A Nonlinear Harmonic Model for Fitting Satellite Images Time Series: Analysis and Prediction of Land Cover Dynamics

Hugo Carrão, Paulo Gonçalves and Mario Caetano

Abstract – Numerous efforts have been made to develop models to fit multispectral reflectances and vegetation indices (VI) time series from satellite images for diverse land cover classes. The common objective of these models is to derive a set of measurable parameters able to characterize and to reproduce the land cover dynamics of natural- and human-induced ecosystems. Good-fitting models should therefore match different waveforms and be insensitive to sharp and localized variations, generally due to atmospheric disturbances. In this paper we propose a model-based approach to identify and predict important dynamics for indiscriminate land cover classes. Our method relies on an original nonlinear harmonic model that remarkably matches intra-annual time series of multispectral reflectances and VIs obtained from satellite images. The proposed model is (i) parsimonious, comprising only 5 parameters; (ii) readily identifiable (in the maximum likelihood sense) from only few observations; (iii) robust to noise; and (iv) versatile, since it can reproduce a wide variety of intra-annual land cover dynamics as a deterministic function of time. To demonstrate the relevance of our approach, we use a time series of MODIS 8-day composite images acquired in Portugal over a one-year period at a 500 m nominal spatial resolution. For 13 different land cover classes, representative of Mediterranean landscapes, we evaluate the data-model adequacy of our model and compare it with several other approaches. We then address a particularly interesting and promising application of our method using rice crops and shrublands as case studies. We not only show that phenological attributes can be accurately estimated from the fitted time series, but we also demonstrate that it is possible to make early predictions of phenological attributes dates and magnitudes from our expected model adjusted to only few anterior observations.

Index Terms – Curve fitting, intra-annual dynamics, land cover, nonlinear harmonic model, phenology, prediction, remote sensing, time series.

I. INTRODUCTION

The spectral, temporal and spatial information contained in satellite images time series offers unique opportunities for monitoring and managing land cover dynamics from local to global spatial scales [1]. Because satellite images time series provide temporal and spatial consistent spectral measurements of Earth’s surface that correlate directly with annual cycles of vegetation growth, i.e. phenology, they offer a significant insight into the response of vegetation to short-term environmental forcing effects, both natural and anthropogenic [2], [3]. Nonetheless, to effectively track continuous intra-annual dynamics of land surface, such as vegetation phenology, error-free remotely-sensed data acquired at appropriate time instants are
stringent conditions rarely met in practice. Indeed, satellite images time series correspond to discrete, possibly non-uniformly sampled measurements of spectral variations, which moreover, can be corrupted by atmospheric, geometric and radiometric disturbances [4]. Therefore, to reduce imprecision on phenological attributes estimation, we must resort to denoising procedures and fill the gaps between satellite images acquisition dates with a pertinent continuous interpolating function [4]. Regarding noise, smoothing techniques (e.g. running averages, running medians or compound smoothers) are commonly used to suppress local artifacts in time series [5], [6]. As for interpolation, more sophisticated mathematical models make it possible to estimate phenological attributes from the functional analysis of continuous time evolutions of land cover spectral responses, e.g. [3], [5], [7]-[10]. Indeed, these functions increase the spatial and temporal consistency of satellite images time series used to derive phenological attributes [11]. Nevertheless, remote sensing estimations of phenological markers are available only at images acquisition dates [4], thus delaying the identification of land cover dynamics and the mitigation of negative stresses on the environment.

In this paper we propose a model-based approach for the identification and prediction of phenological attributes from satellite images time series. Our main goal is to derive phenological attributes in early time to serve as input for decision systems (e.g. forest fire prevention and mitigation operational systems, such as WFAS [12] in the US and PREMFIRE [13] in the EU) and support policy makers within their decision-making frameworks (e.g. agricultural programs, such as the Monitoring Agriculture with Remote Sensing (MARS) programme of the EU [14] and the Agriculture and Resources Inventory Surveys Through Aerospace Remote Sensing (AgRISTARS) in the US [15]). We believe this is an innovative approach: while previous studies have focused only on the identification of phenological key-attributes by fitting the entire annual time series, what we address here is the early prediction of phenological attributes from few anterior observations. Our method relies on an original parsimonious nonlinear harmonic model that we previously proposed [16] to match intra-annual time series of multispectral reflectances and VIs obtained from satellite images. The paper is organized as follows: in Section II, after listing most standard models for fitting satellite images time series, we present the analytic form of our nonlinear model and discuss its identifiability properties. In Section III, we test the robustness and the flexibility of this parametric model on MODIS time series corresponding to multispectral reflectance and VIs measurements acquired in Portugal for 13 land cover classes with distinct intra-annual variations. In this Section, we also focus on a particularly attractive application. From the modelled time series, we identify and predict phenological key-markers that are highly relevant for the underlying land cover classes. We end in Section IV, with concluding remarks and future work.

II. SATELLITE IMAGES TIME SERIES FITTING

Before elaborating on our original model, we start this Section with a brief survey of some of the most standard approaches currently used for modelling satellite images time series. We stress the main advantages and weaknesses of each of these models.
A. State-of-the-art

Because annual phenologies of vegetation have seasonal paths, simple harmonic oscillators and Fourier-series are among the principal models used to fit satellite image time series [3], [10]. However, while simple harmonic oscillators fail at reproducing nonlinear waveforms, Fourier series approaches implicitly assume regularly spaced time series and underlying components that are truly harmonic [3], [5], [11]. Thus, these functions are too constrained to satisfactorily model multispectral reflectances for a wide range of land cover classes. To overcome these difficulties, refined piecewise-defined functions were recently proposed for fitting land cover time series derived from satellite images.

References [5] and [17] presented a new method based on piecewise-defined local Asymmetric Gaussian (AG) functions for extracting seasonality information from NDVI time series data. In this algorithm, local model functions with seven parameters are fitted to data in intervals around maxima and minima in the time series and used to build a global function that describe the annual cycles of vegetation. In the same vein, [9], [11], and [18] proposed piecewise-defined local Double Logistic (DL) functions to identify land cover time trajectories from VIs time series. The local model functions rely only on six parameters, but have the same general form of the former and can also be merged to build a global function that describe the annual seasons of land cover types. Generically, the global AG and DL models functions have the form:

\[ f(t) = \sum_{h=1}^{H} c_{1,h} + \sum_{h=1}^{H} c_{2,h} g(t; A_h) \]  

where \( H \) designates the number of local model functions, i.e. the number of extrema values (minima and maxima) of the time series to be modelled. The linear parameters \( c_{1,h} \) and \( c_{2,h} \) determine, respectively, the base level and the amplitude of the local function \( h \), whereas the nonlinear meta-parameter \( A_h = (a_1, \ldots, a_5) \), determines its shape. The AG type basis function has the form [5]:

\[ g(t; a_1, \ldots, a_5) = \begin{cases} \exp \left[ \frac{t-a_1}{a_3} \right] & \text{if } t > a_1 \\ \exp \left[ \frac{a_1-t}{a_4} \right] & \text{if } t < a_1 \end{cases} \]  

In this function, \( a_1 \) determines the position of the maximum (or minimum) of \( g \); \( a_2 \) and \( a_3 \) (respectively \( a_4 \) and \( a_5 \)) determine the width and flatness of the right (respectively left) half of the function. Accordingly, the AG global model has a total of \( H \times 7 \) parameters. Regarding the DL type basis function [11], it reads:

\[ g(t; a_1, \ldots, a_4) = \frac{1}{1 + \exp \left( \frac{a_1-t}{a_2} \right)} \frac{1}{1 + \exp \left( \frac{a_1-t}{a_4} \right)} \]

where \( a_1 \) and \( a_2 \), determine, respectively, the position of the left and right inflection points, and \( a_3 \) and \( a_4 \) fix the rates of change at those points. Therefore, the DL global function has a total of \( H \times 6 \) parameters.
Albeit superior to standard harmonic analyses, these methods still suffer from severe constrains that hamper their effective use. In particular, these models are not totally versatile as they are always adopted to the upper envelope of the time series [5], [11]. Thus, the fits are subjective when there is no clear seasonality in land cover time series (e.g. urban areas, water or barren) or the time series being modeled are other than VIs. Moreover, growing season variations that do not coincide with the shape of the Gaussian or Logistic curves are not wholly captured [11]. AG and DL functions are not robust to noise and can only fit smoothed time series [5]. When the noise level is high (such as for the corresponding satellite image time series), these methods are generally not successful because there is a difficulty for estimating the time $t_0$ corresponding to the occurrence dates of extrema values (minima and maxima) of the time series to be modelled. More importantly though, since three consecutive annual time series are required to properly estimate the intra-annual functions corresponding to the central year [5], the model parameters are not easily identifiable. Finally, as local contributions of the time limited functions are non-adaptive, phenological attributes for specific land cover classes cannot be early predicted.

B. Nonlinear harmonic model

1) Definition: In this study we recall the model we proposed in a previous work [16] to fit intra-annual response of land cover multispectral reflectances and VIs obtained from satellite image time series. It consists of a trigonometric parsimonious function that describes the nonlinear harmonic solution of a chaotic attractor [19]:

$$S_\theta(t) = M + A\cos(\omega_0 t + \phi + \alpha \cos(\omega_0 t + \phi)), \quad \theta \in \Theta$$

In words, it is a sinusoidal parametric model with an a priori fixed annual frequency $\omega_0 = \frac{2\pi}{T}$, where $T$ denotes a one-year period, and a nonlinear phase term $\alpha \cos(\omega_0 t + \phi)$. The latter plays the role of a time warping (acceleration/deceleration) to account for the different regimes in the intra-annual cycles of land cover types (e.g. growths, cuts, harvests, droughts).

The meta-parameter $\theta = (M, A, \phi, \alpha, \varphi)$ belongs to the parameter space $\Theta = \{(M, A, \phi, \alpha, \varphi) : M \in \mathbb{R}, A \geq 0, \phi \in [0, 2\pi], \alpha \geq 0, \varphi \in [0, 2\pi]\}$, and each component has a specific action in the model response:

- $M$ is a linear parameter that determines the base level of the modelled variable and represents its annual mean;
- $A$ is the amplitude of the sine wave that fixes the peak deviation from the annual mean of the modelled variable;
- $\phi$ is the annual phase that allows to reproduce the specific seasoning of a given land cover class;
- $\alpha$ controls the nonlinear strength. When $\alpha = 0$, the model reduces to a linear harmonic oscillator, whereas $\alpha > 0$ introduces non-symmetry in the waveform;
- $\varphi$ is the annual nonlinear phase that defines the type of non-symmetry of the waveform. This phase allows time to “slow down” and to “accelerate”, in order to reproduce asymmetries in variations (increases vs. decreases) or in steady state durations.
2) Parameters identification: Given a discrete time series \( x = \{ x(t_i) : i = 1, \ldots, N; 1 \leq t_i \leq T \} \), where \( N \) is the number of observations, we want to find the “best” approximation \( s_\theta = \{ s_\theta(t_i) : i = 1, \ldots, N; 1 \leq t_i \leq T \} \) in a maximum likelihood sense. To this end we assume the following observation model:

\[
x = \{ x(t_i) = s_\theta(t_i) + \epsilon(t_i) : i = 1, \ldots, N; 1 \leq t_i \leq T \},
\]

(5)

where \( \epsilon \) is simplistic supposed to be a stationary zero-mean Gaussian noise of unknown variance \( \sigma_\epsilon^2 \). Under the hypothesis of independent noise observations, the likelihood function reads

\[
f(x|s_\theta) = \left(2\pi\sigma_\epsilon^2\right)^{-\frac{N}{2}} \exp\left[-\frac{1}{2\sigma_\epsilon^2} \sum_{i=1}^{N} (x(t_i) - s_\theta(t_i))^2\right],
\]

(6)

and the maximum likelihood estimate \( \hat{\theta} \) becomes

\[
\hat{\theta} = \arg \min_{\theta \in \Theta} \sum_{i=1}^{N} (x(t_i) - s_\theta(t_i))^2.
\]

(7)

Formally, (7) is the solution of the system obtained by differentiating the \( l_2 \)-norm \( \| x - s_\theta \|_2 \) with respect to each component of \( \theta \) and equaling the results to zero. However, because of the trigonometric form defining (4), we are led to a system of nonlinear equations that we can not solve analytically. Considering well-posed problems that involve at least as many observations as unknowns, we then proposed to numerically minimize the approximation error \( \| x - s_\theta \|_2 \) using a nonlinear least-squares optimization routine based on the Levenberg-Marquardt method [20], [21]. Furthermore, as the problem may not be convex, in practice we initialize the solving routine with the heurist initial guess \( \hat{\theta}_0 \):

- \( \hat{M}_0 \) is set to the empirical median of \( x \), i.e. \( (\max\{x(t_i)\} + \min\{x(t_i)\}) / 2 \);
- \( \hat{A}_0 \) is set to the maximum deviation of \( x \), i.e. \( (\max\{x(t_i)\} - \min\{x(t_i)\}) / 2 \);
- \( \hat{\phi}_0, \hat{\alpha}_0, \) and \( \hat{\phi}_0 \) are determined from a least-square regression of the nonlinear phase term

\[
\delta(t_i) = \omega_0 t_i + \phi + \alpha \cos(\omega_0 t_i + \phi)
\]

in (4).

Let us recall that we purposely consider a one-year periodic model. Shorter phenological periods, like the pseudo half-period of bi-annual crops, are accounted for by the non-linearity in \( \delta(t_i) \). Therefore, to cope with intermediate phase shifts, we must properly unwrap the instantaneous phase \( \delta(t_i) = \cos^{-1}(\{x(t_i) - \hat{M}_0\}/\hat{A}_0) \). We then use a geometric criterion to keep phase continuity at global extrema, and to let the angle increase beyond \( 2k\pi, (k = 1, 2, \ldots) \), at local extrema. The parameter set

\[
o = \{\hat{\phi}_0, \hat{\alpha}_0, \hat{\phi}_0\}
\]

that belongs to the parameter space \( O = \{\hat{\phi}_0, \hat{\alpha}_0, \hat{\phi}_0; \hat{\phi}_0 \in [0,2\pi], \hat{\alpha}_0 \geq 0, \hat{\phi}_0 \in [0,2\pi]\} \) is finally initialized to the solution of the minimization problem:

\[
\hat{o} = \arg \min_{o \in O} \sum_{i=1}^{N} \left( (\hat{\delta}(t_i) - \omega_0 t_i) - (\hat{\phi}_0 + \hat{\alpha}_0 \cos(\omega_0 t_i + \hat{\phi}_0)) \right)^2.
\]

1 Although simplistic and sub-optimal in practice, this Gaussian assumption allows for calculating the likelihood function and to express the MLE estimator of \( \hat{\theta} \) in a close form.
Model identifiability: The model’s identifiability and robustness address the following two questions: 1) is it possible to exactly retrieve the parameter values of a given model simply from a finite sampled sequence of its time response? 2) given a model response generated under controlled experimental conditions (notably regarding the sampling process and the Signal to Noise Ratio (SNR)), can the parameters of the model be estimated with acceptable precision? A standard practice for studying identifiability of a parametric model is then to consider multiple known input datasets which are then compared to the corresponding fitted models [22]. The input datasets are model outputs computed with a priori known parameters values. Thus, for different values of the meta-parameter $\theta$ of (4), test signals $s(d,t)$ were synthesized over a one year period ($T = 365$). Experimental conditions are summarized in Table I.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$A$</th>
<th>$\alpha$</th>
<th>$\phi$ (rad)</th>
<th>$\phi'$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.013</td>
<td>$0 \pi/3, 2\pi$</td>
<td>$b, 0, 2\pi$</td>
</tr>
</tbody>
</table>

First, to assess model identifiability as function of $N$, we sampled the synthesized test signals $s(d,t)$ on regularly spaced time lattices with $N = \{5, 6, 8, 12, 16, 24, 46\}$ observations. For each combination of $N$ and the meta-parameter $\theta$ in Table I, we determine the maximum likelihood estimate $\hat{\theta}$ and compute the relative fitting error $e_N = \sqrt{\frac{1}{365} \int_0^{365} (s_{\theta}(t) - s_{\hat{\theta}}(t))^2 dt} \times \frac{100}{P[s_{\theta}]}$, as a measure of agreement between our functional model and the synthesized signal of power $P[s_{\theta}]$. In Fig 1 we present the average fitting error, $<e_N>_{\theta}$, corresponding to different training sample size $N$.

![Fig 1. Average fitting error (%), $<e_N>_{\theta}$, corresponding to different training sample size $N$.](Image)

From Fig. 1, we see that the average fitting error decreases as the number of observations used to fit the model increases. We also checked that this decrease is systematic regardless the magnitude of the parameters, which are perfectly estimated for $N \geq 16$. A closer view at the corresponding time series indicates that this result is not due to methodological constraints, but because of insufficient data. The model fails at matching the time series only when the sampling step of the synthesized signal exceeds the Nyquist rate, i.e. the model parameters are improperly estimated when the discrete time lattice is not dense enough to properly describe the intra-annual variations of the underlying continuous signal.
Next, to demonstrate the denoising properties of our model, we compare the SNR of the observation with that of the corresponding fitted model. To simulate different input SNR values, we added to the synthesized signal $s_\theta(t)$ of fixed power $P[s_\theta]$, a stationary zero-mean white Gaussian noise $\varepsilon(t)$ with variable power $\sigma_\varepsilon^2$. The classical input SNR reads,

$$SNR_{\text{input}} = 10 \log_{10} \frac{P[s_\theta]}{\sigma_\varepsilon^2}, \quad (8)$$

and compares the level of useful signal to the level of corrupting background noise. For each combination of the meta-parameter $\theta$ and of different input SNR values, we determine the maximum likelihood estimate $\hat{\theta}$ and similarly define the output SNR as:

$$SNR_{\text{output}} = 10 \log_{10} \left( \frac{P[s_\theta]}{P[s_{\hat{\theta}}]} \right), \quad (9)$$

We choose to assess identifiability of the model through the SNR gain $\gamma = SNR_{\text{output}} - SNR_{\text{input}}$. Alternatively, we could also compare the exact value of the meta-parameter $\theta$ to its estimate $\hat{\theta}$. However, this approach raises the issue of defining a consistent metric to evaluate the distance between vectors whose components lie in different spaces. In practice, we sampled the observations on a 8-day spaced time lattice corresponding to $N=46$. Fig. 2 shows for four different input SNRs, realizations of noisy time series superimposed to the corresponding fitted models.

From our experiments, we first observed that for a given input SNR, the output SNR does not significantly depend on the values of $\theta$. This led us to the interesting conclusion that the sensitivity of our maximum likelihood estimation procedure is independent of the magnitude of the model parameters. That is the reason why Table II presents only the averaged SNR gain, $< \gamma >_\theta$, corresponding to the same input SNRs. In all the cases, not only the underlying model parameters are reasonably estimated, but also the SNRs systematically increase by more than 10dB! Given also the relatively low
computational cost of the method, a reduction by a factor 10 of the disturbance intensity, is a substantial improvement within the context of satellite imaging.

<table>
<thead>
<tr>
<th>Input time series SNR (dB)</th>
<th>Mean SNR (dB) for model fits</th>
<th>Mean SNR (dB) gain for model fits</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>1.7</td>
<td>11.7</td>
</tr>
<tr>
<td>-5</td>
<td>6.8</td>
<td>11.8</td>
</tr>
<tr>
<td>0</td>
<td>11.6</td>
<td>11.6</td>
</tr>
<tr>
<td>5</td>
<td>17.1</td>
<td>12.1</td>
</tr>
<tr>
<td>10</td>
<td>22.9</td>
<td>12.9</td>
</tr>
<tr>
<td>∞</td>
<td>200.9</td>
<td>-</td>
</tr>
</tbody>
</table>

III. CASE STUDIES

In this section we introduce the proposed model-based approach for phenological attributes identification and prediction. To demonstrate its reliability, we followed a rigorous process in three main steps. Firstly, in Section A, we describe the study area, the remote sensing data and the land cover classes used to assess our method. Then, in Section B, we evaluate the goodness-of-fit of the model to non-smoothed MODIS spectral bands and VIs time series for 13 representative land cover classes in Portugal. In the course of this, we compare the goodness-of-fit of our model with that of two different standard approaches. Finally, in Section C, using rice crops and shrublands as case studies, we show how to accurately estimate phenological key-markers from fitted responses and how to early predict important phenological attributes dates and magnitudes from our expected model adjusted to only few anterior observations.

A. Study area and experimental data set

1) Study area: We focused our experiences on the Portuguese mainland territory. The country has an area of approximately 89 000 km$^2$ and ground altitudes ranging from 0 to 2000 m above sea level. According to [23], in 2000 the agricultural areas occupied 37\% of the country, closely followed by semi-natural areas (33\%) and forest (27\%), while artificial surfaces comprise only 3\% of the total land cover. Portugal is in a transition region featuring diverse landscapes representing both Mediterranean and Atlantic climate environments. The study area is particularly well suited to evaluate data-model adequacy and to gauge the reliability of model-based approaches for identifying and predicting phenological attributes. Indeed, it encompasses natural and managed land cover types with different phenological characteristics, which are subject to different climatic, topographic and human induced forcing effects.

2) Earth observation data: Our study relies on the MOD09A1 product, an 8-day composite of seven surface reflectance images, freely available from MODIS Data Product website (http://modis.gsfc.nasa.gov). This specific product is an estimate of the surface spectral reflectance imaged at a nominal spatial resolution of 500 m as it would have been measured at ground level if there were no atmospheric scattering or absorption. The applied correction scheme compensates for the effects of gaseous and aerosol scattering and absorption, for adjacency effects caused by variation of land cover, for Bidirectional Reflectance Distribution Function (BRDF), for coupling effects, and for contamination by thin cirrus [24]. We used a set of 46 MOD09A1 images for each of the seven spectral bands, covering a full year observation period, from January 2005 to
December 2005. In addition, two VIs were also calculated for each date and used as additional band information, namely the Normalized Difference Vegetation Index (NDVI) and the Enhanced Vegetation Index (EVI).

As complementary information for reference sample land cover type’s identification, we used orthorectified aerial images with four spectral bands in the blue, green, red and infrared wavelengths, and with 50 cm spatial resolution. These aerial images cover the whole Portuguese territory and were acquired during years 2004, 2005 and 2006.

3) Land cover classes: The thirteen land cover classes (Table III) used in this study were derived from LANDEO nomenclature. This nomenclature was recently recommended by The Remote Sensing Unit (RSU) of the Portuguese Geographic Institute (IGP) to be used in regular production of user-driven land cover products for multi-scale environmental monitoring in Portugal [25]. The classes used in this study correspond mainly to vegetated land cover types because we want to demonstrate the relevance of our model-based approach for the identification and prediction of phenological attributes. Nevertheless, to illustrate the flexibility of our model, we also included land cover classes without intra-annual seasonality, namely Water (7) and Continuous artificial areas (11).

For each of the land cover classes presented in Table III, we deterministically collected a reference sample of time series observations. Each observation corresponds to a MODIS pixel with 500 m nominal spatial resolution. Following the lines of [26], we overlaid a co-registered 500 m fishnet corresponding to the MODIS data grid with the aerial images for visual interpretation of reference land cover observations. To guarantee land cover homogeneity within each sample, observations for a given class were selected preferentially in geographic spots located in the centre of 3x3 MODIS pixels of homogeneous land cover. The goal was to reduce intra-class observations dissimilarities due to possible geometric disparities between multitemporal MODIS images. Nevertheless, we endeavored to spread, as much as possible, the observations over the mainland territory, in order to account for possible regional specificities of the classes, and to avoid geographic correlation between adjacent pixels [27].

<table>
<thead>
<tr>
<th>Label</th>
<th>Land cover class</th>
<th>Sample size</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Water</td>
<td>30</td>
</tr>
<tr>
<td>11</td>
<td>Continuous artificial areas</td>
<td>43</td>
</tr>
<tr>
<td>21</td>
<td>Non-irrigated herbaceous crops</td>
<td>40</td>
</tr>
<tr>
<td>22</td>
<td>Irrigated herbaceous crops</td>
<td>33</td>
</tr>
<tr>
<td>23</td>
<td>Rice crops</td>
<td>37</td>
</tr>
<tr>
<td>242</td>
<td>Vineyards</td>
<td>37</td>
</tr>
<tr>
<td>2411</td>
<td>Olive trees</td>
<td>48</td>
</tr>
<tr>
<td>35</td>
<td>Pastures</td>
<td>34</td>
</tr>
<tr>
<td>3121</td>
<td>Agro-forestry areas</td>
<td>42</td>
</tr>
<tr>
<td>34</td>
<td>Shrubland</td>
<td>46</td>
</tr>
<tr>
<td>3111</td>
<td>Cork tree forest</td>
<td>40</td>
</tr>
<tr>
<td>3112</td>
<td>Eucalyptus forest</td>
<td>34</td>
</tr>
<tr>
<td>321</td>
<td>Needleleaf forest</td>
<td>42</td>
</tr>
</tbody>
</table>

As commonly done in functional imagery domain, we analyze the time evolution of measurements within a given pixel. In our case, we consider \( p=9 \) simultaneous measurements corresponding to the reflectances in seven spectral bands and to two VIs. We designate by \( x^{(j)} = \{ x^{(j)}(t) : \ t = 1,...,N; \ 1 \leq t \leq T \} \), \( j=1,...,p \), the \( j \)-th measurement and we group all the
observations related to a pixel in a $p$-dimension vectorial time series noted $\mathbf{x} = (x^{(1)}, x^{(2)}, \ldots, x^{(p)})$. As we use 8-day composite satellite images, we have $N = 46$ measurement points uniformly spaced along a $T = 365$ days period.

B. Data model adequacy

The primary objective of this section is to demonstrate the response of our model-based approach to diverse spectral reflectances and VIs time series for land cover classes of different phenology. We therefore apply our algorithm to the sample set of seven spectral reflectance bands and two VIs time series for each of the land cover classes introduced in Table III, and evaluate its goodness-of-fit. To position our method with respect to the current state-of-the-art alternatives, we compare its performances to those of Asymmetric Gaussian (AG) and Double Logistic (DL) functions presented in Section II.A.

To evaluate models performances, we use the Normalized Root Mean Square Error (NRMSE). This measure was previously used by [11] and [28] to evaluate the fits of different models to time series of satellite images and can be expressed as a percentage, where lower values indicate smaller residual variance and thus a better fit. The NRMSE for each sample individual at spectral band or VI $j$ is defined as:

$$\text{NRMSE}^{(j)} = \frac{\text{RMSE}^{(j)}}{\text{max} \{x^{(j)}(t_i)\} - \text{min} \{x^{(j)}(t_i)\}} \quad \text{for } j = 1, \ldots, p.$$  (10)

The RMSE$^{(j)}$ is calculated as:

$$\text{RMSE}^{(j)} = \sqrt{\frac{\sum_{i=1}^{N} (x^{(j)}(t_i) - s^{(j)}_\theta(t_i))^2}{N - u}}$$  (11)

where $\text{SSE}^{(j)} = \sum_{i=1}^{N} (x^{(j)}(t_i) - s^{(j)}_\theta(t_i))^2$ and $u$ is the number of unknown model parameters. To estimate $s^{(j)}_\theta(t_i)$ we start fitting the available time observations $\mathbf{x} = \{x(t_k); k \neq i; t_k \neq t_i\}$ with a parameter model of expression (4). This standard cross validation process, usually used in longitudinal data analyses, yields an estimate of the expected fitting error when the respective time observation $x(t_i)$ is discarded from the adjusting set. The NRMSE is normalized on the range of the observed sample values at each spectral reflectance band or VI, and adjusted on the residual degrees of freedom of each model to penalize the models with more unknown parameters. Let us recall here that for AG and DL, the number of degrees of freedom depends directly on the number of extrema existing in the time series to be modelled, whereas it is systematically constant with our non harmonic model.

To avoid biased error estimates due to atmospheric noise in the time series, we removed cloudy observations from our sample data set, as similar as in [5] and [11]. This way, the NRMSE becomes also an estimate of the fitting error for the dates $t_i$ identified with clouds and that are never used in the error estimation. Cloudy observations were identified with the surface reflectance quality assessment layer included in MOD09A1 products (see http://modis.gsfc.nasa.gov). Alternatively, we
could also calculate the \( NRMSE \) from the model fitted time series \( s_\theta(t_i) \) and a smoothed observed time series (e.g. \[28\], \[29\]) or an expertly corrected observed time series (e.g. \[3\], \[11\]). However, these corrections on the observed \( x(t_i) \) time series were discarded because they may introduce new unknown errors and dampen important phenological features in the real time series \[30\].

Table IV gives the sample average \( NRMSE \)s corresponding to the nonlinear, AG and DL models’ fits to multispectral reflectance and VIs time series for each land cover class. As a general remark, we can state that AG and DL fits to multispectral reflectances and VIs time series are similar for all land cover classes, and that the nonlinear model fits generally overcome the AG and DL fits. Indeed, the overall average \( NRMSE \) for the nonlinear model (5.25%) is significantly better than the overall average \( NRMSE \)s for the AG (6.97%) and DL (6.59%) functions at the 95% confidence level.

<table>
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<th>Land cover class</th>
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<th>b02</th>
<th>b03</th>
<th>b04</th>
<th>b05</th>
<th>b06</th>
<th>b07</th>
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<th>EVI</th>
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<td>7.42*</td>
<td>2.41*</td>
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</table>

* Significantly different at 5% from the corresponding nonlinear \( NRMSE \).
More formally, a closer view at Table IV shows that the average $NRMSE$s for VIs time series fits are better than the $NRMSE$s for multispectral reflectances time series fits. These results are mainly due to the fact that not all clouds are identified by the MODIS surface reflectance quality assessment layer, and some sharp and localized atmospheric disturbances still pollute the data classified as clear. Accordingly, because VIs are less sensitive than spectral reflectance bands to cloud contamination and also minimize many contamination problems present in the spectral wavelength bands, such as those associated with aerosol influences [31], the residuals of the smoothed curves are smaller for VIs.

Now, looking at land cover classes, we see that the nonlinear model fits significantly overcome the AG and DL fits for Water (7) and Continuous artificial areas (11) because these functions often fail at reproducing time series without apparent seasonal waveforms [5], [11]. Still, as AG and DL functions are not fully versatile and their use is generally constrained to VIs time series of land cover classes with seasonal waveforms [5], [9], [11], [17], [18], we expected those functions to perform as good as the nonlinear model for those datasets. Conversely, we note that the nonlinear model fits to VIs time series for agricultural cover types, such as Rice crops (23), are significantly superior to the respective DL and AG fits. From Fig. 3, we remark that the AG and DL fits describe the EVI data as good as the nonlinear fits in the time interval around the maximum. At the limbs, however, the fits are less good than the nonlinear fits. The reason is that because AG and DL functions are piecewise, they cannot reproduce the global seasonal pattern of the class from a single yearly time series set of observations. We remark that neither AG nor DL functions are able to depict properly the local maximum (respectively local minimum) in the left half (respectively right half) of the observed curves. This drawback was presented before by [5], which propose the use of three consecutive years of data to correctly model the limbs of the seasons for the central year. However, if we do not have data for the surrounding years, this problem makes difficult the use of AG and DL functions for fitting satellite images time series.

Fig. 3. Observed EVI time series for three Rice crops (23) individuals and the respective nonlinear, AG and DL fits. For visual convenience, we arbitrarily connected the observations with dotted straight lines.

Importantly also, Fig. 3 shows that the nonlinear model fits present identical time patterns for all individuals of the same land cover class, in opposition to AG and DL fits. We verify this also for natural vegetated land cover classes. For example, AG and DL fits to EVI time series for Needleleaf forest (321) individuals are considerably heterogeneous (Fig. 4). On the
other hand, the nonlinear fits to the same individuals have consistent time trajectories and fits are independent of data disturbances not depicted by the quality assessment layer. This outcome is extremely important for phenological attributes identification and land cover classes’ characterization through time series analysis of satellite images.

We also perceive that the border values of the annual curves fitted by AG and DL functions do not match (Fig. 4). We expected the waveforms of natural land cover classes to present similar EVI values at the beginning and ending times of the year because we are dealing with individuals that have recurring seasonal patterns across different years – unless some unexpected environmental forcing effect occurs during the modelled year, which is never the case here. However, because AG and DL functions are piecewise, they cannot reproduce reliable fits using a single yearly time series set of observations.

The nonlinear model fits to VIs time series for most natural vegetated cover types with slow time variations, also present significantly lower fitting residuals. This is the case of, e.g. models fits to EVI time series for Eucalyptus forest (3112) observations (Fig. 5). As pointed by [11], the reason is that AG and DL functions often fail at reproducing time series that do not correspond to the shape of the Gaussian or Logistic curves.

C. Phenological key-markers identification and prediction
1) **Identification:** Now that we confirmed the ability of the nonlinear model to fit multispectral reflectances and VIs time series for diverse land cover classes, we will use the modelled curves to effectively identify phenological measures. This is a common approach and several methods for deriving annual phenological metrics from smoothed time series have been proposed in the literature, e.g. [9], [32]-[35]. These methods aim at overcoming data noise corruptions and at providing a more accurate continuous time evolution of land cover phenological characteristics that are necessarily discrete due to sampled satellite image measurements [4].

Following the lines of [9], we use the first and second derivatives of our modelled VIs time series to identify the inflection points of the intra-annual curves and, accordingly, the unbiased phenological attributes estimates. The number and position of the inflection points depend on the seasonal characteristics of the land cover class being modelled and on the respective physical measure used to derive the phenological attributes.

To illustrate our approach, we use the Rice crops (23) NDVI time series observations from our sample database. For example, the minimum of the second derivative of the modelled time series is the time for maximum greenness (i.e. heading) $t^M$ of the crop, while the maximum (respectively the minimum) of the first derivative corresponds to the onset $t^{OnM}$ (respectively to the offset $t^{OfM}$) time of maturity. In the sequence, we identify the times for transplant $t^T$ and harvesting $t^H$ as the second derivative maxima. An identification example of phenological attributes using a Rice crops (23) NDVI time series sample observation is shown in Fig. 6.

To evaluate the accuracy of our analytical method, we compared the model-based estimated phenological times for the Rice crops (23) observations of our sample database to the empirical reference dates provided in Portuguese Agricultural Calendar [36]. More formally, we followed a rigorous process in two main steps: 1) we fitted each NDVI time series sample observation (corresponding to geographically distant pixels) with our smooth model; 2) we estimated three specific phenological key-dates for each sample observation through the inflection points of the first and second derivatives of each continuous fitted curve. The estimated key-dates, expressed in week of the year (WOY) are: the rice crops transplant date $t^T$ (between the 15th and the 19th WOY), the maximum greenness time $t^M$ (between the 24th and the 28th WOY), and the harvesting date $t^H$ (between the 33rd and the 37th WOY). Fig. 7 shows the estimated Rice crops (23) sample probability distributions for the three phenological dates. In all the cases, not only the underlying phenological measures are reasonably
estimated, because the mean dates correspond to the theoretical references of [36], but also the sample estimated phenological key-dates are normally distributed and have very small standard deviations from the respective expected date.

![Fig. 7. Estimated Rice crops (23) sample probability distributions for: a) Transplanting time $t^T_i$; b) Maturity time $t^M_i$; c) Harvesting time $t^H_i$. Vertical bars represent the limits for the phenological reference key-dates provided in [36] for rice crops in Portugal.]

2) **Prediction:** Suppose now that we want to predict characteristic phenological attributes of a specific class from only a series of anterior images of the same year. Let us call $x = \{x(t_i); i = 1, ..., n; 1 \leq t_i < T\}$ the observed intra-annual time series corresponding to the evolution of a reflectance or of a VI in a given pixel of class $C_l$. To predict the date of interest $t_m$, with $t_n < t_m \leq t_N$, we start fitting the available observations $x$ with a parameter model of expression (4). To cope with censured data $x(t_k); t_k > t_n$ that may mislead the initial guess $\hat{\theta}_0$, we initialize the solving procedure of Section II.B.2 with an average meta-parameter $\overline{\theta}_l$ representative of the class $C_l$. To this end, the class model meta-parameter must have been previously and empirically determined from an accurate training sample set. Once estimated the fitting function $s_{\hat{\theta}}(t)$, its evaluation for $t > t_n$ serves as a prediction for the phenological development of the pixel area, and forecasted milestone dates associated to this particular land cover class simply derive from the variation analysis of the path $s_{\hat{\theta}}(t); t > t_n$.

Let us stress that this approach holds only with global models that uniformly fit the time series over a full year period. Analytic models of the form of (4) naturally meet this requirement: structurally constrained by definition, they are consistently identifiable from a time incomplete set of observations, while sufficiently versatile to yield patterns able to match a broad spectrum of land cover classes and physical measurements. That is not the case though with models such as Asymmetric Gaussian (AG) or Double Logistic (DL) functions whose parameters are piecewise constant and hence unpredictable from one segment to the next.

We illustrate the relevance of our model-based predictive approach with two significant case studies. The first application deals with rice crops phenological attributes prediction. Reference [29] state that essential rice crop’s phenological stages for evaluating crop productivity and crop management are the planting, the heading and the harvesting dates. Indeed, vegetation magnitudes at the heading are related to potential yield provided that no accident occurs after that stage [4]. As it seems unrealistic to predict the planting date from phenological arguments only (it is mainly determined by human decisions), our analyses focus on the prediction of heading and harvesting dates. The second application concerns the prediction of shrubland phenological attributes. Our goal here is to predict both the time and the magnitude of vegetation minimum
greenness before the Portuguese fire season, which officially starts in June (20th WOY, [37]). Indeed, because fire is sensitive to the quantity of green vegetation, prescribed burning activities can benefit from the assessment of the minimum vegetation greenness extent and timing [38]. Therefore, if these phenological attributes are forecasted before fire season, then it will become easier to plan fire-fighting activities and to allocate surveillance infrastructures in due time. The main difference in these prediction case studies is related to political constrains. On the one hand, agricultural monitoring undergoes no political constraints, and prediction can be repeated at any time during the crop growing season in order to progressively refine phenological attributes estimates. On the other hand, planning of an efficient forest fire prevention policy imposes an official date \( t \) for predicting important phenological attributes of natural vegetation.

\textbf{a) Rice crops (23) case study:} In Section III.C.1, we considered a set of NDVI-time series sample observations representative of rice crops, and for each of these we estimated the phenological dates of maturity \( t_M^M \) and harvesting \( t_H^H \). We now use these estimates as the target values that we want to predict.

We remove from our dataset all observations that lie beyond the current date \( t_n \) and we apply our prediction procedure to all time series censured in this way. We note \( t_n^{M\hat} \) (respectively \( t_n^{H\hat} \)) the predicted values to make their conditional dependence on observations up to date \( t_n \) explicit. The prediction error \( eT_n^M = t_M^M - t_n^{M\hat} \) (respectively \( eT_n^H = t_H^H - t_n^{H\hat} \)) is a random variable whose statistics (mean, variance...) assess the relevance of our model through the quality of its prediction.

In Fig. 8, we present the Root Mean Square Prediction Errors (RMSPE) for the dates of maturity \( t_M^M \) and for the dates of harvesting \( t_H^H \) as functions of \( t_n \). RMSPEs were computed for all Rice crops (23) sample individuals and for

\[ t_n = t_M^M - k, \quad k = 1, ..., 10 \text{ weeks (respectively } t_n = t_H^H - k, \quad k = 1, ..., 10 \text{ weeks).} \]

From our experiments, we notice that the prediction errors for both \( t_M^M \) and \( t_H^H \) estimates decrease as \( k \) decreases, i.e. as \( t_n \rightarrow t_M^M \) (respectively \( t_n \rightarrow t_H^H \)). More importantly though, with six weeks of antecedence (i.e. \( k = 6 \)), we are able to predict the dates of maturity \( t_M^M \) with \( \pm 1.1 \) weeks of precision; in the same way, the RMSPE for forecasted dates of harvesting \( t_H^H \) is of approximately \( \pm 1.3 \) weeks when forecasting seven weeks ahead (i.e. \( k = 7 \)).
Assume now that $NDVI^M$ denotes the maximum greenness for Rice crops (23) sample observations at the estimated phenological dates of maturity $t^M$, and $NDVI^M_n$ refers to the forecasted maximum greenness values based on a model fitted on observations up to date $t_n$. Considering $k = 6$ for all observations, we compute the sample estimated probability distribution for the maximum greenness prediction error $eN^M_n = NDVI^M - NDVI^M_n$, and present the results in Fig. 9.

![Fig. 9. Estimated Rice crops (23) sample probability distribution for $eN^M_n$, where $t_n = t^M - 6$.](image)

From Fig. 9, we see that the estimated sample probability distribution for the maximum greenness prediction errors is normal and approximately zero mean, with a standard deviation of 0.05. Moreover, considering that the average sample $NDVI^M$ is 0.79, then the estimated standard deviation is only 6% that value. This result confirms that our model-based prediction approach provides 6 weeks ahead precise crop development information for time-specific crop management.

**b) Shrublands (34) case study:** Now, consider that we want to predict the minimum greenness value $NDVI^m_n$ for Shrubland (34) observations and its respective date $t^m$ of occurrence based on a model fitted only to $t_n$ past observations of the $NDVI$ time series. In our case, the prediction process is limited to $t_n = 20$ past observations to respect the time constrain on the forest fire prevention program in Portugal. Following the same strategy as for rice crops, we estimate the reference phenological dates of minimum greenness $t^m$ and the respective minimum greenness $NDVI^m$ for Shrubland (34) sample observations using our analytical method based on the first and second derivatives of the modelled time series. In the sequence, we calculate the estimated sample probability distributions for the prediction errors $eT^m_{20} = t^m - t^m_{20}$ and $eN^m_{20} = NDVI^m - NDVI^m_{20}$ (Fig. 10).

![Fig. 10. Estimated Shrubland (34) sample probability distributions for: a) $eT^m_{20}$; b) $eN^m_{20}$.](image)
The minimum greenness time prediction errors $e_{T_{20}}^m$ for Shrubland (34) sample observations have an estimated zero mean normal probability distribution with a standard deviation of ±2 weeks. Considering that the sample average $t^m$ is 10 weeks after the prediction date $t_{20}$, we can state that the estimated standard deviation for the prediction error is small and that the prediction is accurate. Moreover, the minimum greenness prediction errors $e_{N_{20}}^m$ have an estimated zero mean normal probability distribution with a standard deviation of 0.03. We remark that this value is only 9% the observed average $\overline{NDVI_{20}}$, thus confirming that $\hat{NDVI}_{20}$ is precise and can effectively be used for fire-fighting planning strategies.

In Fig. 11, we present the $NDVI$ time series model fits for two Shrubland (34) observations and the respective predicted time series $NDVI(t), t > t_{20}$. The visual analysis of forecasted time series suggest that the model-based predictions do not mislead the planning actions to be performed for Shrubland (34) observations with divergent greenness magnitudes.

In Fig. 11, we present the $NDVI$ time series model fits for two Shrubland (34) individuals and respective model predictions for $t_{20}$. The rectangles represent the standard deviations for $e_{T_{20}}^m$ and $e_{N_{20}}^m$.

IV. CONCLUSION

In this paper we proposed and evaluated a model-based approach for the identification and prediction of vegetation phenological attributes. The method relies on a consistent, whereas very simple, nonlinear parametric harmonic model that we developed in a related work [16] to fit intra-annual multispectral reflectances and VIs time series from satellite images. The identifiability study revealed that the model parameters are easily and perfectly estimated from a single yearly time series set of observations, and that the sensitivity of the estimation procedure is independent of parameters magnitude. We demonstrated that the model parameters are reliably identified from continuous signals sampled on different time lattices, as long as the intra-annual time variations of the underlying signals are captured by the quantization step. Moreover, the noise power of model fits to time series synthesized with different SNR values is always lower than the noise power of the respective input data, thus indicating that the model is able to diminish disturbances intensity in time series.

The goodness-of-fit confirmed that the nonlinear harmonic model is sufficiently able to match multispectral reflectances and VIs time series derived from satellite images for disparate land cover classes. Moreover, the nonlinear harmonic model can be used on all time series data sets without supervision and prior testing, because it does not need to be tuned for specific
land cover classes and remotely-sensed measurements. From a general viewpoint, the nonlinear harmonic model confirmed to be more versatile than AG and DL functions to yield a broad spectrum of physical measurements describing the intra-annual time evolution of land surface characteristics. In particular, the nonlinear harmonic model performed even better than the other fitting algorithms for some unexpected situations, such as for modelling VIs time series of vegetated land cover classes, which are the focus of AG and DL functions. The nonlinear harmonic method, as outlined in this paper, holds the additional advantage that the time series is fitted over a full year period. Piecewise functions, such as AG and DL, fail at accurately matching the global waveform of numerous time series because they rely on the linear combination of local and independent intra-annual functions.

The original model-based applications presented here for the identification and prediction of phenological key-attributes revealed extremely successful under particularly difficult experimental conditions. The identification of phenological key-markers using first and second derivatives of the nonlinear harmonic model proved reliable and functional for the study cases. From our experiments, the estimated key-attributes perfectly match the respective reference values for the study area. More important though, the prediction approach revealed that the model is consistently identifiable from a time incomplete set of observations and flexible enough to allow for forecasts of different phenological attributes for a broad spectrum of land cover classes. This particularly interesting and promising application allowed for the prediction up to ten weeks ahead of important phenological attributes for agricultural management and pre-season planning of forest fire fighting activities and surveillance from only few anterior observations.

Furthermore, we remark that the model can be applied in numerous different applications. Because the five model parameters contain information about the continuous intra-annual variation of cover types’ reflectances, we can use the model for reducing the dimensionality of satellite images time series and the model parameters for multitemporal land cover classification.

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REFERENCES


