BAYESIAN IMAGE SEGMENTATION WITH MEAN SHIFT

Huiyu Zhou 1, Gerald Schaefer 2, M. Emre Celebi 3 and Minrui Fei 4

1Queen’s University Belfast, Belfast, BT3 9DT, United Kingdom
2Loughborough University, Loughborough, LE11 3TU, United Kingdom
3Louisiana State University, Shreveport, LA 71115, United States
4Shanghai University, Shanghai 200072, PR China

ABSTRACT

Image segmentation plays a key role in many image content analysis applications, and a lot of effort has aimed at improving the performance of established segmentation algorithms. In this paper, we present a mean shift-based combined Dirichlet process mixture (MDP)/Markov Random Field (MRF) image segmentation algorithm. Our method incorporates a mean shift process to iteratively reduce the difference between the mean of cluster centres and image pixels within the standard MDP/MRF procedure. Experimental results show that the proposed segmentation technique outperforms the classical MDP/MRF algorithm.

Index Terms— Image segmentation, mean shift, Dirichlet process mixture, Markov Random Field.

1. INTRODUCTION

Image segmentation plays a key role in many image content analysis applications. Consequently, a lot of effort has aimed at improving the performance or robustness of established segmentation algorithms, where image observations (e.g., features or cues) are usually combined with necessary priors (or regularisation) in order to discover or merge homogeneous regions [1]. Among such feature-based image partitioning schemes, Gabor features [2] and wavelet-based features [3] are two examples of widely used descriptors. Graph-based methods (e.g. [4]) perform segmentation by formulating the image segmentation problem as a graph partitioning problem. Dirichlet process mixture (MDP) models [5] are one of the outstanding approaches that apply a Bayesian scheme for clustering problems with an unknown number of groups. Technically speaking, the number of classes is determined by investigating the likelihood of similarity according to the observations. MDP models have been shown to provide remarkable adaptivity and flexibility in finding the optimal number of classes.

MDP models in combination with Markov Random Field (MRF) have recently been proposed for image segmentation [6], where MRF priors with spatial smoothness constraints on cluster labels are employed to handle incoherent segments and jagged boundaries [7]. The combination of MDP and MRF utilises Markov Monte Carlo sampling to estimate state vectors of the model from input data, leading to very promising segmentation outcomes. However, the established MDP/MRF approach still needs to be improved in the case of image clutter or noise.

In this paper, we apply a mean shift procedure to the MDP/MRF model so as to enhance the segmentation performance while avoiding ambiguity and uncertainty in inhomogeneous regions. The mean shift procedure is used to locate density extrema or modes of a specific distribution by an iterative process, resulting in merging of pixels with approximate local densities. Those pixels that are further from the centres of the clusters will be given lower weights in the MDP/MRF computation. This in turn significantly reduces the influence of clutter or noise on the estimation of posterior probabilities while enhancing the clustering performance.

2. DIRICHLET PROCESS MIXTURE MODELS AND CLUSTERING

Image segmentation aims to group a set $x_1, ..., x_i$ of inputs (local image features) into individual classes. Let’s denote the number of classes by $N_C$, then the class assignment of input $x_i$ is represented by an indicator $S_i (i = 1, ..., N_C)$. A class in image clustering will be of a form with finite mixture models

$$p(x) = \sum_{j=1}^{N_C} c_j p_j(x), \quad (1)$$

where $c_j = \Pr\{S = j\}$ for an input drawn randomly from the entire model, and $\sum_j c_j = 1$, and $p_j$ indicates a single probability distribution. This model may have a two-stage generative process for the input $x$: $x \sim p_S$, and $S \sim (c_1, ..., c_{N_C})$. If the distribution $p_j(x)$ can be parameterised as $p_j(x) = p_j(x|\theta_j)$, then we can parameterise the distribution shown in Eq. (1) with $c_1, ..., c_{N_C}$ and $\theta_1, ..., \theta_{N_C}$ ($\theta$ is a parameter value of the clusters).

MDP models draw a random prior $G$ from a Dirichlet Process (DP). Combining it with a parametric likelihood $F(x|\theta)$,
this results in the following form:
\[
\begin{align*}
\mathbf{x}_i & \sim F(\mathbf{x}|\theta_i), \\
\theta_i & \sim G, \\
G & \sim DP(\alpha G_0),
\end{align*}
\] (2)
where $\alpha$ is a scalar and $G_0$ is the base measure of the process. In a Bayesian framework, $F(\cdot|\cdot)$ is considered to be the posterior estimation of the data. Prior and posterior in Eq. (2) refer to the same model class and the posterior parameters are dynamically updated during the process. A conditional distribution for property 3 of Eq. (2) can be computed as
\[
p(\theta_{n+1}|\theta_1, \ldots, \theta_n) = \frac{1}{n+\alpha} \sum_{i=1}^{n} \Delta_{\theta_i}(\theta_{n+1}) + \frac{\alpha}{n+\alpha} G_0(\theta_{n+1}),
\] (3)
where $\Delta_{\theta_i}$ is the Dirac measure centered at $\theta_i$.

3. DIRICHLET PROCESS MIXTURES WITH MARKOV RANDOM FIELDS

Markov random fields (MRFs) are stochastic models that characterise local spatial interactions in data. MRFs are used with MDP in order to enforce spatial constraints during the segmentation process. An MRF consists of random variables defining an undirected and weighted graph with sites, edges, and weights.

Let an MRF distribution $\Xi$ be decomposed into a site-wise term $\mathcal{S}$ and the remaining interaction term $\mathcal{R}$. The MRF distribution can be written as follows to generate MRF of $\mathbf{x}_i$,
\[
\Xi \propto \mathcal{S}(\theta_1, \ldots, \theta_n)\mathcal{R}(\theta_1, \ldots, \theta_n),
\] (4)
with the site $\mathcal{S}(\theta_1, \ldots, \theta_n) := \frac{1}{Z_{\mathcal{S}}} \exp(-\sum_{i} H_i(\theta_i))$ and the remaining term $\mathcal{R}(\theta_1, \ldots, \theta_n) := \frac{1}{Z_{\mathcal{R}}} \exp(-\sum_{C \in \mathcal{C}_1}H_C(\theta_C))$, where $Z_{\mathcal{S}}$ indicates the partition function, singleton cliques $C = \{i\}$ and $C_2 := \{C \in \mathcal{C}||C| \geq 2\}$ ($\mathcal{C}$ is the set of all cliques), and $H(\cdot)$ is a cost function that defines a distribution by $\Xi(\theta_1, \ldots, \theta_n) := \frac{1}{Z_{\mathcal{H}}} \exp(-H(\theta_1, \ldots, \theta_n))$ with a normalisation term $Z_{\mathcal{H}}$.

Omitting intermediate steps, we arrive at the following form for the posterior estimation [6]:
\[
\Xi(\theta_i|\theta_{-i}) \propto \sum_{k=1}^{N_C} \mathcal{R}(\theta_i|\theta_{-i}) n_{ki}^{-1} \delta_{\theta_k}(\theta_i) + \frac{\alpha}{Z_H} G_0(\theta_i),
\] (5)
where $\theta_{-i} := \{\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n\}$, $n_{ki}^{-1}$ is the number of values accumulated in group $k$, $\delta$ is the Kronecker symbol, and $\theta_k^*$ denotes the parameters in class $k$.

4. MDP/MRF WITH MEAN SHIFT

A Gibbs sampler can be used for implementing MDP models (see Eq. (2)). There are two steps in this sampling algorithm, an assignment step and a parameter update step. The assignment of inputs $\mathbf{x}_i$ to cluster $k$ relies only on the current state of the model. If $\mathbf{x}_i$ falls in the known classes, then $\mathbf{x}_i$ is assigned to cluster $k$. Otherwise, a new cluster will be created. Image segmentation is a clustering problem where two class labels can only be identical or different. We have
\[
H(\theta_i|\theta_{-i}) = \frac{1}{n} \sum_{j=1}^{n} K_H(\theta_i - \theta_j),
\] (6)
where $K_H(\theta) = |H|^{-1/2} K(|H|^{-1/2}\theta)$ and $K$ is the $d$-variate kernel function. Assuming $H = h^2 I$ ($h^2$ refers to variance estimates and $I$ is identity matrix), we have
\[
H(\theta_i|\theta_{-i}) = \frac{1}{nh^d} \sum_{j=1}^{n} K_H\left(\frac{\theta_i - \theta_j}{h}\right),
\] (7)
which indicates that image segmentation can be reached subject to the measurements by the mean of the square error between the density and the estimate. Using the established mean shift algorithm [8], we iteratively calculate the difference between the mean of the cluster centres and image pixels until this difference is less than a pre-defined threshold:
\[
m(\theta_i) = \frac{\sum_{j=1}^{n} \theta_j g\left(\frac{\|\theta_i - \theta_j\|^2}{h}\right)}{\sum_{j=1}^{n} g\left(\frac{\|\theta_i - \theta_j\|^2}{h}\right)} - \theta_i.
\] (8)
Given an image, we extract features by formulating histogram bins from the image. Each histogram is described by a vector $\mathbf{r}_i = (r_{i1}, \ldots, r_{IN})$, where $N$ is the number of the bins.

The initial class probability $q_{i0}$ can be represented as [6]
\[
q_{i0} \propto \int_{\Omega_0} F(\mathbf{r}_i|\theta_i) G_0(\theta_i) d\theta_i = \frac{D_{G}(\mathbf{r}_i + \beta \pi)}{D_{F}(\mathbf{r}_i) D_{G}(\beta \pi)},
\] (9)
where $D_{F}$ is the multinomial partition function and $D_{G}$ is a normalisation term.

Let $k = 1, \ldots, N_C$, then we have the class probability
\[
q_{ik} \propto \frac{n_k^{-1} \exp\left(-H(\theta_k^*|\theta_{-i})\right)}{D_{F}(\mathbf{r}_i)} F(\mathbf{r}_i|\theta_k^*)
\]
\[
= \frac{n_k^{-1}}{D_{F}(\mathbf{r}_i)} \exp\left(\frac{1}{n h^2} \sum_{j=1}^{n} K_H\left(\frac{\theta_i - \theta_j}{h}\right) + \sum_{j} h_{ij} \log(\theta_k^*)\right).
\] (10)
Therefore, the proposed mean shift based MDP/MRF segmentation algorithm can be summarised as follows:

Step 1. Initiate the algorithm with a single cluster $\theta_1^*$. 

Step 2 Generate random samples from data indices.

Step 3 Compute cluster probabilities $q_{i0}$ and $q_{ik}$ using Eqs. (9) and (10) ($k$ is a random index).

Step 4 Assign observations $x_i$ to cluster $k$ based on the value of $k$.

Step 5 Update cluster parameters $\theta^*_{ik}$ by sampling $\theta^*_{ik} \sim G_0(\theta^*_{ik}) \prod_i F(x_i|\theta^*_{ik})$.

5. EXPERIMENTAL RESULTS

We evaluated our proposed image segmentation algorithm on a subset (50 images in total) of the Berkeley Segmentation dataset [9] which also contains manual border delineations that serve as ground truth. Fig. 1 shows some examples of the test database together with their ground truth segmentations.

For our experimental evaluation, we used a PC with Intel(R) Core(TM)2 CPU (2.66GHz) and 2GB RAM. The algorithms that we compared are the conventional MDP/MRF algorithm from [6] and our proposed MDP/MRF with mean shift algorithm. In the final stage of both algorithms, morphological processing is employed for smoothing the segmentation outcomes and removing small isolated areas.

Examples of the segmentations obtained by both algorithms are given in Fig. 2 which shows the original images together with the results of the two methods and the ground truth segmentation. It can be observed that while the algorithms produce similar results, the segmentations produced by the classical MDP/MRF algorithm subjectively are less smooth than those by the proposed mean shift based MDP/MRF approach. This is due to the fact that our algorithm takes into account mean values in the sampling so their outcomes are smoothed. Clearly, smoother borders are more realistic and also conform better to the manual segmentations.

To obtain quantitative results, we record, for each image segmentation, the number of True Positives $TP$ (the number of pixels that were classified both by the algorithm and the manual segmentation as object pixels), True Negatives $TN$ (the number of pixels that were classified both by the algorithm and the ground truth as non-object pixels), False Positives $FP$ (the number of instances where a non-object pixel was falsely classified as part of an object by an algorithm) and False Negatives $FN$ (the number of instances where an object pixel was falsely classified as non-object by an algorithm). From this we can then calculate recall and fall-out, defined as

$$\text{recall} = \frac{TP}{TP + FN},$$

and

$$\text{fall-out} = \frac{FP}{FP + TN}.$$ 

as measures for segmentation quality as is suggested in [10]. We also calculate the overall accuracy, defined as

$$\text{accuracy} = \frac{TP + TN}{TP + FN + TN + FP}.$$ 

In Table 1 we list recall, fall-out and accuracy values for both algorithms averaged over all test images. It can be seen that both algorithms have similar recall values but that the proposed MDP/MRF with mean shift algorithm outperforms the classical MDP/MRF algorithm in terms of fall-out and accuracy by about 5%.

6. CONCLUSIONS AND FUTURE WORK

In this paper we have introduced a new mean shift-based MDP/MRF image segmentation algorithm. The proposed method incorporates a mean shift process to iteratively reduce the difference between the mean of the cluster centres...
and image pixels within the standard MDP/MRF procedure. Experimental results have shown that the proposed segmentation technique outperforms the classical MDP/MRF algorithm.

References