Noncoherent Multiple Symbol Detection for MIMO Ultra-Wideband Systems

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Abstract—In this paper, we investigate noncoherent Multiple-Input Multiple-Output (MIMO) ultra-wideband (UWB) systems where the signal is encoded by Differential Space-Time Block Code (DSTBC). Considering the specific signal format of DSTBC-UWB system and employing the property of DSTBC, a noncoherent multiple symbol detection (MSD) scheme is developed by generalized likelihood ratio testing (GLRT) approach. Although the proposed MSD scheme can enhance the performance of DSTBC-UWB system, the complexity of the exhaustive search based MSD exponentially increases with observation window size. To decrease the computational complexity, the original MSD metric is transformed into another equivalent form which can be implemented by sphere decoding (SD) for DSTBC-UWB system. Moreover, a suboptimal Decision-Feedback (DF) based MSD with lower complexity than SD based MSD is proposed to further reduce the computational complexity.

I. INTRODUCTION

Noncoherent ultra-wideband (UWB) impulse radio systems are popularly used in the view of their limited complexity induced by obviating the complicated treatments on UWB channel, however, they suffer from a performance degradation caused by energy inefficiency and/or noisy template [1]. Recently, there has been increasing interest in using noncoherent multiple symbol detection (MSD) to detect differentially encoded UWB signal [2]–[4]. Although MSD scheme can obtain a solid performance improvement compared with symbol-by-symbol differential detection (DD), the complexity of MSD exponentially increasing with observation window size is intractable. The authors devise the application of sphere decoding (SD) algorithm to fulfill a low complexity MSD for UWB in [3]. Herein, taking into account the Binary Pulse Amplitude Modulation (BPAM) signaling format, reconstruction of MSD metric is performed to fit SD framework.

The noncoherent Multiple-Input Multiple-Output (MIMO) UWB system is proposed in [5], where the signal is encoded by Differential Space-Time Block Code (DSTBC) and a symbol-by-symbol DD scheme is employed to exploit both the multipath diversity and spatial diversity. This DSTBC-UWB system can provide performance improvement compared with differential SISO-UWB system. However, the DD scheme for DSTBC-UWB system has low energy efficiency because the information between the two sequential space-time symbols is only used.

In this paper, the DSTBC-UWB system is further investigated. In order to further enhance the system performance, the MSD scheme is extended to DSTBC-UWB system. Considering the specific signal structure of DSTBC-UWB system, the MSD metric is derived by generalized likelihood ratio testing (GLRT) approach. Nevertheless, the exhaustive search of the original MSD scheme involves prohibitive computational complexity when the observation window size becomes large.

To decrease the computational complexity, we resort to SD algorithm and transform the original MSD metric into an equivalent form which can be implemented by SD with a lower complexity. Since the model of SD in [3] is specific to differential SISO-UWB signaling, it cannot be employed for DSTBC-UWB system. The reconstruction model of SD metric proposed in this paper can be considered as a general approach. Although SD scheme can facilitate the implementation of MSD, its polynomial expected complexity could be unaffordable when observation window size is rather large. This motivates us to propose the suboptimal Decision-Feedback (DF) based MSD scheme which is much simper as well as more effective.

The rest of this paper is organized as follows. The system model of DSTBC-UWB system is described in Section II. In Section III, the proposed MSD scheme is developed by GLRT approach. The SD based MSD scheme is derived in section IV. The section V illustrates the DF based MSD scheme. Section VI shows simulation results. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

A. Transmit Signal Model

A peer to peer MIMO-UWB communication system includes a transmitter equipped with \( N_t \) antennas and a receiver equipped with \( N_r \) antennas. Being consistent with [5], we focus on the system with \( N_t = 2 \) transmit antennas which is desirable in the practical case. In the system DSTBC scheme is employed to encode the original information bit. The input bit sequence is divided into blocks of 2 bits. These bit blocks are fed into the encoder and the encoder maps each 2 bits into a space-time block codeword which is drawn from a codeword book

\[ \Omega = \{ U^{(1)}, U^{(2)}, U^{(3)}, U^{(4)} \} \]

where the codewords

\[ U^{(1)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad U^{(2)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad U^{(3)} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad U^{(4)} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \]

are constructed according to the property \( U^{(m)} U^{(m)^T} = \lambda I \) for all \( U^{(m)} \in \Omega \) [6], where \( \lambda \) is the transpose operator. When \( N_r > 2 \) we can construct codewords according to this property, hence the proposed scheme can be extended to system with larger \( N_r \). The bits-to-codeword mapping is Gray mapping such that \( 00 \rightarrow U^{(1)}, 01 \rightarrow U^{(2)}, 11 \rightarrow U^{(3)} \) and \( 10 \rightarrow U^{(4)} \). Then the information-bearing codeword is differentially encoded to obtain the transmitted symbol, this operation is expressed as

\[ D_k = D_{k-1} U_k, \quad k = 1, 2, \ldots, \infty \]

where the 2 × 2 matrix \( D_k \) is the \( k \)th transmitted symbol which will be transmitted over 2 transmit antennas during 2 frame durations, \( U_k \in \Omega \) is the \( k \)th information-bearing symbol.
(codeword). The initial transmitted symbol $D_0$ is the reference symbol which is set as

$$D_0 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}. \quad (3)$$

It is worth noting that $D_k$ meets the property

$$D_k D_k^T = D_k^k = 2I. \quad (4)$$

The $p^{th}$ row and $n^{th}$ column entry of $D_k$, denoted by $d_{p,2k+n}$, is transmitted by the $p^{th}$ transmit antenna during the $n^{th}$ frame duration of the $k^{th}$ transmitted symbol, where $p,n = 1, 2$. Hence, the signal radiated by the $p^{th}$ transmit antenna is given by

$$s_p(t) = \frac{E_b}{2} \sum_{k=0}^\infty \sum_{n=1}^2 d_{p,2k+n} \omega(t-(n-1)T_f-kT_s)$$

$$= \frac{E_b}{2} \sum_{j=1}^l d_{p,j} \omega(t-(j-1)T_f), \quad (5)$$

where $\omega(t)$ is the monopulse pulse waveform of duration $T_o$ with normalized energy, $T_f$ is the frame duration (pulse repetition interval), $T_s = 2T_f$ is the duration of one transmitted symbol, $E_b$ is the energy used to transmit one bit information and normalization factor 2 insures the same transmission power level as in the single-antenna case. Note the second equation of (5), where a single time index $j = 2k+n$ is introduced to replace the double index $(k,n)$, correspondingly, $d_{p,2k+n}$ is rewritten as $d_{p,j}$.

### B. Received Signal Model

The channel impulse response (CIR) between the $p^{th}$ transmit antenna and the $q^{th}$ receive antenna is modeled as

$$h_{q,p}(t) = \sum_{l=0}^{L-1} \alpha_{l,q,p}^T \delta(t - \tau_{l,q,p}^T), \quad (6)$$

where $\delta$ is the Dirac delta function, $L$ is the number of resolvable multipath components (MPCs), $\alpha_{l,q,p}^T$ and $\tau_{l,q,p}^T$ are the gain and delay of each MPC, respectively. The received signal at the $q^{th}$ receive antenna is given by

$$r_q(t) = \sum_{p=1}^2 s_p(t) \otimes h_{q,p}(t) + n_q(t)$$

$$= \frac{E_b}{2} \sum_{p=1}^2 \sum_{j=1}^l d_{p,j} g_{q,p}(t-(j-1)T_f) + n_q(t), \quad (7)$$

where $\otimes$ stands for convolution, $n_q(t)$ denotes zero mean AWGN process with two-sided power spectral density $N_0/2$, and $g_{q,p}(t)$ is the overall channel response between the $p^{th}$ transmit antenna and the $q^{th}$ receive antenna which can be defined as

$$g_{q,p}(t) = \omega(t) \otimes h_{q,p}(t) = \sum_{l=1}^L \alpha_{l,q,p}^T \omega(t - \tau_{l,q,p}^T). \quad (8)$$

With $T_o$ denoting the maximum delay spread of all $g_{q,p}(t)$, the Intersymbol Interference (ISI) is avoided by letting $T_f \geq T_o$. For noncoherent reception, all $g_{q,p}(t)$ are not known to the receiver.

### III. MSD for DSTBC-UWB

In this section, we derive MSD metric for DSTBC-UWB system by using GLRT optimization criterion. The length of observation window at the receiver is set to $M$ symbols duration, during which we assume that the channel remains invariant. Since the UWB channel is quasi-static in typical indoor environments [7], this assumption is justifiable. The objective of MSD is to jointly detect $M-1$ information-bering symbols over the observation of $M$ received symbols.

For notational simplicity but without loss of generality, we consider the case that recovering the first $M-1$ information-bering symbols which are given by a set $\{r_q(t)\}_{q=1}^{M-1},$ where $0 \leq t \leq MT_s$. Due to the differentially encoding operation (2), the detection of $U$ will involve the $M$ transmitted symbols $D = [D_0, D_1, \ldots, D_{M-1}]$ and $D$ is a function of $U$ ($D = D(U)$). The close analysis of the received signal at the $q^{th}$ antenna is performed to derive the detection metric for $r_q(t)$, and then the detection metrics of $N$ received signals are combined. The GLRT detection performs maximization of the log-likelihood metric

$$\Lambda \left(r_q(t) \mid D(U) = \{g_{q,p}(t)\}_{p=1}^2 \right) = \int_{-\infty}^{\infty} \int_0^{(M-1)T_s} r_q(t) \tilde{x}_q(t) dt - \int_{-\infty}^{\infty} \int_0^{(M-1)T_s} (\tilde{x}_q(t))^2 dt$$

not only over $D(U)$ but also over all infinite-energy functions $\{g_{q,p}(t)\}_{p=1}^2$ with assumed duration $T_I$. In (9), $\tilde{x}_q(t)$ is the received signal hypothesis

$$\tilde{x}_p(t) = \sqrt{E_b} \sum_{j=1}^{2M} \sum_{p=1}^2 \tilde{d}_{p,j} \tilde{g}_{q,p}(t-(j-1)T_f) + n_q(t)$$

corresponding to the trial symbols $\tilde{D} \{U\}$ and hypothetic finite-energy functions $\{\tilde{g}_{q,p}(t)\}_{p=1}^2$. Substituting (10) into (9), with some straightforward manipulations, (9) is rewritten as (11) which is shown at the top of the next page. The second equation in (11) is obtained by using the relationship

$$\sum_{j=1}^{2M} \tilde{d}_{1,j} \tilde{d}_{2,j} = 0, \quad (12)$$

which results from (4). It is noted in (11) that $\tilde{g}_{q,p}(t)$ is only interrelated with $\{\tilde{d}_{p,j}\}_{j=1}^{2M}$ and $\tilde{g}_{q,p}(t)$ do not correlate with each other. In (11), we define metric

$$\Lambda_p \left( r_q(t) \mid \tilde{d}_{p,j} = \{\tilde{d}_{p,j}\}_{j=1}^{2M}, \tilde{g}_{q,p}(t) \right)$$

which has the same format as the metric of differential SISO-UWB system. Therefore, the log-likelihood metric is reexpressed as the addition of two equivalent SISO metrics (the third equation of (11)). This expression will facilitate the solution to GLRT optimization problem of DSTBC-UWB system.

The GLRT based detection strategy on $D(U)$ is given by

$$\hat{D}(\hat{U}) = \arg\max_{\hat{D}(\hat{U})} \{ \Lambda \left( r_q(t) \mid D(U), \{g_{q,p}(t)\}_{p=1}^2 \right) \}$$

$$= \arg\max_{\hat{D}(\hat{U})} \left\{ \sum_{p=1}^2 \Lambda_p \left( r_q(t) \mid \tilde{d}_{p,j} = \{\tilde{d}_{p,j}\}_{j=1}^{2M}, \tilde{g}_{q,p}(t) \right) \right\}$$

which boils down to an addition of two segments. The $p^{th}$ segment is only associated to the channel $g_{q,p}(t)$. Therefore, we can employ the result for GLRT optimization problem of differential SISO-UWB system to each segment. First, we keep $D(U)$ fixed and find each $\tilde{g}_{q,p}(t)$ that maximize $\Lambda_p \left( r_q(t) \mid \tilde{d}_{p,j} = \{\tilde{d}_{p,j}\}_{j=1}^{2M}, \tilde{g}_{q,p}(t) \right)$, respectively. The optimum $\tilde{g}_{q,p}(t)$ is obtained by using variational techniques [3]

$$\tilde{g}_{q,p}(t) = \frac{1}{M \sqrt{E_b}} \sum_{j=1}^{2M} \tilde{d}_{p,j} r_q(t+(j-1)T_f). \quad (14)$$
with waveforms as 

According to the differential encoding operation (2) and code-

received signals. 

independent received signals, the detection strategy can be 

Then, substituting \( \bar{g}_{q,1} \) and \( \bar{g}_{q,2} \) into (11) and ignoring the 

irrelevant factor yield 

\[
\Lambda \left( r_q(t) \right) = \sum_{p=1}^{2} \lambda_p \sum_{j=1}^{2M} \frac{1}{\sqrt{E_b}} \int_0^{T_f} d_{p,j} \bar{g}_{q,p}(t) r_q(t + (j - 1) T_f) dt - M E_b \int_0^{T_f} \left( \sum_{p=1}^{2} \bar{g}_{q,p}(t) \right) dt
\]

\[
= \sum_{p=1}^{2} \left\{ \int_0^{T_f} \left( \sum_{j=1}^{2M} d_{p,j} \bar{g}_{q,p}(t) r_q(t + (j - 1) T_f) dt - M E_b \right) \right\}
\]

After that, maximization of (15) over all possible \( \bar{D}(U) \) di-

rectly give out the estimation of \( \bar{D}(U) \). Since the detection of 

information-bearing symbols \( U \) is the final objective, the rela-

tion between the detection metric and \( U_k \) should be revealed.

(15) can be simplified to a compact matrix form (see Appendix)

\[
\Lambda \left( r_q(t) \right) = \sum_{u=1}^{M-1} \sum_{v=0}^{u-1} \text{Tr} \left( \bar{D}_v^T \bar{D}_u \bar{R}_q^{u,v} \right)
\]

where \( \text{Tr} (\cdot) \) denotes the trace of a matrix and \( \bar{R}_q^{u,v} \) is a \( 2 \times 2 \) 

matrix whose entries are the results of correlation between the 

\( u^\text{th} \) received symbol waveform and the \( v^\text{th} \) received symbol 

waveform. Upon defining vector of received continuous time 

waveforms as \( \bar{r}_q(t) = [r_q(t + 2kT_f), r_q(t + (2k+1)T_f)] \),

\( \bar{R}_q^{u,v} \) can be expressed as

\[
\bar{R}_q^{u,v} = \int_0^{T_f} \left( \bar{r}_q(t) \right)^T \bar{r}_q(t) dt.
\]

According to the differential encoding operation (2) and code-

word property \( \bar{u}_k^T \bar{U} = 1 \), we have

\[
\bar{D}_v^T \bar{D}_u = \prod_{k=v+1}^u \bar{U}_k, \quad v < u.
\]

Substituting (18) into (16) and considering there are \( N_r \)

independent received signals, the detection strategy can be 

expressed as eventually

\[
\bar{U} = \arg \max_{\bar{U} \in \Omega^{M-1}} \Lambda \left( \left\{ r_q(t) \right\}_{q=1}^{N_r} \right) \quad \text{for} \quad \Lambda \left( \left\{ r_q(t) \right\}_{q=1}^{N_r} \right) \]

with

\[
\Lambda \left( \left\{ r_q(t) \right\}_{q=1}^{N_r} \right) = \sum_{u=1}^{M-1} \sum_{v=0}^{u-1} \text{Tr} \left( \prod_{k=v+1}^u \bar{U}_k \right) \bar{R}_{u,v},
\]

where \( \bar{R}_{u,v} = \sum_{q=1}^{N_r} \bar{R}_q^{u,v} \) is the result of combining all \( N_r \)

received signals.

When \( M = 2 \), (19) degenerates into DD scheme for DSBSC-

UWB system which also appears in [5]. As \( M \) increases, 

the proposed MSD scheme can offer an additional detection 

gain compared with DD. However, the detection strategy (19) 
is maximum likelihood (ML) sequence detection that has 
an exhaustive search process, and the size of search space 
grows exponentially with \( M - 1 \). When \( M \) gets large, the 
computational complexity becomes intolerable. In next section, 
we develop SD based MSD for DSTBC-UWB to reduce the 
computational complexity.

IV. SD BASED MSD FOR DSTBC-UWB

Due to the simple search of lattice points within a sphere of 

radius \( C \), SD has polynomial expected complexity, and 
can achieve or approximate ML performance. However, 
the existing SD scheme [8] cannot be straightforward applied 
to the proposed MSD for DSTBC-UWB. Thus, we must make some 
equivalent reconstructions on the detection strategy (19) to 
enable SD. The proposed reconstruction model of SD metric 
is more general than the one used in [3] which is specific to 
BPAM signaling format. In this section, \( \Lambda \left( \left\{ r_q(t) \right\}_{q=1}^{N_r} \right) \)

denoted as \( \Lambda \left( \bar{U} \right) \) for notational simplicity.

Since SD minimizes the distance between given point and 
all possible lattice points, our detection strategy must transform 
to a minimization problem. The detection strategy described by 
(19) is equivalent to

\[
\bar{U} = \arg \min_{\bar{U} \in \Omega^{M-1}} \Lambda \left( \bar{U} \right)
\]

where \( \Lambda \left( \bar{U} \right) = -\Lambda \left( \bar{U} \right) \). The possible negative values 
of \( \Lambda \left( \bar{U} \right) \) do not satisfy the property of distance which is 
minimized in SD. We construct a new detection metric as

\[
\Phi \left( \bar{U} \right) = \Lambda \left( \bar{U} \right) + \sum_{u=1}^{M-1} \sum_{v=0}^{u-1} \text{Tr} \left( I \left( \bar{R}_{u,v} \right) \right)
\]

\[
= \sum_{u=1}^{M-1} \sum_{v=0}^{u-1} \text{Tr} \left( I \left( \bar{R}_{u,v} \right) - \left( \prod_{k=v+1}^u \bar{U}_k \right) \bar{R}_{u,v} \right) \geq 0
\]

where \( I \) is the \( 2 \times 2 \) matrix whose entries are all 1, \( \bar{R}_{u,v} \)
is the \( 2 \times 2 \) matrix whose entry is the absolute value of the 
corresponding entry in \( \bar{R}_{u,v} \). Then, SD examines those points 
that lie inside a sphere with radius \( C \):

\[
\Phi \left( \bar{U} \right) \leq C.
\]

In order to implement SD, the detection metric accounting for 
the first \( i \) symbols \( \bar{U}_1, \bar{U}_2, \cdots, \bar{U}_i \) is given by

\[
\Phi_i = \sum_{u=1}^i \sum_{v=0}^{u-1} \text{Tr} \left( I \left( \bar{R}_{u,v} \right) - \left( \prod_{k=v+1}^u \bar{U}_k \right) \bar{R}_{u,v} \right),
\]

and bearing in mind that \( \Phi_0 = 0, \Phi_{M-1} = \Phi \left( \bar{U} \right) \). The 
relationship between \( \Phi_{i-1} \) and \( \Phi_i \) has a recursive form

\[
\Phi_i = \Phi_{i-1} + \lambda_i,
\]
where
\[
\lambda_i = \sum_{v=0}^{i-1} \text{Tr} \left( I (R_{u,v}) - \left( \prod_{k=v+1}^{M+1} \tilde{U}_k \right) R_{i,v} \right).
\] (26)

Now, the necessary condition for (23) can be expressed as
\[
\Phi_i \leq C, \quad 0 \leq i \leq M - 1.
\] (27)

Since the computation of \( \Phi_i \) only involves first \( i \) symbols and \( \Phi_i \) is nonnegative, SD algorithm can check the condition in (27) one by one and the detection metrics are recursively computed according to (25). The proposed SD based MSD (SDMSD) for DSTBC-UWB is detailed as a pseudocode form.

### Table I
**SDMSD for DSTBC-UWB**

<table>
<thead>
<tr>
<th>Function SDMSD_DSTBC_UWB (Ω, M, {R_{u,v}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( C = \infty ) / initial radius /</td>
</tr>
<tr>
<td>2. ( t = 1 ) / search starts from the 1st layer /</td>
</tr>
<tr>
<td>3. ( m_i = 0 ) / the index of codeword /</td>
</tr>
<tr>
<td>4. ( q_i = 0 )</td>
</tr>
<tr>
<td>5. &lt; loop &gt;</td>
</tr>
<tr>
<td>6. ( m_i = m_{i+1} ) / chose the next codeword from ( \Omega ) /</td>
</tr>
<tr>
<td>7. ( \tilde{U}_i = \tilde{U}^{(m_i)} )</td>
</tr>
<tr>
<td>8. if ( m_i \leq 4 ) / the finite codebook size is considered /</td>
</tr>
<tr>
<td>9. ( \Phi_i = \Phi_{i-1} + \lambda_i ) / compute ( \Phi_i ) according to (25) /</td>
</tr>
<tr>
<td>10. if ( \Phi_i &lt; C )</td>
</tr>
<tr>
<td>11. ( t = i + 1 ) / move to the next layer /</td>
</tr>
<tr>
<td>12. if ( i &lt; M - 1 )</td>
</tr>
<tr>
<td>13. ( \tilde{U} = \tilde{U} ) / the best point so far /</td>
</tr>
<tr>
<td>14. ( C = \Phi_{M-1} ) / update radius /</td>
</tr>
<tr>
<td>15. end</td>
</tr>
<tr>
<td>16. else</td>
</tr>
<tr>
<td>17. ( i = i ) / stay this layer for the next codeword /</td>
</tr>
<tr>
<td>18. end</td>
</tr>
<tr>
<td>19. if ( i = 1 ) / search ends /</td>
</tr>
<tr>
<td>20. break</td>
</tr>
<tr>
<td>21. else</td>
</tr>
<tr>
<td>22. ( \tilde{U} = \tilde{U} ) / move back to the prior layer /</td>
</tr>
<tr>
<td>23. end</td>
</tr>
<tr>
<td>24. end</td>
</tr>
<tr>
<td>25. end</td>
</tr>
<tr>
<td>26. goto &lt; loop &gt;</td>
</tr>
</tbody>
</table>

### V. DF based MSD for DSTBC-UWB

In this section, the DF based MSD (DFMSD) with lower complexity than SDMSD is proposed to further reduce the computational complexity of MSD for DSTBC-UWB. The notion of DFMSD is to detect the \( k^{th} \) information-bearing symbol \( \tilde{U}_k \) by substituting the estimate of previous \( M - 2 \) symbols, \( \tilde{U}_{k-M+2}, \tilde{U}_{k-M+1}, \ldots, \tilde{U}_{k-1} \), into the original MSD metric (19). The DFMSD scheme for DSTBC-UWB is formalized as
\[
\hat{U}_k = \arg \max_{U_k} \Lambda \left( \tilde{U}_k \right)
\] (28)

with
\[
\Lambda \left( \tilde{U}_k \right) = \sum_{v=k-M+1}^{k-1} \text{Tr} \left( \left( \prod_{\mu=v+1}^{M+1} \tilde{U}_\mu \right) \tilde{U}_k R_{k,v} \right).
\] (29)

The observation window size of DFMSD still preserves \( M \), however, the observation window slides one symbol duration down each time. Since one codeword is detected each time, the size of search space is reduced (equals to the size of codeword book). Thus, the DFMSD has the lowest complexity than original MSD and SDMSD. However, additional analog delay lines with longer delays is required for DFMSD scheme compared with symbol-by-symbol DD scheme. Its performance is validated by simulations in next section.

### VI. Simulation Results

In this section, simulations and comparisons are conducted to validate the proposed schemes. In all cases, the MIMO-UWB channels are generated according to IEEE 802.15.3a CM2 model [7]. The used impulse shape is the second derivative of a Gaussian function \( \omega(t) = \frac{A_H}{\sqrt{\pi}} \exp \left( -2\pi \left( \frac{t}{\tau_m} \right)^2 \right) \), where \( A_H \) is the energy normalized parameter and \( \tau_m = 0.2877 \) ns is the pulse shaping parameter. And its duration is \( \tau_T = 0.5 \) ns. We assume there is no ISI in the system, thus, the frame duration \( \tau_T \) is set to 100 ns which is larger than the maximum excess delay of the channel. The integration interval is \( \tau_T = 20 \) ns.

**TEST 1: The BER performance of SDMSD for DSTBC-UWB system.**

The bit error rate (BER) of SDMSD is investigated. The BER performances of ideal Rake reception for Alamuti space-time coded MIMO-UWB system [9] and DD scheme for SISO-UWB [10] system are also evaluated and plotted as the performance benchmarks. Fig.1 shows that the performance of DSTBC-UWB system outperforms that of differential SISO-UWB system. Compared with DD for DSTBC-UWB [5], the proposed SDMSD scheme can further improve the performance of DSTBC-UWB system by offering an additional detection gain which increases with \( M \). We can observe that the performance of proposed scheme can get closer to that of Rake receiver at the price of more computational complexity. The proposed scheme with \( N_t = 2, N_r = 2 \) and \( M = 10 \) even outperforms the ideal Rake for SISO-UWB system when \( E_b/N_0 \geq 14 \) dB. Thus, the lost performance due to lack of channel state information (CSI) can be partially recovered by the proposed scheme with more antennas and more computational complexity.

**TEST 2: The BER performance of DFMSD for DSTBC-UWB system.**

In this case, we test the BER performance of DSTBC-UWB systems with DFMSD scheme. All systems are equipped with \( N_t = 2, N_r = 1 \) antennas. The results are shown in Fig. 2. We can find that DFMSD almost have the same BER with SDMSD except that DFMSD slightly underperforms SDMSD when \( E_b/N_0 \) is small. This slight performance gap is due to the error propagation effect of DFMSD at low \( E_b/N_0 \). Since the search space of DFMSD dose not increase with observation window size \( M \), we can increase \( M \) to enable the BER get...
close to ideal Rake reception. The performance gap between the proposed DFMSD for DSTBC-UWB with \( M = 20 \) and ideal Rake reception for Alamouti space-time coded MIMO-UWB is within 3 dB at BER=10\(^{-6}\).

VII. CONCLUSION

Considering the specific signal format of DSTBC-UWB system, we derive the MSD metric for DSTBC-UWB based on GLRT optimization criterion. The resultant MSD metric is an addition of equivalent MSD metrics for SISO-UWB system, moreover, it is eventually formalized to a compact matrix form by employing the property of the used space-time codeword. In order to implement our proposal even with large observation window size, we reformulate the MSD metric and apply SD and DF to it, respectively. The SDMSD scheme is developed by a general method which is divided into two steps: the transformation to minimization problem and the adjustment to satisfy nonnegativity. The DFMSD has the lowest complexity than original MSD and SDMSD and it is also beneficial to the BER performance of DSTBC-UWB system. The simulations validate our proposals.

APPENDIX

Expanding (15) and using the fact that \( \tilde{d}_{p,j} \) take values in \{±1\} to drop the item independent on \( d_{p,j} \), we have

\[
\Lambda \left( r_q (t) \mid \tilde{D} \left( \overline{U} \right) \right) = \sum_{p=1}^{2M} \sum_{j=2}^{M-1} \sum_{i=1}^{2} \tilde{d}_{p,j} \tilde{d}_{p,i} \\
\times \int_{0}^{T_f} r^q (t + (j - 1) T_f) r^q (t + (i - 1) T_f) \, dt.
\]

(30)

According to the parity of index \( j \), the items in (30) are classified as

\[
\Lambda \left( r_q (t) \mid \tilde{D} \left( \overline{U} \right) \right) = I_{even} + I_{odd}.
\]

(31)

where

\[
I_{even} = \sum_{p=1}^{M-1} \sum_{u=1}^{2} \sum_{i=1}^{2} \tilde{d}_{p,2u} + \tilde{d}_{p,2u-1} \\
\times \int_{0}^{T_f} r^q (t + (2u + 1) T_f) r^q (t + (i - 1) T_f) \, dt.
\]

(32)

\[
I_{odd} = \sum_{p=1}^{M-1} \sum_{u=1}^{2} \sum_{i=1}^{2} \tilde{d}_{p,2u+1} + \tilde{d}_{p,2u} \\
\times \int_{0}^{T_f} r^q (t + (2u + 2) T_f) r^q (t + (i - 1) T_f) \, dt.
\]

(33)

Note that the transmitted symbol meets

\[
\sum_{p=1}^{2} \tilde{d}_{p,2k+2} \tilde{d}_{p,2k+1} = 0,
\]

(34)

which results from (4). Taking (34) into consideration, (32) can be rewritten as

\[
I_{even} = \sum_{p=1}^{M-1} \sum_{u=1}^{2} \sum_{i=1}^{2} \tilde{d}_{p,2u+2} \tilde{d}_{p,i} \\
\times \int_{0}^{T_f} r^q (t + (2u + 1) T_f) r^q (t + (i - 1) T_f) \, dt.
\]

(35)

Substituting (35), (33) into (31) and then letting \( d_{p,i} = d_{p,2u+n} \)

\((u = 1, 2, \cdots, u - 1 \text{ and } u = 1, 2)\), we have

\[
\Lambda \left( r_q (t) \mid \tilde{D} \left( \overline{U} \right) \right) = \sum_{u=1}^{M-1} \sum_{n=1}^{2} \sum_{n'=1}^{2} \sum_{v=0}^{n-1} x \sum_{n''=1}^{n} \sum_{u=1}^{M-1} \sum_{n''=1}^{2} \\
\times \int_{0}^{T_f} r^q (t + (2u + n') T_f) r^q (t + (i - 1) T_f) \, dt.
\]

(36)

Using \( R_{q,v}^u \) defined in (17), we may reexpress (36) as a compact matrix form

\[
\Lambda \left( r_q (t) \mid \tilde{D} \left( \overline{U} \right) \right) = \sum_{u=1}^{M-1} \sum_{v=0}^{2} \sum_{n=1}^{2} \sum_{n'=1}^{2} \sum_{n''=1}^{n-1} \sum_{v'=1}^{n''-1} \\
\times \int_{0}^{T_f} r^q (t + (2u + n') T_f) r^q (t + (i - 1) T_f) \, dt.
\]

(37)

where \( 1 = \{1, 1\}^T \) is the 2 × 1 summation vector, \( \circ \) is the Hadamard product. Using the relationship between Hadamard product and matrix trace \[11\]

\[
1^T (\tilde{D}^T \tilde{D}_u \circ R_{q,v}^u) = \text{Tr} \left( \tilde{D}^T \tilde{D}_u R_{q,v}^u \right).
\]

(38)

(37) is reformulated as (16).

REFERENCES