Abstract — Power distribution systems are now integrating Smart Grid technologies allowing for widespread implementation of demand response and direct load control. This paper will present a service restoration problem formulation and solution algorithm for power distribution systems which incorporates load curtailment. A ranking based heuristic search algorithm is proposed which prioritizes the multi-objective service restoration problem and returns a new system configuration and load curtailment scheme to restore power to out-of-service customers following fault isolation. A numerical example is presented to illustrate the mechanics of the proposed algorithm.

I. INTRODUCTION

In modern power distribution systems, increasing attention and investment is being focused on Smart Grid technologies allowing generation, network, and load to interact in real time using existing and new communication infrastructures [1]. These technologies facilitate new control and automation applications. Among these applications is broader implementation of demand response and direct load control via smart metering and intelligent appliances [2].

Power distribution systems serve as the direct link to electric power for the majority of consumers. These systems are operated as unbalanced networks where loading levels may vary across each phase. In addition, they are generally operated in a radial configuration for ease of protection and system design. Normally closed sectionalizing switches (ss) and normally open tie switches (ts) are available to reconfigure distribution networks.

Meanwhile, the availability of demand side management is increasing. Direct load control allows utility operators to automatically curtail customer power demands by controlling air conditioning and water heater temperature set points and other intelligent appliances. Customers participating in load curtailment (LC) may be compensated through discounted electricity rates while still being guaranteed a base load condition. A similar concept is demand response which allows customers to automatically control power consumption based on market signals.

One application which can benefit from the Smart Grid paradigm is service restoration. Service restoration is typically formulated to maximize out-of-service load restored after fault isolation e.g. [3, 4]. This goal is achieved through network reconfiguration and determining the status of each switch in the network. Work has extended formulations to include network operating constraints and priority customer considerations e.g. [5]. This paper will further extend the service restoration problem formulation to include load curtailment during outage conditions.

By curtailing the load of participating customers, systems can increase capacity to restore priority and non-priority load. The addition of LC significantly increases problem complexity. Yet, this approach has the human centric objective of ensuring a maximum number of customers will receive at least a minimum amount of service during an outage. Specifically, this paper will present:

- a service restoration problem formulation incorporating load curtailment
- selection indices used for ranking switches and loads
- a heuristic, ranking based, service restoration algorithm
- a numerical example to illustrate the algorithm

II. PROBLEM FORMULATION

The service restoration problem with priority customers is formulated as a constrained non-differentiable multi-objective optimization problem. The objectives considered are: (1) to maximize priority load restored, (2) to maximize total load restored, (3) to minimize number of switching operations, and (4) to minimize the amount of LC.

A feasible solution satisfies electrical constraints expressed as the three-phase power flow equations. Current flows, feeder loading and voltage magnitudes must lie within acceptable operating ranges. Also, switch voltage differentials, load curtailment limits and a radial network structure must be satisfied after reconfiguration. In summary:

\[
\begin{align*}
\max_{\mathbf{I}_{x,1}} & \sum_{k \in \mathcal{N}_p} |I_{x,1}| \\
\max_{\mathbf{I}_{x,1}} & \sum_{k \in \mathcal{N}_g} |I_{x,1}| \\
\min_{\mathbf{n}_{op}} & \sum_{k \in \mathcal{N}_p} |I_{x,1}| \\
\min_{\mathbf{n}_{op}} & \sum_{k \in \mathcal{N}_g} |I_{x,1}| \\
\end{align*}
\]
subject to:

\[
f(V, g_k, u_k) = 0 \quad (5)
\]

\[
\left| I^*_{nf} \right| \leq \left| I^\text{max}_{nf} \right| \quad \forall \ k \in N \quad (6)
\]

\[
P_f + Q_f \leq (S^\text{max}_f)^2 \quad \forall \ f \in F \quad (7)
\]

\[
\left| V^*_{k} \right| \leq \left| V^\text{max}_{k} \right| \quad \forall \ k \in N, \ p = a, b, c \quad (8)
\]

\[
\Delta V_f = \left( \left| V^*_{f} \right| - \left| V^*_{j} \right| \right) \leq \left| V^\text{max}_{k} \right| \quad \forall \ j \in g^\text{slow}_k \quad (9)
\]

\[
\left| I^*_{ts, k} \right| \leq \left| I^\text{max}_{ts, k} \right| \quad \forall \ k \in N_{LC}, \ p = a, b, c \quad (10)
\]

\[
g_k \in G_k \quad (11)
\]

where:

\( g_k \): post-fault network configuration, \( g^\text{slow}_k \) set of \( ts \) closed \n
\( G_k \): set of possible radial configurations \n
\( u_k \): load curtailment scheme \n
\( U \): set of possible load curtailments \n
\( N_{pr} \): set of restored priority customers \n
\( N_{ts} \): set of restored buses \n
\( N_{LC} \): set of buses participating in load curtailment \n
\( I_{LC,k} \): total load current at bus \( k \) \n
\( n_{op} \): number of switch operations to arrive at \( g_k \) \n
\( I_{LC,k} \): total load curtailment current at bus \( k \) \n
\( f(V, g_k, u_k) \): three-phase power flow equations \n
\( V \): node voltage vector \n
\( V^*_{k} \): voltage at bus \( k \), phase \( p \) \n
\( I^*_{k} \): current flow entering bus \( k \), phase \( p \) \n
\( S_f = P_f + jQ_f \): total apparent power entering feeder \( f \) \n
\( S^\text{max}_f \): max capacity of feeder \( f \) or its supplying transformer, whichever is least \n
\( I^*_{ts, k} \): load to be curtailed by customers at bus \( k \), phase \( p \) \n
\( I^\text{max}_{ts, k} \): maximum available load curtailment of customers at bus \( k \), phase \( p \)

The service restoration problem is NP-complete with discrete and continuous variables related to switch status and network voltage and current flows, respectively. The addition of LC increases the complexity of the problem. An upper bound on the search space can be calculated as the set of all possible post-fault network configurations along with all possible LC schemes. Letting \( n_t \) be the number of \( ts \), \( n_s \) be the number of \( ss \), and \( n_{LC} \) be the number of candidate buses for LC with two loading levels each, the upper bound on dimension of the search space is:

\[
(2^{n_s}) \sum_{k=1}^{n_t} \binom{n_t}{k} \binom{n_s}{k-1}
\]

For example, an actual 416 bus system is presented in [5]. The system contains 53 \( ss \), 30 \( ts \), and 208 loads. If all \( ts \) are candidates for curtailment, the search space increases by a factor of \( 2^{208} \). Thus, to facilitate computation of a restoration plan, a ranking based heuristic search method is proposed.

The solution algorithm solves the multi-objective optimization problem by prioritizing the objectives in the order presented. A pareto optimal solution is then found which favors the objectives in this order. That is, no improvement in a lower priority objective is accepted if it degrades the optimality of higher priority objective. This method is chosen over other multi-objective optimization solutions such as a weighted sum objective function or evolutionary algorithm [6].

Total load restored is given priority over LC to improve reliability indices such as system average interruption duration index (SAIDI). The minimization of switching operations is prioritized based on the fact that network switches are inherently more expensive than LC hardware and that repeated operation reduces lifetime and increases required maintenance. To assist the heuristic search, analytically-based search indices are proposed.

III. SWITCH SELECTION INDICES

In this work, four switch indices will be utilized. A new index, \( I^\text{avail}_{ts} \), is proposed which quantifies capacity release available through LC. The three remaining indices are adapted from [5]. The indices allow the restoration algorithm to systematically distinguish between network switches based on graph theoretical arguments and analytically determined criteria. In summary, the indices are:

- \( I^\text{available}_{ts} \): the LC available to each \( ts \)
- \( I_M \): the spare capacity of each \( ts \)
- \( Z_{path} \): the electrical distance of each \( ts \) to other buses
- \( I_{ss} \): the amount of load each \( ss \) can transfer to a \( ts \)

The impact of LC will be to increase the spare capacity of each \( ts \). As such, the index \( I_M \) is reviewed.

Each feeder has a maximum amount of current it can sustain before a branch becomes overloaded or a protection device operates. For each \( ts \), the spare capacity is as follows:

\[
I_M = \min_{k \in CC_p} \left( I^\text{max}_{k} - I^*_{k} \right) \quad (13)
\]

where \( C_p \) is the set of upstream buses on the path from the primary side of each \( ts \) to the substation. In addition to \( I_M \), the critical branch at which this value is found is stored as \( b_{ts} \).

The LC available for a \( ts \), \( I^\text{available}_{ts} \), is defined as the maximum amount of load which may be curtailed from upstream buses to increase \( I_M \). It is computed as the minimum over all phases of the total load which may be curtailed from \( ss \) located on the path downstream from branch, \( b_{ts} \) to the primary/substation side of each \( ts \):

\[
I^\text{available}_{ts} = \sum_{k \in CC_p \cup N_{LC}} I^\text{max}_{ts, k} \quad (14)
\]

\[
I^\text{available}_{ts} = \min_{p} I^\text{available}_{ts, p} \quad (17)
\]
where  is the set of all buses downstream of  to the  
In this work, each candidate for load curtailment has only one level of load control,  
For illustration, a single phase sample distribution system is shown in Figure 1. The system consists of 22 buses located along a main feeder with three laterals. A fault occurs at the denoted location and is assumed to be isolated. The out-of-service area is located downstream of the fault. Assuming critical branches for each candidate  as shown, for  and for  

IV. SERVICE RESTORATION ALGORITHM

Load curtailment should be considered in cases where full restoration of out-of-service loads,  is not attainable and/or to possibly reduce the number of required, e.g. from [5]. Therefore, the service restoration algorithm in [5] is run first and required stored. If multiple out-of service areas exist the following algorithm works sequentially on each out of service area in decreasing order of priority load served.

Step 1: Build candidate tie and sectionalizing switch list

Step 2: Build candidate load curtailment list

Step 3: Select and operate one candidate  and implement LC to attempt full restoration of out-of-service area.

Step 4: Select and operate candidate switch pairs ordered by transferable load,  and implement LC.

Step 5: Determine which non-priority and then priority loads not to restore by opening  and implementing LC.

Step 6: Return new system configuration and load curtailment scheme

The details of each step are now presented.

Step 1: Candidate Switch Lists

Candidate  and  are identified as:

•  from energized feeders that can connect directly into the out-of-service area
•  located in the out-of-service area

The spare capacity, , associated with each  is determined.

Step 2: Candidate LC List

Candidates for LC are participating customers on the path upstream from each candidate  to the corresponding critical branch. Participating customers in the out-of-service areas are not considered. With each candidate , is stored , and a LC options vector , where  is the total number of participating customers in the network. , contains the values of each candidate LC available to a given as in its corresponding row, if the LC customer is not a candidate. If LC candidates are associated with multiple ,  and  are updated as LC is enacted for each .

For each , different combinations of customers can be curtailed to meet LC requirements. When required, the minimum amount of these LC options which is greater than the requirement, e.g. capacity release needed, is selected:

\[
I_{LC} = \min_c e^T I_{LC,n}^o
\]

s.t.  \( e^T I_{LC,n}^o \geq \text{required} \)  

where  is a vector composed of 1’s and 0’s used to compute a partial sum of .

Step 3: Select One 

This step attempts to operate one  and implement available LC to restore the entire out-of-service area. If this is not possible, the candidate  with the largest  is chosen and the algorithm proceeds to Step 4. Specifically, an initial  is selected by the following steps:

S3.1: Create candidate list with  Order  by increasing  \( I_{LC} \geq I_{LC} \) , if empty go to S3.5.

S3.2: Select the next on the list, set as . If none, go to S3.5. Operate , run power flow and check constraints.

• If all constraints are satisfied, go to Step 6.

• If a voltage violation occurs and other  exist, order the  by and select according to [5].

• If an overload violation exists, record the amount as . This represents the minimum amount of LC required for one .

S3.3: Determine and implement  \( I_{LC} \geq Overl \) according to (18) and continue, else go to S3.5

S3.4: Run power flow, check constraints, and record

• If all constraints are satisfied, go to Step 6.

• Else undo and go to S3.2

S3.5: Select with largest  and set as . Run power flow, store  and go to Step 4.

Step 4: Select Candidate Switch Pairs

In this step, switch pairs and load curtailment are sequentially selected to alleviate the overload of . For a given , candidates are those lying in the out-of-service area and in the path from the secondary side of the towards the substation. Switches are not paired together if their operation would create a constraint violation.
The algorithm first attempts to select one additional switch pair \((ts_A, ss_A)\) with \(I_{ts1}^{ss} \geq I_{ts1}^{ss_A}\) and \(I_{ts1}^{ss} \geq Ovrld\) to alleviate the overload of \(ts_1\) similar to [5]. If this cannot be achieved and only one additional switch pair is to be utilized, then load curtailment is necessary (but not sufficient):

- Determine (minimal) LC for \(ts_1\) and the operation of \((ts_A, ss_A)\); i.e. \(I_{LC}^{ts} \geq Ovrld - I_{ts1}^{ss}\).
- Determine (minimal) LC for \(ts_A\) to allow for a new \((ts_A, ss_{new})\) pair with \(I_{ss_{new}}^{ts} \geq Ovrld\); i.e. \(I_{LC}^{ts} \geq I_{ss_{new}}^{ts} - I_M^{ts}\).
- Determine combination of LC on \(ts_1\) and \(ts_A\) would allow for a new \((ts_A, ss_{new})\) pair; \(I_{ts1}^{ss} \geq I_{ss_{new}}^{ts} - I_M^{ts}\) and \(I_{LC}^{ts} \geq Ovrld - I_{ss_{new}}^{ts}\).
- Select the option with minimal LC from above and check constraints.

If the entire out-of-service area cannot be served, an additional switch pair is needed. Select the \((ts, ss)\) including LC yielding the largest \(I_MAX\) transferred away from \(ts\) and repeat the process. If no additional pairs are available, proceed to Step 5.

**Step 5: Determine Loads Not to be Restored**

If the entire out-of-service area cannot be restored through Steps 3 and 4 without constraint violations, then non-priority and, if necessary, priority loads must be dropped. This is accomplished by opening \(ss\) according to the method described in [5].

**Step 6: Return Feasible Control Scheme**

To implement the service restoration plan, the load curtailment scheme should be executed before any switching operations are performed. Then, \(ss\) identified in Step 4 and Step 5 are opened. Next, all \((ts_i, ss_i)\) pairs are operated and \(ts_1\) is closed.

**V. NUMERICAL ILLUSTRATION**

The system shown in Figure 1 is used to demonstrate the service restoration algorithm. An initial power flow is run to calculate the post-outage system states. The candidate switches are identified as \(ts_1, ts_2, ss_2, ss_3\) and \(ss_4\). The spare capacity, critical branch, and available LC of each tie switch are calculated. \(I_{LC,ts1}^{ss}\) and \(I_{LC,ts2}^{sw}\) for \(ts_1\) and \(ts_2\) are:

\[
I_{LC,ts1}^{ss} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T
\]

\[
I_{LC,ts2}^{ss} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T
\]

Here, 80 different settings of load curtailment and radial network reconfiguration are possible: 8 settings correspond to \(n_{ops} = 1\), operating one \(ts\); 48 correspond to \(n_{ops} = 3\), operating a tie switch and a switch pair; 24 settings correspond to \(n_{ops} = 2\), operating a tie switch and a sectionalizing switch, where not all load can be restored.

**Table 1. Numerical values for switch indices corresponding to Figure 1.**

<table>
<thead>
<tr>
<th>ts1</th>
<th>ts2</th>
<th>ss1</th>
<th>ss2</th>
<th>ss3</th>
<th>ss4</th>
</tr>
</thead>
<tbody>
<tr>
<td>I_{ds}</td>
<td>I_{ds}^{out}</td>
<td>I_{ms}</td>
<td>I_{ms}</td>
<td>I_{ms}</td>
<td>I_{ms}</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>17</td>
<td>10</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>7</td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For this example, \(I_{ds} = 17\) and additional index values may be seen in Table 1. By utilizing the switch selection indices, the proposed analytically-based heuristic can quickly identify potentially infeasible options and significantly reduce the effective search space. With respect to the numerical example, in Step 3, utilizing \(I_{ds} + I_{ds}^{out}\) the algorithm quickly avoids all 8 settings corresponding to \(n_{ops} = 1\). In Step 4, the use of \(I_{ms} + I_{ms}^{out}\) eliminates 36 settings (or \(3!\)) of the possible \(n_{ops} = 3\) settings and orders the remaining 12 options with respect to the minimal LC. Then, control options are selected sequentially in increasing order of LC if needed. For this case, the algorithm determines that the entire out-of-service area can be restored by operating \(ts_1, (ts_2, ss_3)\) and \(I_{LC,3}\).

**VI. CONCLUSIONS**

The problem of service restoration in power distribution systems including load curtailment has been addressed. With the inclusion of LC, it is expected that previous service restoration options can be improved with respect to either the total load restored and/or the number of required switch operations. A multi-objective problem formulation has been presented together with switch selection indices based on graph theory and power flow analysis. The proposed method relies on ranking the switch indices to eliminate infeasible solutions while identifying sets of non-inferior optimal solutions. A sample implementation was presented to demonstrate the mechanics of the proposed method. It is anticipated that the work can both justify the implementation of Smart Grid technologies in distribution systems and benefit from the same technological advancements.

**REFERENCES**


