EARLY WARNINGS IN ECOSYSTEMS: COMPARISON BETWEEN DIFFERENT INDICATORS

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The task of providing early indicators of catastrophic regime shifts in ecosystems is fundamental in order to design management protocols for those systems. Here we address the problem of lake eutrophication (the overenrichment with nutrients leading to algal blooms) using a simple spatial model. We discuss and compare different spatial and temporal early signals announcing these catastrophic events. In particular we consider the spatial standard deviation and its associated patch structure of turbid water regions. The patch sizes exhibit a power law distribution when the lake is close to the eutrophic transition. We also analyse the spatial and temporal early warnings in terms of the amount of information required by each and their respective forewarning times. We then provide a link between spatial and temporal indicators and their interplay. From the consideration of different remedial procedures than can be followed after these early signals we conclude that some of these indicators are, unfortunately, not early enough to avoid the undesired shift to the eutrophic state.

Keywords: Alternate stable states; regime shift; spatial early warnings.

1. Introduction

The problem of providing early warning signals of catastrophic shifts in ecosystems has been recently addressed by different methods. On the one hand, lake eutrophication [Carpenter & Brock, 2006] and pollutants [Brock & Carpenter, 2006] have been approached using minimal models. The procedure employed consists in using time series to construct indicators that rise sharply in advance of regime shifts. More specifically, the temporal variance rises prior to a catastrophic regime shift. No specific knowledge of the mechanisms underlying the regime shift is needed to construct the indicator. On the other hand, the problem of desertification in arid ecosystems and how vegetation patchiness changes with different grazing pressures was analysed using a cellular automaton model [Kéfi et al., 2007]. The conclusion of their work was that patch-size distributions may be a warning signal for the onset of desertification.

Water quality of lakes and reservoirs provide a well-studied example [Scheffer, 1998; Carpenter et al., 1999; Ludwig et al., 2003; Carpenter, 2003], on which we focus the present work. Our ultimate aim here is to provide qualitative as well as quantitative tools both to anticipate catastrophic changes and to help design management protocols for a number of similar ecosystems. In the process of searching for the most appropriate indicators for those shifts, we will discuss different spatial and temporal early signals of these events. In fact, this serves to link both kind of indicators.

In section II we describe the model we employ to study the signals providing early warning of abrupt changes in the quality of the lake ecosystem. The results obtained with this model are discussed in section III, where a detailed comparison between the early signals of spatial and temporal analyses is performed. Finally in section IV we present our conclusions and suggestions for other ecosystems where these signals could be used to anticipate dramatic changes.

2. The Model

We consider a two dimensional model of a lake that is the spatial version of the mean field model of [Carpenter et al., 1999], describing the change over time of some property, $s$, that characterizes its state (for example the water turbidity). We represent the lake by a square lattice of $L \times L$ sites identified by their coordinates $(x, y)$. Obviously lakes of arbitrary shape can be studied by embedding them into a square lattice like the one above, with appropriate boundary conditions. Similarly, we could generalize the problem to the case where several quantities determine the status of the lake by considering that $s$ is a vector. In what follows, for simplicity, we consider the simple schematic square shape and that a single variable $s$ is sufficient to represent the lake status. The evolution equation for the quantity $s$ is then given by:

$$\frac{\partial s(x, y; t)}{\partial t} = a(x, y; t) - bs(x, y; t) + r f[s(x, y; t)] + D \nabla^2 s(x, y; t),$$

(1)

where $a(x, y; t)$ represents an environmental factor that promotes $s$, for instance the phosphorus loading rate, varying both from point to point and in time, $b$ represents the rate at which $s$ decays in the system, i.e. the nutrient removal rate, $r$ is the rate at which $s$ recovers, i.e. a recycling parameter, and $f$ is a Hill function:
1. The time series produced by the model: \( f(s) = s^a/(s^a + h^a) \). We have also included a diffusion term as in [van Ness and Scheffer, 2005] with diffusion coefficient \( D \).

We have taken the same parameter value set as in [van Ness and Scheffer, 2005], namely: \( b = r = h = 1 \), the only difference being that, instead of \( q = 4 \) we take \( q = 8 \), as in [Carpenter, 2005]. For the diffusion coefficient we consider three values: \( D = 0.0, 0.1 \) and \( 1 \). We consider that at each time \( t \), \( a(x, y; t) \) fluctuates around an average value \( \bar{a}(t) \) in the interval \( [\bar{a}(t) - \Delta, \bar{a}(t) + \Delta] \) where \( \Delta \) represents the effect of mechanical stirring of the lake waters (wind, currents, animals). We have taken \( \Delta = 0.125 \) and have verified that the results do not depend much on this value. Furthermore, we have assumed that \( \bar{a}(t) \) varies in steps of \( \delta a = 0.001 \) per time step. This value of \( \delta a \) was estimated from [Carpenter, 2005] to approximately represent one year in the evolution of lake Mendota, in Wisconsin.

In order to make quantitative comparisons between the different signals, we calculate the following quantities from the time series produced by the model:

1. The spatial variance of \( s(x, y; t) \), \( \sigma_s^2 \), defined as

\[
\sigma_s^2 = \langle s^2 \rangle - \langle s \rangle^2 = \frac{\sum_{x,y=1}^{L} s(x, y; t)^2 - \left( \sum_{x,y=1}^{L} s(x, y; t) \right)^2}{L^2},
\]

where \( \langle s \rangle \) stands for the spatial average of the variable \( s \). Note that this quantity requires knowledge of the status of the lake at all points in a grid covering it. Later we will show that such detailed information is not required to make accurate predictions on the times at which the catastrophic changes are expected to occur.

2. Similarly to non-spatial models, the temporal variance \( \sigma_t^2 \), at an arbitrary point, say \( (x, y) = (0, 0) \), is defined as

\[
\sigma_t^2 = \langle s(0, 0; t)^2 \rangle - \langle s(0, 0; t) \rangle^2 = \frac{\sum_{t=t-\tau}^{t} s(0, 0; t)^2 - \left( \sum_{t=t-\tau}^{t} s(0, 0; t) \right)^2}{\tau}
\]

for temporal bins of size \( \tau \) as \( a(t) \) is varied. In this case the information required is restricted to only one place in the lake, but taken over a period of time.

3. The patchiness or cluster structure, which is very illuminating to understand the behavior of \( \sigma_s \). Clusters of high (low) \( s \) are defined as those connected regions of sites \( (x, y) \) such that

\[ s(x, y) > \langle s \rangle; \quad s(x, y) < \langle s \rangle, \]

respectively. If the low and high values have different visual properties, for example, these regions can be easily detected.

4. Spatial correlations, in particular the two-point correlation function of the values of \( s \) for pairs of cells at \((x_1, y_1)\) and \((x_2, y_2)\), separated a given distance \( d \), which is given by

\[
C_2(d) = \langle s(x_1, y_1)s(x_2, y_2) \rangle - \langle s(x_1, y_1) \rangle \langle s(x_2, y_2) \rangle
\]

requires knowledge of the value of the measured quantity at several pairs of positions in the lake.

In the following section we look at the results obtained using this model, in particular at the evolution of the measured observable \( \langle s(t) \rangle \), and at the four quantities defined above.

3. Results

In accordance with previous studies, we have found that, as \( a(t) \) goes over a critical value, which lies around 0.63 for our set of parameters, there is a sharp transition in the spatial average of the measured quantity, \( \langle s(t) \rangle \), as seen in fig. 1. As it has been noted before, once the system has undergone the transition, it is difficult to return it to its original, non turbid, state, as there is a hysteresis loop, clearly depicted in this figure.

![FIG. 1: Average value of the measured observable, \( \langle s(t) \rangle \), as a function of the average nutrient input rate, \( a(t) \).](image)

In the case of non spatial models, it has been pointed out that this transition is accompanied by a peak in the temporal variance of \( s \) [Brock & Carpenter, 2006; Carpenter & Brock, 2006]. We have checked that our model reproduces this behavior and will be illustrated later.

Fig. 2 is a plot of both \( \langle s(t) \rangle \), and \( \sigma_s \). It shows that the spatial variance \( \sigma_s \) is also an early signal: it increases by 0.015 around \( a(t) \approx 0.63 \). In other words, since \( \delta a = 0.001 \) per year, the difference of 0.015 corresponds to a forewarning of approximately 15 years. We should also note that \( \sigma_s \) peaks at \( a(t) = 0.644 \) while the lake is still in a mixed state, before reaching the alternative state at \( a(t) \approx 0.66 \). It is possible to make a preliminary study of the consequences of different remedial actions after the early signal discussed above. In fig. 3 we show that, if the increase in...
FIG. 2: $\langle s(t) \rangle$ (blue) and $\sigma_s$ (green), as a function of the average nutrient input rate, $\bar{a}(t)$.

FIG. 3: $\langle s(t) \rangle$ for two remedial actions: decrease the average nutrient input rate, $\bar{a}(t)$ after reaching the load that fires the early warning (full line) or keep it constant (dashed line).

$a(t)$ is stopped immediately, the growth in $\langle s(t) \rangle$ continues and the lake passes to the alternative state. If a much more drastic action is taken, reducing $a(t)$ at the same rate it was increasing until the detection of the warning signal, then $\langle s(t) \rangle$ also starts decreasing. However, it follows a path close, but not exactly the same as when it was increasing, so it remains in a mixed state for a long time. This shows that an even more drastic course of action might be needed to take the lake to a safe situation. Therefore this early warning signal just represents a time where it is still possible to act, not when the situation is easily reversible.

In order to assess the practical difficulty of estimating $\sigma_s$, we have performed calculations over different grid sizes. In fig. 4 we notice that the signal does not depend qualitatively on the number of points on the grid that are considered in order to estimate $\sigma_s$.

In fig. 5 $\sigma_s$ is compared to $\sigma_t$ in the region close to the catastrophic transition, and we notice that it rises earlier, so it works better than $\sigma_t$ as a warning signal for the upcoming transition in $s(t)$.

The reason for this is clear. When estimating the temporal variance one must consider past values in the time series, which correspond to situations where the lake is far from undergoing a transition. The spatial variance considers only the present values, so if a signal announcing a change is present, it is not obscured by averaging it with data where these indications are not present.

It is also interesting to find the reasons leading to the rise in $\sigma_s$ that are the basis for this early warning signal. As we notice in fig. 6, $s(x, y; t)$ starts developing spatial fluctuations around this time, that take the form of a distribution of clusters.

The appearance of those clusters of turbid water might therefore be considered as an alternative spatial early signal for an impending catastrophic shift in the status of the lake. It is remarkable that this cluster distribution follows a power law, as illustrated in fig. 7 for the value $\bar{a}(t) = 0.644$ (at the peak of both $\sigma_s$ and $\sigma_t$). For $\bar{a}(t) = 0.624$, i.e. when $\sigma_s$ has about doubled its initial value, but is still only around a quarter of its maximum value, the distribution is not a power law but resembles an exponential.

It is clear that the spatial heterogeneities that precede the catastrophic transition can be visualized in other ways.
For instance, the two point correlation function $C_2(d)$ is plotted in fig. 8 for the same times considered in fig. 4.

Notice that the correlation found at the peak of $\sigma_s$ ($t = 269$) completely disappears 50 years before or after the maximum in the spatial standard deviation.

4. Conclusions
We have considered several possible early warnings for eutrophication transitions in lakes. To do this we have considered a model that describes the evolution of an observable $s$ in time and in points on a grid on the surface of the lake. The observable $s$ considered was the Phosphorus concentration, which dominates the eutrophication process in lakes. As a first step we have checked that the behavior of the temporal standard deviation $\sigma_t$, which has been suggested [Brock & Carpenter, 2006; Carpen-
As an early signal, is confirmed by our model. We have then considered the spatial $\sigma_s$ by measuring samples of $s$ on a grid of points on the lake surface. It was found that a grid containing few points might be sufficient for the purpose of extracting an appropriate signal, and that a significant growth in $\sigma_s$ could serve as an early warning of an imminent transition. The spatial variance appears to have an advantage over the temporal one, as $\sigma_t$ appears to be delayed with respect to $\sigma_s$, since the former employs data in times where the fluctuations are still small.

When studying the origin of the rise in $\sigma_s$, we found that it is due to the appearance of spatial patterns, in the form of clusters of clear and turbid water. Then concluded that the visualization of the onset of those patches, for example by aerial or satellite imaging of the lake surface, may be an effective way of anticipating an eutrophication transition. However, it appears that, unfortunately, all these warning signals considered are not early enough. They do not provide enough anticipation to the upcoming changes in the lake status that could make simple measures effective to avoid them from occurring. It turns out that, in general, when the early warning is released, the lake could be in a situation where very drastic actions might be required in order to avoid a shift to the catastrophic alternative state. The decision to enter into a water quality management program, with its associated costs, could be based on economic consideration, which we do not discuss here. It is worth to remark that the quantitative details of our conclusions depend on the choice of parameter values employed in our model. We have verified that the qualitative behavior of our results do not depend strongly on those values. Furthermore our model as presented in this work is schematic in the sense that the quality of the lake’s water is dependent of a single parameter, the amount of Phosphorus in solution, in our case. In real cases other environmental factors might be playing an important role in the evolution, and the model and its predictions could be much more complicated. Nevertheless it appears that our main conclusions should hold in the more realistic case: spatial variance of critical quantities is an earlier signal than the temporal variance, and its associated cluster structure of the patterns formed in the eutrophication process could be the fastest detectable warning that a catastrophic change is about to occur. It seems plausible that the range of applications of our model could be extended to other ecosystems. In fact the appearance of characteristic patterns of vegetation, in the form of a power law of the patch size distribution, has been considered by [Kéfi et al., 2007] as a warning signal of imminent desertification. This suggests that a model similar to the one presented here, which also leads to power laws in the distribution of sizes of lake regions with turbid water, might be amenable to describe the desertification process.

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References


