-buffering zone inside. The phases are represented by the different colors as shown in Fig. 3. Note that there is no other phase except these six.

Interestingly multi-reentrant transition zone appears as shown in Fig. 3, which is caused by the mixed interaction among the people with a variety of neighbors. The network properties summarized in the Table II for the node 2, 7, and 33, show that the system easily produces the cascade when the hub is the zealot, as expected. This is, however, not enough for achieving the larger cascade. Additional important conditions are whom the zealot connects and where he/she is located. The zealot needs to have small clustering coefficient (CC), and its neighbors should have small degree. High CC means that one’s neighbors know each other very well and they favor to share a common strategy, which means that they can convert their strategy at the same time. Therefore, high clustering around one individual can be an obstacle to large cascades. Furthermore, one’s neighbors with small-degree can be easily influenced by the zealot’s opinion since its influence is reciprocally proportional to the neighbors’ degree. We find that these effects are well observed in the karate network, as shown in the Table II.

On the other hand, the case of the network with strong community structures can be a different story. We investigate several target nodes in other social networks: Les Miserables network [8] ($N = 77$), dolphin network [9] ($N = 62$), and coauthorship network of network scientists [10] ($N = 379$, only considering the giant connected component), based on high $k^i$, small CC, and small $\bar{k}_{nn}$. We find that small CC, small $\bar{k}_{nn}$, and even high $k^i$ do not promise large cascade size since cascades often stop after converting several nodes in a community even when they start from the hubs. Therefore, the size of assigned community or the centrality can be also important factors for the cascades when the network has high modularity, which requires an in-depth study on the role of the modularity on the cascading in complex networks.

<table>
<thead>
<tr>
<th>node</th>
<th>$k^i$</th>
<th>CC</th>
<th>$\bar{k}_{nn}$</th>
<th>BC</th>
<th>Closeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>17</td>
<td>0.11</td>
<td>3.8</td>
<td>387.1</td>
<td>1.82</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>1.00</td>
<td>10.3</td>
<td>66.0</td>
<td>2.27</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>0.24</td>
<td>6.6</td>
<td>217.7</td>
<td>1.79</td>
</tr>
</tbody>
</table>

TABLE II. Network properties. $\bar{k}_{nn}$ means the average neighbor degree. “BC” means the betweenness centrality, and “closeness” represents closeness centrality, respectively [7].
V. SUMMARY AND DISCUSSION

To summarize, we considered costly bilingualism model in a population with one zealot, and explored how the compatibility influences on the cascading behavior of one strategy, extending the interaction range from local to global. We found that superior strategy does not necessarily propagate. In the parameter region where this phenomenon occurs, the reentrant phase transition occurs. We found that it is caused by the local interaction with one’s neighbors rather than the long-range one. We applied the model to real-world social network and showed how the network properties take effects in the cascades. We have learned the lessons that if the zealot locates at the node with high degree, this is good for the larger cascade. Furthermore, we showed that the small clustering coefficient and small average neighbors degree enhance the cascade. Finally, we demonstrated that the community structure makes it hard to predict the cascade size. If a network has high modularity, the community structure should be considered carefully, which is remained for future study.

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[5] We have used the parallel updating process that all individuals simultaneously update their current strategy. The sequential updating process also gives the same result in regular lattices. However, in the case of complex networks, the final result can be changed according to the updating process.