Iterative Robust Minimum Variance Beamforming

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Abstract—Based on worst-case performance optimization, the recently developed adaptive beamformers utilize the uncertainty set of the desired array steering vector to achieve robustness against steering vector mismatches. In the presence of large steering vector mismatches, the uncertainty set has to expand to accommodate the increased error. This degrades the output signal-to-interference-plus-noise ratios (SINRs) of these beamformers since their interference-plus-noise suppression abilities are weakened. In this paper, an Iterative Robust Minimum Variance Beamformer (IRMVB) is proposed which uses a small uncertainty sphere (and a small flat ellipsoid) to search for the desired array steering vector iteratively. This preserves the interference-plus-noise suppression ability of the proposed beamformer and results in a higher output SINR. Theoretical analysis and simulation results are presented to show the effectiveness of the proposed beamformer.

Index Terms—Adaptive arrays, array signal processing, interference suppression, robustness.

I. INTRODUCTION

The Minimum Variance (MV) beamformer [1] has superior performance on interference-plus-noise suppression compared to conventional/data-independent beamformers so long as the desired array steering vector and the array covariance matrix are known or can be estimated accurately. In practice, signal source movement and array imperfections such as steering direction errors, array calibration errors, etc., are unavoidable and they cause steering vector mismatches. If the statistics about interferences and noise are available, adaptive beamformers can be robust against the mismatches [2]–[6]. However, in applications like passive sonar and wireless communications, the array data usually contains the desired signal. Thus, adaptive beamformers can degrade rapidly with steering vector mismatches.

Many methods like the eigenspace-based approach [7] and diagonal loading approach [8], [9] have been proposed to improve the robustness of adaptive beamformers against steering vector mismatches. Nevertheless, the eigenspace-based beamformer is not efficient at low signal-to-noise ratios (SNRs) and/or when the number of signal-plus-interferences is large or unknown. One shortfall with the diagonal loading method is that there is no systematic way to determine the optimal loading factor. Based on worst-case performance optimization and the uncertainty set of the desired array steering vector, the beamformers of Shahbazpanahi et al. [3], Li et al. [4], Vorobyov et al. [5], Lorenz and Boyd [6] are all robust MV beamformers which combat steering vector mismatches effectively [10]. In fact, [4]–[6] lead to the same beamforming weight when the uncertainty set of the desired array steering vector is a sphere. When large steering vector mismatches occur, a large uncertainty set is required to describe the increased error of the desired array steering vector. The robustness of the beamformers of [3]–[6] against large steering vector mismatches can be obtained but at the expense of reduced output SINRs due to the degradation of their interference-plus-noise suppression abilities. Unlike [3]–[6], the beamformer of Yu et al. [11] imposes magnitude response constraints by using the array weight autocorrelation sequence to achieve robustness against large steering direction errors.

Different from [3]–[6], [11], this paper proposes an IRMVB which uses a small uncertainty sphere (and a small flat ellipsoid) to search for the desired array steering vector iteratively. In this way, the interference-plus-noise suppression ability of the proposed beamformer can be preserved by preserving its degrees-of-freedom (DOFs) and by using the corrected desired array steering vector, the proposed IRMVB achieves higher output SINR than the beamformers of [3]–[6], [11]. Different from [10] which uses only one stopping criterion, the proposed IRMVB of this paper applies two stopping criteria. Due to one of the stopping criteria, the calculated steering vector by the proposed IRMVB is not allowed to converge to the steering vectors of the interferences. This problem was not dealt with in [10]. Unlike [10], this paper proposes another IRMVB which uses a small flat ellipsoid to search for the desired array steering vector iteratively. Theoretical analysis and simulation results show the effectiveness of the proposed method.

This paper is organized as follows. The data model and some background on adaptive beamforming are given in Section II. In Section III, we introduce the beamformer of Li et al. [4] and present the proposed IRMVB algorithms using spherical and flat ellipsoidal uncertainty sets separately.
in Sections III-A and III-B, respectively. The theoretical result of this paper (Theorem 1) is given in Section III-A which shows the output SINR improvement by the proposed IRMVB (using spherical uncertainty set) with each iteration and the proof is deferred to the Appendix. We discuss the design of the stopping criteria in the proposed IRMVB in Section III-C and show the simulation results in Section IV where the performance of the proposed IRMVBs are compared with the existing beamformers. Conclusions are given in Section V.

II. DATA MODEL

The output of a narrowband beamformer is

$$y(k) = w^H x(k)$$  \hspace{1cm} (1)

where $k$ is the time index, $w$ is a complex $N \times 1$ beamforming weight, $N$ is the number of array elements, $(\cdot)^H$ is a Hermitian transpose operator, and $x(k)$ is the received array snapshot vector given by

$$x(k) = z(k) s_0 + i(k) + n(k)$$  \hspace{1cm} (2)

$$= z(k) + i(k) + n(k)$$  \hspace{1cm} (3)

where $z(k)$, $i(k)$, and $n(k)$ are the desired signal, interference, and noise components, respectively. $z(k)$ and $s_0$ are the desired signal waveform and desired array steering vector, respectively. By maximizing the beamformer’s output SINR subject to (s.t.) a unity gain response to the desired signal, i.e.,

$$\max_w \text{SINR} = \frac{\sigma_0^2 |w^H s_0|^2}{w^H R_{in} w} \text{ s.t. } w^H s_0 = 1$$  \hspace{1cm} (4)

where $\sigma_0^2 = \mathbb{E}\{|z(k)|^2\}$ is the desired signal power, $\mathbb{E}\{\cdot\}$ is an expectation operator, $|\cdot|$ is an absolute operator, and $R_{in} = \mathbb{E}\{i(k) + n(k))(i(k) + n(k))^H\}$ is the interference-plus-noise covariance matrix, we get

$$\min_w w^H R_{in} w \text{ s.t. } w^H s_0 = 1.$$  \hspace{1cm} (5)

In practice, $R_{in}$ is usually not available as it requires an infinite number of snapshots and that the desired signal is often present in the snapshots. Instead, the sample array covariance matrix

$$R = \frac{1}{N_s} \sum_{k=1}^{N_s} x(k)x^H(k)$$  \hspace{1cm} (6)

is used where $N_s$ is the number of snapshots collected. The optimal solution to (5) with $R_{in}$ replaced by $R$ is the MV beamformer given by

$$w = \frac{R^{-1} s_0}{s_0^H R^{-1} s_0}.$$  \hspace{1cm} (7)

The array output power $P = w^H R w$ obtained with (7) is

$$P = \frac{1}{s_0^H R^{-1} s_0}.$$  \hspace{1cm} (8)

III. PROPOSED ITERATIVE ROBUST MINIMUM VARIANCE BEAMFORMER

The beamformer of Li et al. [4] achieves robustness against steering vector errors by maximizing the array output power $P$ or equivalently, minimizing the denominator of (8) and solving

$$\min_s s^H R^{-1} s$$  \hspace{1cm} (9a)

s.t. $|s - s_0|^2 \leq \epsilon_1$  \hspace{1cm} (9b)

once, assuming that the desired array steering vector $s_0$ is located in a sphere (9b) centred at the presumed one $s_0$ where $\epsilon_1$ is a user parameter that specifies the Euclidean distance between a steering vector $s$ and $s_0$. $\|\cdot\|$ is a vector norm.

Assume strong duality is achieved in (9), let $g \geq 0$ be the Lagrange multiplier that corresponds to the inequality constraint (9b). Due to the complementary slackness condition in the Karush-Kuhn-Tucker conditions, either $g = 0$ and $|s - s_0|^2 < \epsilon_1$ will occur or $g > 0$ and $|s - s_0|^2 = \epsilon_1$ will occur. In the first case, the constraint (9b) is inactive and the optimal solution is the eigenvector of $R$ corresponding to its maximum eigenvalue provided that the presumed desired array steering vector is near to the true one, the interferences are well-separated from the mainlobe region, and that the desired signal is dominant. Otherwise, the constraint (9b) is active and the solution will occur at the boundary of the constraint set under the assumption that $|s_0|^2 > \epsilon_1$. By applying the Lagrange multiplier methodology to (9), we get

$$l = s^H R^{-1} s + g(|s - s_0|^2 - \epsilon_1).$$  \hspace{1cm} (10)

Setting the differentiation of (10) with respect to $s$ to zero gives the calculated desired array steering vector as

$$s_0 = (g^{-1} R^{-1} + I)^{-1} s_0$$  \hspace{1cm} (11)

$$= s_0 - (I + gR)^{-1} s_0$$  \hspace{1cm} (12)

where $I$ is an identity matrix and $g$ is obtained by replacing $s$ with $s_0$ in the constraint $|s - s_0|^2 = \epsilon_1$ and solving

$$|s_0 - s_0|^2 \triangleq \|(I + gR)^{-1} s_0\|^2 = \epsilon_1,$$  \hspace{1cm} (13)

after which $s_0$ is found by (12) with the obtained $g$ from (13).

In practice, signal source movement, antenna array motion, etc., can result in a large error or uncertainty in the desired array steering vector [2], [12]. To enhance the output SINRs of adaptive beamformers in the presence of large steering direction errors, we propose an IRMVB which uses a smaller uncertainty sphere (and a smaller flat ellipsoid) than that used by [4] to search for the desired array steering vector iteratively.

A. Spherical Uncertainty Set

The concept of the proposed IRMVB (with spherical uncertainty set) is shown in Fig. 1. When there is a steering direction error, the desired array steering vector $s_0$ (corresponding to the desired signal direction $\theta_0$) and the presumed one $s_0$ (corresponding to the presumed desired signal direction $\theta_0$) do not coincide. If this error is large, the uncertainty sphere (green sphere) of size $\epsilon_1$ used in the beamformer of [4] has to be large. This consumes the DOFs of the beamformer of
IRMVB is insensitive to a wide range of $\varepsilon_2$ where all the tested $\varepsilon_2$ values are smaller than the coarse $\varepsilon_1$ estimates for all the simulation scenarios.

The proposed IRMVB algorithm (with spherical uncertainty set) is summarized. Let the steering vector determined at the $i$th iteration of the proposed algorithm and the corresponding Lagrange multiplier be $\hat{s}_0^i$ and $g^i$, respectively.

1. At $i = 0$, initialize $\hat{s}_0^0 = s_0$.
2. When $i \geq 1$, solve equation (13) with $\hat{s}_0^{i-1}$ and $\varepsilon_2$ (instead of $s_0$ and $\varepsilon_1$, respectively) to obtain $g^i$. Find $\hat{s}_0^i$ by (12) with the obtained $g^i$ and $\hat{s}_0^{i-1}$ (instead of $g$ and $s_0$, respectively). Obtain $\varepsilon_0^i = \sqrt{N}\hat{s}_0^i/\|\hat{s}_0^i\|$ so that $\|\hat{s}_0^i\| = \sqrt{N}$. We use $\varepsilon_2 = 0.1$; this will be discussed later in Section IV (Simulation Results).
3. Check if the stopping criteria in (30) are reached. The stopping criteria in (30) are discussed in Section III-C (Design of Stopping Criteria). If (30) is satisfied, go to step (4). If (30) is not satisfied, assign $\hat{s}_0^i$ to $\hat{s}_0^i$ and repeat step (2).
4. Use the converged $\hat{s}_0^{i-1}$ to replace $s_0$ in the MV beamformer (7) to obtain the proposed IRMVB weight.

The proposed IRMVB works by searching for a steering vector at each iteration to approach the desired array steering vector. The theoretical result of this paper (Theorem 1) shows that the proposed IRMVB (with spherical uncertainty set) can increase the output SINR with each iteration. This is achieved because the generalized angle $\hat{\theta}$ between the calculated steering vector of the proposed IRMVB and the desired array steering vector is reduced with each iteration. To see this, Lemma 1 is required [14], [15].

**Lemma 1:** In the presence of steering vector errors ($s_0$ is used instead of $s_0$) and assuming that the theoretical array covariance matrix $R$ is available, the output SINR of the MV beamformer, i.e., $w = (\bar{s}_0^i R^{-1}s_0)^{-1}R^{-1}s_0$ is

$$\text{SINR}_o = \frac{\text{SINR}_{\text{opt}} \cos^2(\hat{\theta}; R_{\text{in}}^{-1})}{1 + \sin^2(\hat{\theta}; R_{\text{in}}^{-1})[2\sin^2(\hat{\theta}; R_{\text{in}}^{-1}) + \text{SINR}_{\text{opt}}]}$$

(14)

where $\text{SINR}_{\text{opt}} = \sigma_0^2 R_{\text{in}}^{-1}s_0^H s_0$ is the optimal SINR. Here, $\hat{\theta}$ is the generalized angle between the presumed desired array steering vector $s_0$ and the true one $s_0$; the cosine-squared of which is given by

$$\cos^2(\hat{\theta}; R_{\text{in}}^{-1}) = \frac{\|s_0^H R_{\text{in}}^{-1}s_0\|^2}{\|s_0\|^2 \|R_{\text{in}}^{-1}s_0\|^2}$$

(15)

in the space $\mathcal{H}(R_{\text{in}}^{-1})$ defined by the inner product between $s_0$ and $s_0$, i.e., $s_0^H R_{\text{in}}^{-1}s_0$. It is appropriate to mention that $0 \leq \cos^2(\hat{\theta}; R_{\text{in}}^{-1}) \leq 1$ due to Schwarz inequality. $\|x\|_R = \sqrt{x^H R^{-1} x}$ is the extended vector norm-squared, i.e., the length of $x$ in $\mathcal{H}(R_{\text{in}}^{-1})$ is $(x^H R_{\text{in}}^{-1} x)^{1/2}$ where $x \in \mathbb{C}^N$. When $\cos^2(\hat{\theta}; R_{\text{in}}^{-1}) = 0$, it means that the vectors $s_0$ and $s_0$ are orthogonal in $\mathcal{H}(R_{\text{in}}^{-1})$; when $\cos^2(\hat{\theta}; R_{\text{in}}^{-1}) = 1$, it means that the vectors $s_0$ and $s_0$ are aligned perfectly in that one is a scalar multiple of the other.

One important observation from Lemma 1 is that (14) is a monotonically increasing function of $\cos^2(\hat{\theta}; R_{\text{in}}^{-1})$.

**Theorem 1:** Let the steering vector found by the proposed IRMVB at the $i$th iteration be $\hat{s}_0^i$ and its scaled version be
\( s_0 \) (to prevent scaling ambiguity). Let the generalized angle between \( s_0^{-1} \) and \( s_0 \) be \( \theta^{-1} \) and that between \( s_0^{-1} \) and \( s_0 \) be \( \theta \), respectively. Assuming that the interferences are not located near the protected mainlobe region, this paper shows that

\[
\cos^2(\hat{\theta} + \mathbf{R}_{inv}^{-1}) < \frac{\|s_0^{-1}\mathbf{R}_{inv}^{-1} - s_0\|^2}{\|s_0^{-1}\|^2} < \cos^2(\hat{\theta} + \mathbf{R}_{inv}^{-1})
\]

which means that the generalized angle between the calculated steering vector of the proposed IRMVB and the true one is reduced with each iteration, thereby increasing the output SINR of the IRMVB (with proof in the Appendix).

Next, the proposed IRMVB which uses a small flat ellipsoid to search for the desired array steering vector iteratively is presented before the design of the stopping criteria is discussed.

### B. Flat Ellipsoidal Uncertainty Set

As considered in [4], [6], if there is prior information, the uncertainty set of the desired array steering vector can be made tighter by modelling it as a flat ellipsoid, i.e., \( s = B\hat{u} + \hat{s}_0 \) where \( B \) is a \( N \times L \) matrix with full column rank (\( L < N \)) and \( u \) is a \( L \times 1 \) vector. The beamformer of Li et al. [4] solves

\[
\begin{align*}
\min_{u} & \quad (Bu + \hat{s}_0)^H \hat{R}^{-1} (Bu + \hat{s}_0) \\
\text{s.t.} & \quad \|u\| \leq (\varepsilon_1^f)^{1/2}
\end{align*}
\]

once with \((\varepsilon_1^f)^{1/2} = 1\) where the superscript “f” denotes the case for flat ellipsoid constraint. In contrast, the proposed IRMVB solves (17) iteratively using \((\varepsilon_2^f)^{1/2} \ll 1\) in place of \((\varepsilon_1^f)^{1/2}\). Let

\[
\hat{R} = B^H \hat{R}^{-1} B, \quad \hat{s}_0 = B^H \hat{R}^{-1} s_0,
\]

and applying the Lagrange multiplier methodology to (17),

\[
\tilde{f} = u^H \hat{R} u + \hat{s}_0^H u + u^H \hat{s}_0 + \hat{g}(u^H u - \varepsilon_1^f)
\]

where \( \hat{g} \geq 0 \) is the Lagrange multiplier. Setting the differentiation of (19) with respect to \( u \) to zero gives

\[
\check{u} = -(\hat{R} + \hat{g} I)^{-1} \hat{s}_0.
\]

If \( \|\hat{R}^{-1/2} \hat{s}_0\|^2 > \varepsilon_1^f, \) \( \hat{g} > 0 \) is the root of the constraint equation

\[
\|\check{u}\|^2 = \|\hat{R} + \hat{g} I\|^{-1} \hat{s}_0\|^2 = \varepsilon_1^f.
\]

Otherwise, \( \hat{g} = 0 \). With the obtained \( \check{u} \) in \( \hat{u} \) of (20), the calculated desired array steering vector is

\[
\hat{s}_0 = B \check{u} + \hat{s}_0.
\]

The proposed IRMVB algorithm (with flat ellipsoidal uncertainty set) is summarized. Let the steering vector determined at the \( i \)th iteration of the proposed algorithm and the corresponding Lagrange multiplier be \( \hat{s}_i \) and \( \hat{g}_i \), respectively. Given \( B \), let \( \hat{s}_0 = B^H \hat{R}^{-1} \hat{s}_0 \) where \( \hat{s}_0 \) is the updated presumed desired array steering vector at the \( i \)th iteration of the proposed algorithm.

1) At \( i = 0 \), initialize \( \hat{s}_0 = \hat{s}_0 \) and \( \hat{s}_0 = B^H \hat{R}^{-1} \hat{s}_0 \).

2) When \( i \geq 1 \), if \( \|\hat{R}^{-1} \hat{s}_i\|^2 \leq \varepsilon_2^f \), set \( \hat{g}_i = 0 \). If \( \|\hat{R}^{-1} \hat{s}_i\|^2 > \varepsilon_2^f \), solve equation (21) with \( \hat{g}_i^{-1} \) and \( \varepsilon_2^f \) (instead of \( s_0 \) and \( \varepsilon_1^f \), respectively) to obtain \( \hat{g}_i \). Calculate \( \check{u} \) by (20) using \( \hat{s}_i^{-1} \) and \( \hat{g}_i^{-1} \) (instead of \( \hat{s}_0 \) and \( \hat{g}_0 \), respectively). Next, obtain \( \hat{s}_i \) by (22) with \( \check{u} \) and \( \hat{s}_0 \) respectively. Obtain \( \hat{s}_i^p = \sqrt{N} \hat{s}_i^0 / \|\hat{s}_i^0\| \) so that \( \|\hat{s}_i^p\| = \sqrt{N} \). We set \( \varepsilon_2 = 0.1 \) (same as \( \varepsilon_2 \)).

3) Check if the stopping criteria in (30) are reached. If (30) is satisfied, go to step (4). If (30) is not satisfied, assign \( \hat{s}_i \) to \( \hat{s}_0 \) to update the presumed desired array steering vector, calculate \( \hat{s}_i = B^H \hat{R}^{-1} \hat{s}_i \) and repeat step (2).

4) Use the converged \( \hat{s}_0^{-1} \) to replace \( s_0 \) in the MV beamformer (7) to obtain the proposed IRMVB weight.

### C. Design of Stopping Criteria

If the iterative process is allowed to continue and if the interference power \( \sigma_i^2 \) is higher than the desired signal power \( \sigma_0^2 \), i.e.,

\[
\sigma_i^2 = \frac{1}{s^H(\hat{\theta}_i)\hat{R}^{-1}s(\hat{\theta}_i)} > \frac{1}{s^H(\theta_0)\hat{R}^{-1}s(\theta_0)} = \sigma_0^2
\]

where \( \hat{\theta}_i \in \Theta_i \) (the set of the interferences’ DOAs), then the output SINR will eventually decrease as the calculated steering vector converges to the steering vectors of the interferences. This is because the proposed IRMVB seeks to find a solution to minimize its objective function and

\[
s^H(\hat{\theta}_i)\hat{R}^{-1}s(\hat{\theta}_i) < s^H(\theta_0)\hat{R}^{-1}s(\theta_0)
\]

is achieved when the calculated steering vector by the proposed IRMVB converges to the steering vectors of the interferences. Thus, stopping criteria are required to interrupt the iterative algorithm once the desired array steering vector is reached.

To help shed light on the design of the stopping criteria in the proposed IRMVB, the sensitivity analysis in optimization problems is used [16]. Consider the standard optimization problem subject to perturbations \( a, b \):

\[
\begin{align*}
\min_{x} & \quad f(x) \\
\text{s.t.} & \quad p(x) \leq a, \quad q(x) = b
\end{align*}
\]

where \( f(\cdot), p(\cdot), q(\cdot) \) are functions and the variable \( x \in C^N \). This corresponds to the standard optimization problem when \( a = 0 \) and \( b = 0 \), i.e., no perturbation. The Lagrangian associated with the standard optimization problem is

\[
L = f(x) + \alpha p(x) + \beta q(x)
\]

where \( \alpha \geq 0 \) and \( \beta \) are the dual variables or Lagrange multipliers. If there is strong duality and dual optimum is achieved, let \((\alpha^*, \beta^*)\) be optimal for the dual of the standard optimization problem, then, for all \( a \) and \( b \),

\[
h^*(a, b) \geq h^*(0, 0) - \alpha^* a - \beta^* b
\]

where \( h^*(a, b) \) is the optimal value of the perturbed problem. Assuming \( h^*(a, b) \) is differentiable at \( a = 0, b = 0 \), the
optimal Lagrange multipliers $\alpha^*$ and $\beta^*$ are related to the gradient of $h^*$ at $a = 0$, $b = 0$ as

$$
\alpha^* = -\frac{\partial h^*(0,0)}{\partial a}, \quad \beta^* = -\frac{\partial h^*(0,0)}{\partial b}.
$$

(28)

The optimal $\alpha^*$ and $\beta^*$ are the local sensitivities of the optimal value $h^*(0,0)$ with respect to the constraint perturbations. In other words, they are measures of how active the constraints are at the optimal $x^*$. Suppose $p(x^*) = 0$, then the inequality constraint is active and $\alpha^*$ indicates how active this constraint is. If $\alpha^*$ is large, the impact on the optimal value is large even if the constraint is tightened or relaxed slightly. If $\alpha^*$ is small, the constraint can be tightened or relaxed slightly without much impact on the optimal value.

We can make use of the previous analysis to design the stopping criteria of the IRMVB. There is one inequality constraint in (9) and (17). Without loss of generality, we use (9) (with spherical uncertainty set) for subsequent discussion. At any $i$th iteration of the proposed IRMVB algorithm, if the Lagrange multiplier $g^i$ is large, it suggests that the objective function value may be decreased further since a slight adjustment in the constraint will have a large impact on the optimal value. In contrast, if $g^i$ is small, it suggests that the optimal value is reached as a slight adjustment in the constraint will not impact the optimal value much.

Further insights can be gained from considering the $s^H(\theta) R^{-1} s(\theta)$ spectrum (reciprocal of the power spectrum). At high desired signal’s SNRs, a clear trough/minimum is formed at the desired signal direction. The proposed IRMVB starts at the presumed desired array steering vector. By minimizing the objective function, the steering vectors calculated by the proposed IRMVB approach the desired array steering vector $s_0$ iteratively and the Lagrange multiplier $g$ is decreased at each iteration. When the desired array steering vector is reached, this minimum of $s^H(\theta) R^{-1} s(\theta)$ is reached and the corresponding Lagrange multiplier $g$ is very small. As the proposed IRMVB uses a small sphere, the current steering vector obtained by the proposed IRMVB is very near to that of the next iteration. Again, the next Lagrange multiplier $g$ will also be very small. Therefore, we can trigger the stopping of the proposed algorithm once $|g^i - g^{i-1}| \leq \delta$ is satisfied where $\delta$ is a threshold to determine that the difference between two consecutively obtained $g$ values is small enough such that the two corresponding calculated $\hat{s}_0^i$’s are very near to $s_0$.

On the contrary, at low desired signal’s SNRs, the previous stopping criterion does not work well as the $s^H(\theta) R^{-1} s(\theta)$ spectrum does not show a clear trough/minimum at the desired signal direction and there are other troughs/minimums corresponding to the interferences with higher powers. Hence, the Lagrange multiplier can be large when the proposed IRMVB reaches the desired array steering vector because there seems no trough/minimum corresponding to the desired signal and that the optimal value can be further reduced with more iterations. If the iterative algorithm continues, the calculated steering vector will converge to the steering vectors of the interferences instead and the output SINR will be very poor. Hence, an additional stopping criterion is needed. We propose to stop the algorithm when the inner product between $\hat{s}_0^i$ calculated by the proposed IRMVB at the $i$th iteration and $s_0$ is equal or less than the inner product between $s_t$ and $s_0$ (presumed desired array steering vector). $s_t$ is a steering vector that corresponds to an angle of $\theta_t + \Delta \theta$ or $\theta_t - \Delta \theta$, whichever results in a smaller inner product with $s_t$. $\Delta \theta$ is related to the DOA uncertainty range of the desired signal and it is usually given as a system design requirement for a specific application [17]. For example, in wireless communications, a coarse knowledge of the beamforming scenario can be available from field trials or measurement campaigns [3] and information such as the average mobility rate of the mobile users can be exploited to obtain $\Delta \theta$. It is assumed that no interference will arrive in the DOA uncertainty region of the desired signal $[\theta_0 - \Delta \theta, \theta_0 + \Delta \theta]$ where the desired signal is expected to arrive from. Note that the DOA uncertainty range of the desired signal is also used to implement the beamformers of [11], [18]. The second stopping criterion prevents the angle $\theta^i$ between $\hat{s}_0^i$ calculated by the proposed IRMVB and $s_0$ from exceeding $\Delta \theta$ where $0^\circ \leq \Delta \theta < 90^\circ$, i.e.,

$$
\cos \theta^i = \frac{|s_0^H s_0|}{|s_0^i||s_0|} = \min\{\cos(\theta_0 \pm \Delta \theta)\}.
$$

(29)

Note that the $\theta^i$ used in the second stopping criterion differs from the generalized angle $\theta^i$ of (15). The former ($\theta^i$) is the angle between the calculated steering vector of the proposed IRMVB $\hat{s}_0^i$ and the presumed desired array steering vector while the latter ($\theta^i$ of (15)) is the angle between $\hat{s}_0^i$ and the true desired array steering vector.

The second stopping criterion can be used when the desired array steering vector suffers from steering direction error or array calibration errors or both. For brevity sake, Fig. 2 illustrates the concept of the second stopping criterion for the case of array calibration errors only where the desired array steering vector lies in the green sphere of size $\epsilon_1$ centred at the presumed one $s_0$. Although $\Delta \theta$ is not directly related to the non-angular distortions of the desired array steering vector (which is difficult to estimate in practice), the second stopping criterion ensures that at low SNRs of the desired signal, the calculated steering vector by the proposed IRMVB does not go out of the dotted cone in Fig. 2 which describes the DOA uncertainty region of the desired signal $[\theta_0 - \Delta \theta, \theta_0 + \Delta \theta]$. In this way, the final converged steering vector by the proposed IRMVB can be ensured to be still near to the desired array steering vector. In the same case at high desired signal’s SNRs, the first stopping criterion in (30) is in effect.

In summary, the proposed IRMVB stops upon reaching

$$
(g^i < 1 \text{ and } |g^i - g^{i-1}| \leq \delta) \text{ or } \left( \frac{|s_0^H s_t|}{|s_0^i||s_0|} \leq \frac{|s_0^H s_0|}{|s_t||s_0|} \right).
$$

(30)

The first stopping criterion is triggered only when the Lagrange multiplier $g$ starts to become small, i.e., $g^i < 1$. At high SNRs of the desired signal, we can expect that the first stopping criterion be triggered before the second one. Note that the knowledge of the desired signal’s SNR is not required to implement the proposed IRMVB and it can be easily implemented by solving (13) or (21) with a Newton’s method
Fig. 2. Concept of the second stopping criterion in the proposed IRMVB in the presence of array calibration errors only. Due to calibration errors, the desired array steering vector is located in the green sphere of size $\varepsilon_2$ centred at the presumed one $\hat{s}_0$. The proposed IRMVB uses a small red sphere of size $\varepsilon_1$ to search for $s_k$ iteratively. At low SNRs of the desired signal, the second stopping criterion prevents the steering vectors calculated by the proposed IRMVB from going out of the dotted cone which describes the DOA uncertainty region of the desired signal $[\theta_0 - \Delta \theta, \theta_0 + \Delta \theta]$.

(similar to [4]) while [5], [18]–[21] require specialized interior point method solvers, e.g., [22]. From the simulation results, the IRMVB always converges quickly with few iterations.

IV. SIMULATION RESULTS

The proposed IRMVB is tested on a uniform linear array of 10 isotropic elements with a $0.5\lambda$ spacing where $\lambda$ is the wavelength of the impinging signals. The noise is spatially white Gaussian with unit variance. The desired signal is always present in the array snapshots with DOA and SNR of $[96^\circ, 0\text{dB}]$, respectively but it is presumed to be at $\theta_0 = 90^\circ$. There is a steering direction error of $6^\circ$ and we assume the DOA uncertainty region of the desired signal is given as $[83^\circ, 97^\circ]$ where $\Delta \theta = 7^\circ$. There are 2 interferences with DOAs and interference-to-noise ratios (INRs) of $[110^\circ, 30\text{dB}]$ and $[120^\circ, 30\text{dB}]$, respectively. The abbreviation “BF” in the plots stands for “Beamformer” and 100 Monte Carlo trials are used to obtain each output SINR point. The proposed IRMVB uses the spherical constraint unless stated otherwise, with $\delta = 0.01$ and $\varepsilon_2 = 0.1$ (this choice is discussed later).

A. Output SINRs of Beamformers Versus Number of Snapshots

In the first example, the output SINR of the proposed IRMVB versus the number of snapshots is shown in Fig. 3(a). The beamformer of Li et al. [4] is tested with optimal $\varepsilon_1 = 8.5$. The beamformer of Shahbazpanahi et al. [3] is tested with diagonal loading $1 = 16$ added to $R$ and optimal $R_s$ loading $= -7$. The beamformer of Yu et al. [11] is tested with optimal relative regularization factor $= 0.1$. The beamformer of Hassanien et al. [18] is tested with the optimal number of eigenvectors of dominant eigenvalues $= 4$ and diagonal loading $= 10$ is used to control the sidelobes. The same DOA uncertainty region is used in [11], [18]. The MV beamformer is tested with $s_0$ in (7). The other parameters remain the same.

Overall, the proposed IRMVB achieves the best output SINR in Fig. 3(a). When the number of snapshots is 500 or more, the output SINR is about $0.3\text{dB}$ from the optimal SINR. The performance of the beamformer of [18] is very near to that of the proposed IRMVB and their normalized beampatterns in Fig. 3(b) at 100 snapshots show that they are able to find the desired array steering vector by pointing their mainlobes to the desired signal direction at $96^\circ$ instead of the presumed one (shown as a dashed vertical line). On the other hand, the beamformers of [3], [4] point their mainlobes towards the presumed desired signal direction. The beamformer of [11] forms a broad mainlobe with a controlled response

1The rule of thumb for choosing the diagonal loading added to $R$ is $10\text{dB}$ to $12\text{ dB}$ above the noise level [3].
ripple according to the DOA uncertainty range and inevitably includes more noise in the beamformer output.

B. Output SINRs of Beamformers Versus SNR

In the second example, the output SINR of the proposed IRMVB versus the desired signal’s SNR is shown in Fig. 4. The number of snapshots is 100. The other parameters remain the same. The performances of the proposed IRMVB and the beamformer of [18] are very similar at high SNRs but the proposed IRMVB outperforms the latter at low SNRs. This is because the orthogonal matrix projection operation in [18] increases its noise power at low SNRs. The proposed IRMVB also outperforms the beamformer of [4] due to its improved interference-plus-noise suppression ability derived from the use of a small uncertainty sphere iteratively.

C. Output SINRs of IRMVBs Versus Choice of $\varepsilon_2$

In the third example, the effect of $\varepsilon_2$ on the output SINR of the proposed IRMVB at different desired signal’s SNRs ($-10$dB and $6$dB) is shown. The choices of $\varepsilon_2$ are 0.01, 0.1, 0.5, and 1. The other parameters remain the same as the second example. Note that the stopping criteria in (30) are not imposed in the IRMVBs as the purpose is to study their behaviour as the iterative processes continue. The iteration index at which the stopping criteria in (30) would have stopped the IRMVB algorithm (if they were implemented) are indicated at the corresponding output SINRs using different markers in the plots.

From Fig. 5, the optimal output SINR of the proposed IRMVB is insensitive to $\varepsilon_2$ provided that $\varepsilon_2 \ll \varepsilon_1$. This is an attractive property. A small $\varepsilon_2$ reduces the sensitivity of the proposed IRMVB to output SINR degradation if the IRMVB algorithm is not stopped precisely at the optimal output SINR but it also increases the number of iterations to reach the optimal output SINR. In contrast, a large $\varepsilon_2$ allows the IRMVB to converge quickly to the optimal output SINR but it is important to stop the IRMVB algorithm precisely as the output SINR can rapidly decrease with further iterations.

At low desired signal’s SNR, i.e., $-10$dB in Fig. 5(a), the second stopping criterion in (30) is in effect. At high desired signal’s SNR, i.e., 6dB in Fig. 5(b), the first stopping criterion in (30) is in effect. From Figs. 5(a) and 5(b), the proposed stopping criteria in (30) are effective in stopping the IRMVBs at nearly the optimal output SINRs at various SNRs and $\varepsilon_2$ values. Finally, we recommend $\varepsilon_2 = 0.1$ in the proposed IRMVB (plotted in red with a circle marker in Figs. 5(a) and 5(b)). Compared to other $\varepsilon_2$ values, $\varepsilon_2 = 0.1$ offers both robustness against output SINR degradation and fast convergence where the proposed IRMVB typically stops between 10 – 20 iterations for a large steering direction error of $6^\circ$. 

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Fig. 4. Optimal SINR and output SINRs of the proposed IRMVB, the beamformer of Li et al. [4], the beamformer of Shahbazpanahi et al. [3], the beamformer of Yu et al. [11], the beamformer of Hassanien et al. [18], and the MV beamformer. There is a steering direction error of $6^\circ$.

Fig. 5. Optimal SINR and output SINRs of the proposed IRMVBs with $\varepsilon_2 = 0.01, 0.1, 0.5$, and $1$ at SNRs $= -10$dB and $6$dB, respectively. There is a steering direction error of $6^\circ$. No stopping criteria are imposed in these IRMVBs. For each $\varepsilon_2$, a marker is used to indicate the iteration index at which the proposed stopping criteria in (30) would have stopped at and the corresponding output SINR if they were implemented.
D. Output SINR of IRMVB versus Choice of $\Delta \theta$

In the fourth example, we discuss, using Fig. 6, the effect of $\Delta \theta$ in the second stopping criterion of (30) which affects the iteration index at which the proposed algorithm is stopped, at low SNR = $-10$ dB. Fig. 6 is actually the red curve with circle marker of Fig. 5(a). Like in [11], [18], the DOA uncertainty range $[\theta_0 - \Delta \theta, \theta_0 + \Delta \theta]$ is the region where the desired signal is expected to arrive from. Fig. 6 shows an interesting observation where the output SINR of the proposed IRMVB increases as $\Delta \theta$ increases to $8^\circ$ even though the steering direction error is $6^\circ$. This is because after $\Delta \theta > 6^\circ$, the generalized angles between the subsequent steering vectors calculated by the proposed IRMVB and the desired array steering vector continue to decrease which, in turn, increases the output SINR of the proposed IRMVB. Refer to (14) and (15). Though the steering direction error of $6^\circ$ is unknown, Fig. 6 shows that the proposed IRMVB is rather robust to the choice of $\Delta \theta$, i.e., the difference in the output SINRs with $\Delta \theta = 6^\circ$ and $\Delta \theta = 8^\circ$ is less than 1 dB.

E. Output SINRs of Beamformers With Array Calibration Errors

In the fifth example, only array calibration errors (sensor amplitude, phase, and position errors) are considered by perturbing each element of the steering vector of each impinging signal with a zero-mean circularly symmetric complex Gaussian random variable, i.e., $s = s + \Delta s$ with $\|\Delta s\|^2 = 0.2\|s\|^2$ where $s$ and $\Delta s$ are the true and presumed array steering vectors, respectively. The perturbing Gaussian random variables are independent of each other. The beamformer of Li et al. [4] uses optimal $\varepsilon_1 = 5.5$. The beamformer of Shahbazpanahi et al. [3] uses optimal $R$, loading $= -5.5$. There are no steering direction error but we still use $\Delta \theta = 3^\circ$. The DOA uncertainty region $[87^\circ, 93^\circ]$ is used in the proposed IRMVB, the beamformer of Yu et al. [11], and the beamformer of Hassanien et al. [18], and the MV beamformer. There are array calibration errors.

F. Output SINRs of Beamformers With Flat Ellipsoidal and Spherical Constraints

In the sixth example (based on an example in [4]), we compare the output SINRs of the proposed IRMVB using flat ellipsoidal and spherical constraints, respectively in the presence of steering direction errors. Finite snapshot effect is not considered here. The desired signal is at $100^\circ$ but it is presumed to be at $102^\circ$. We assume $\Delta \theta = 3^\circ$. There are 8 interferences with DOAs of $[15^\circ, 30^\circ, 45^\circ, 60^\circ, 80^\circ, 115^\circ, 125^\circ, 140^\circ]$, all at INR $= 50$ dB. The beamformer of Li et al. [4] uses optimal $\varepsilon_1 = 6.5$ (for spherical constraint). For the proposed IRMVB and the beamformer of Li et al. [4], both using flat ellipsoidal constraints, let $s(\theta_0) - s(\theta_0 - \Delta \theta)$ and $s(\theta_0) - s(\theta_0 + \Delta \theta)$ be the first and second columns of $B$, respectively. In Fig. 8(a), the proposed IRMVB with spherical constraint outperforms the beamformer of Li et al. [4] with spherical constraint. At high SNRs ($\geq 30$ dB), the proposed IRMVB with flat ellipsoidal constraint outperforms the beamformer of Li et al. [4] with flat ellipsoidal constraint which, in turn, outperforms the proposed IRMVB with spherical constraint. As the proposed IRMVB with flat ellipsoidal constraint has the best output SINR at SNR $= 35$ dB, we examine the beam-patterns of the beamformers at SNR $= 35$ dB in Fig. 8(b). The beam-patterns of the proposed IRMVB and the beamformer of Li et al. [4], both using flat ellipsoidal constraints, are similar. However, the output SINR of the proposed IRMVB with flat ellipsoidal constraint is better than the latter’s due to its superior interference rejection ability as a result of
using a small flat ellipsoid to search for the desired array steering vector. This is evident from the deeper nulls formed at the interferences’ DOAs by the proposed IRMVB with flat ellipsoidal constraint. This example suggests that in some cases, if there exists prior information about the beamforming scenario at hand, using the proposed IRMVB with the small flat ellipsoid to search for the desired array steering vector can be more beneficial in achieving a higher output SINR.

G. Power Spectrums of Beamformers With Array Calibration Errors

In the final example, we show the DOA estimation performance of the proposed IRMVB based on an “imaging” example in [4] via the power spectrum. There are 5 signals with DOAs of \([55^\circ,75^\circ,90^\circ,100^\circ,130^\circ]\) and SNRs of \([30, 15, 40, 35, 20]\)dB, respectively. There are array calibration errors. Dotted vertical lines indicate the impinging signals’ DOAs. Dotted horizontal line indicates the 0dB gain level.

V. CONCLUSION

An IRMVB which uses a small uncertainty sphere (and a small flat ellipsoid) to search for the desired array steering vector iteratively has been proposed. By preserving its DOFs and in turn, its interference-plus-noise suppression ability, and by using the corrected desired array steering vector, the proposed IRMVB achieves higher output SINR than the worst-case performance optimization based beamformers. Theoretical analysis and simulation results have been presented to support the effectiveness of the proposed beamformer.

APPENDIX I

PROOF OF THEOREM 1

We assume that the interferences are not located near to the protected mainlobe region and the theoretical array covariance matrix \( \mathbf{R} \) is used in place of \( \mathbf{R} \). Let the steering vector calculated by the proposed IRMVB at the \( i \)th iteration be \( \hat{s}_0^i \). We set \( s_0^0 = s_0 \) and at the 1st iteration \((i = 1)\), \( s_1^0 = s_1 \). As noted in [4], normalization is done to remove the scaling ambiguity, thus \( s_1^i = \sqrt{N}s_1^i/||s_1^i|| \) so that \( ||s_1^i|| = \sqrt{N} \). Let the
generalized angle between $s_0$ and $s_0$ be $\hat{\theta}^0$ and that between $s_1$ and $s_0$ be $\hat{\theta}$. We set out to prove that

$$\cos^2(\hat{\theta}; R_0^{-1}) = \frac{|s_1^H R^{-1}_0 s_0|^2}{|s_1||s_0|^2} \geq \frac{|s_1^H R^{-1}_0 s_0|^2}{|s_0||s_0|^2} = \cos^2(\hat{\theta}; R_0^{-1})_s = s_1^H R^{-1}_0 s_0 = s_1^H [U_s\ U_n] \begin{bmatrix} A_1^{-1} & 0 \\ 0 & \sigma_n^{-2}I \end{bmatrix} [U_s\ U_n]^H U_s c$$

(31)

where $|x|^2 = x^H x$ is the extended vector norm-squared and the output SINR in (14) is a monotonically increasing function of $\cos^2(\hat{\theta}; R_0^{-1})$, meaning that the proposed IRMVB increases the output SINR after one iteration. We apply eigendecomposition on $R$ in (11), so

$$s_1 = U \begin{bmatrix} \frac{\lambda_1}{\sqrt{\lambda_1}} \\ \vdots \\ \frac{\lambda_K}{\sqrt{\lambda_K}} \end{bmatrix} = U^H s_0$$

(32)

where the columns of $U$ are the eigenvectors of $R$ and $\lambda_k$ where $k = 1, \ldots, N$ are the corresponding eigenvalues.

In the presence of $K$ strong signals and spatially white Gaussian noise, $\lambda_1 \geq \lambda_2 \geq \ldots \lambda_K \gg \lambda_{K+1} = \ldots = \lambda_N$, $\frac{\lambda_k}{\sqrt{\lambda_k}} \approx 1$ for $k = 1, 2, \ldots, K$ and $\frac{\lambda_k}{\sqrt{\lambda_k}} \approx 0$ for $k = K + 1, K + 2, \ldots, N$. Thus, (32) can be approximated as

$$s_1 \approx U_s U_s^H s_0$$

(33)

where the columns of $U_s$ are the eigenvectors corresponding to the largest $K$ eigenvalues of $R$. The desired array steering vector $s_0$ is spanned by $U_s$, i.e., $s_0 = U_s c$ where $c$ is a coefficient vector. Since the space of the interferes is a subspace of $U_s$, the interference-plus-noise covariance matrix $R_{in}$ can be expressed as [15]

$$R_{in} = R + \sigma_n^2 I = [U_s\ U_n] \begin{bmatrix} A_1 & 0 \\ 0 & \sigma_n^{-2}I \end{bmatrix} [U_s\ U_n]^H$$

(34)

where $R_1$ is the interference covariance matrix, the $K \times K$ matrix $A_1$ may not be a diagonal matrix, $\sigma_n^2$ is the variance of the white noise, and the columns of $U_n$ are the eigenvectors corresponding to the smallest $N - K$ eigenvalues of $R$. Working on the left hand side (LHS) of the inequality of (31),

$$s_1^H R_{in}^{-1} s_0 = s_1^H [U_s\ U_n] \begin{bmatrix} A_1^{-1} & 0 \\ 0 & \sigma_n^{-2}I \end{bmatrix} [U_s\ U_n]^H U_s c$$

(35)

$$= \sqrt{N}\frac{s_1^H U_s}{|U_s U_s^H s_0|} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} A_1^{-1} & 0 \\ 0 & \sigma_n^{-2}I \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}^H U_s c$$

(36)

and

$$\|s_1\|^2 = s_1^H R_{in}^{-1} s_1$$

(38)

Finally, the LHS of the inequality of (31),

$$\frac{|s_1^H R_{in}^{-1} s_0|^2}{\|s_1\|^2} = \frac{\sqrt{N}\frac{s_1^H U_s}{|U_s U_s^H s_0|} |U_s U_s^H s_0|^2}{\|s_1\|^2}$$

(39)

$$= \frac{\sqrt{N}\frac{s_1^H U_s}{|U_s U_s^H s_0|} |U_s U_s^H s_0|^2}{\|s_1\|^2}$$

(40)

Working on the right hand side (RHS) of the inequality of (31),

$$\frac{\|s_0\|^2}{\|s_0\|^2} \leq \|s_0\|^2 = \|s_0\|^2$$

(41)

and

$$\|s_0\|^2 = \|s_0\|^2$$

(42)

Finally, the RHS of the inequality of (31) is

$$\frac{|s_1^H R_{in}^{-1} s_0|^2}{\|s_0\|^2} \leq \frac{|s_1^H R_{in}^{-1} s_0|^2}{\|s_0\|^2} = \cos^2(\hat{\theta}; R_0^{-1})$$

(43)

Since $\sigma_n^{-2} |U_s^H s_0|^2 |s_0|^2 \geq 0$,

$$\cos^2(\hat{\theta}; R_0^{-1}) = \frac{|s_1^H R_{in}^{-1} s_0|^2}{\|s_0\|^2} \leq \frac{|s_1^H R_{in}^{-1} s_0|^2}{\|s_0\|^2} = \cos^2(\hat{\theta}; R_0^{-1})$$

(46)

this shows that the proposed IRMVB does improve the output SINR after one iteration as the generalized angle between the calculated steering vector and the desired array steering vector is reduced. For subsequent iterations ($i > 1$), similar reasoning applies for $s_0^i$ and $s_0^{i-1}$. Therefore, we have Theorem 1, i.e., $\cos^2(\hat{\theta}; R_0^{-1}) \geq \cos^2(\hat{\theta}; R_1^{-1})$, indicating that the IRMVB reduces the generalized angle between its calculated steering vector and the desired array steering vector with each iteration, thereby increasing its output SINR.

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