Range of Influence of Physical Impairments in Wavelength-Division Multiplexed Systems

Houbing Song and Maëtit Brandt-Pearce
Charles L. Brown Department of Electrical and Computer Engineering
University of Virginia
Charlottesville, VA 22904
Email: song@virginia.edu, mb-p@virginia.edu

Abstract—In wavelength-division multiplexed (WDM) systems, system performance is adversely affected by severe physical impairments due to fiber loss, dispersion and nonlinearity. Fiber modeling is a prerequisite for the development of physical impairment mitigation techniques to improve system performance. The distance between two interacting symbols in time and wavelength, i.e., the range of influence (RoI) of each physical impairment, plays an important role in the development of these mitigation techniques. This paper develops a two-dimensional (time and wavelength) discrete-time input-output model of WDM systems based on the Volterra series transfer function (VSTF) method. This model takes into account multiple channel effects, fiber loss, and frequency chirp, which are ignored in the current literature. Furthermore, with this model we define coefficients that capture intersymbol interference (ISI), self phase modulation (SPM), interchannel cross phase modulation (IXPM), interchannel four wave mixing (IFWM), cross phase modulation (XPM) and four wave mixing (FWM) to characterize the impact of these impairments individually on the system output. By investigating these impairment coefficients, the RoIs for different physical impairments are determined.

I. INTRODUCTION

In wavelength-division multiplexed (WDM) systems, severe physical impairments are inevitable due to fiber loss, dispersion and nonlinearity. Intersymbol interference (ISI), interchannel interference (ICI), self phase modulation (SPM), interchannel cross phase modulation (IXPM), interchannel four wave mixing (IFWM), cross phase modulation (XPM) and four wave mixing (FWM) together with noise, adversely affect system performance [1] [2] [3] [4].

Fiber modeling is a prerequisite for the development of physical impairment mitigation techniques to improve system performance. A desirable model of WDM systems is two-dimensional (time and wavelength), discrete-time, and able to characterize physical impairments individually. Such a model has the potential to facilitate the development of physical impairment mitigation techniques and fully exploit the advantages of digital communications and digital signal processing (DSP).

The physical impairments acting on a symbol in a WDM system are caused by other symbols located in neighboring channels and neighboring time slots. If two symbols are close to each other in time and/or wavelength, there are strong physical impairments imposed on one another. If two symbols are far enough from each other in time and/or wavelength, these impairments all weaken. Therefore, there exists a range of influence (RoI) for physical impairments that must be carefully determined to reduce the complexity of signal processing techniques, such as electrical equalization and constrained coding.

The purpose of this paper is to develop a two-dimensional (2D) discrete-time input-output model of physical impairments in WDM systems, specifically multichannel multipulse multispan systems with periodic dispersion management and amplification, and determine the RoIs for various physical impairments. To our knowledge, this is the first work to identify the RoI of physical impairments in WDM systems. In addition to point-to-point links, the technique proposed in this paper can be used on all-optical networks to predict the crosstalk due to switched lightpaths sharing a fiber. In Section II, we develop the model and introduce ISI, SPM, IXPM, IFWM, XPM and FWM coefficients to characterize the impact of these impairments on the system output. In Section III, the RoIs of these physical impairments in WDM systems are determined. Section IV concludes the paper.

II. TWO-DIMENSIONAL MODEL OF PHYSICAL IMPAIRMENTS IN WDM SYSTEMS

This section describes our 2D analytical model of multichannel multipulse multispan systems. Fig. 1 shows a schematic of a typical F-channel WDM system with periodic dispersion compensation and amplification. At the transmitter, a bank of laser diodes and a WDM multiplexer convert the data of the kth channel into the corresponding transmitted optical signal s(t) = ∑ ∑ k=1 l s(t) and then launch it into the optical fiber serving as the communication channel. s(t) is transmitted through the fiber span by span and becomes the receiver input signal r(t). To highlight the origins of the physical impairments, the predetection optical filtering, photodetection, and postdetection electrical filtering are not considered in this paper.

Our 2D model is based on the Volterra series transfer function (VSTF) method for a single channel single span case [5] and extended from the one-dimensional model for a single channel multipulse multispan case [6]. The VSTF method expresses the nonlinear Schrödinger (NLS) equation that models the propagation of optical pulses inside single-mode fibers (SMF) as a polynomial expansion in the frequency.
domain. Retaining only the first-order and the third-order Volterra kernels, the frequency-domain output of the fiber at length $L$ is given as [5]

$$A(\omega, L) \approx H_1(\omega, L)A(\omega, 0) + \int H_3(\omega_1, \omega_2, \omega - \omega_1 + \omega_2, L)A(\omega_1, 0)A^\ast(\omega_2, 0)A(\omega - \omega_1 + \omega_2, 0)d\omega_1 d\omega_2$$

(1)

where

$$H_1(\omega, L) = \exp(-\frac{\alpha}{2}L + i\frac{\beta_2}{2}\omega^2 L),$$

(2)

$$H_3(\omega_1, \omega_2, \omega - \omega_1 + \omega_2, L) = \frac{i\gamma}{4\pi^2}H_1(\omega, L)\int_0^L \exp(-\alpha z + i\beta_2(\omega_1 - \omega)(\omega_2 - \omega))dz.$$ (3)

$A(\omega, z)$ is the Fourier transform of $A(t, z)$, which is the slowly varying complex envelope of the propagating field, $t$ is measured in a frame of reference moving with the pulses at the group velocity, and $z$ is the propagation distance measured along the fiber. $H_1(\omega, L)$ and $H_3(\omega_1, \omega_2, \omega - \omega_1 + \omega_2, L)$ are the first-order and the third-order Volterra kernels, which are the linear and nonlinear transfer functions of an optical fiber of length $L$, respectively. $\alpha$ is the attenuation constant, a measure of total power loss from all sources during transmission of optical signals inside the fiber; $\beta_2$ is the group-velocity dispersion (GVD) parameter, a measure of chromatic dispersion that induces pulse broadening; $\gamma$ is the nonlinear parameter.

Suppose the system under consideration consists of $N$ spans, each of length $L$. The output of each span after dispersion compensation and amplification becomes the input of the next span. In the frequency domain, the combined effects of dispersion compensation and amplification can be represented by

$$H_1^{-1}(\omega, L) = \exp(\frac{\alpha}{2}L - i\frac{\beta_2}{2}\omega^2 L).$$ (4)

Applying (1) followed by (4) span by span yields the multi-span VSTF expression of a periodic dispersion managed and amplified system.

We define $S(\omega)$ and $R(\omega)$ as the input and the output of the fiber in the frequency domain, which correspond to $s(t)$ and $r(t)$ in the time domain. For notational simplicity, we define the effective width of the input pulse in the $j$th channel as $\tilde{T}_j^2 = \frac{T_0^2}{1+\Delta f^2}$, where $C_j$ is the chirp parameter, which governs the frequency chirp imposed on the pulse in the $j$th channel, and $T_0$ is the half-width of the pulse in the $j$th channel at the 1/e intensity point. In the case of $K$ successive independent modulated chirped Gaussian input pulses and $F$ equally spaced channels, the input field to the fiber is given as

$$S(\omega) = \sqrt{\frac{2\pi}{\Delta f}} \sum_{k=0}^{K-1} \sum_{l=0}^{F-1} a_{fk}A_{f}\tilde{T}_f$$

(5)

$$\exp\left[-\frac{(\omega - f\Delta T_f^2)^2}{2} - i(\omega - f\Delta)kT_s + i\Phi_{fk}\right],$$

where $a_{fk}$ is the modulating symbol; $A_f = \sqrt{T_f}$ is the peak amplitude of the Gaussian pulse in the $f$th channel, where $P_f$ is the launched peak power; $\Delta$ is the channel spacing; $T_s = \frac{1}{n-1}$ is the symbol period, where $R_s$ is the symbol rate; $\Phi_{fk}$ is the phase of the $k$th pulse in the $f$th channel.

Substituting (5) into the multispans VSTF expression yields the output of the fiber $R(\omega)$, which is a triple integral. By simplifying the triple integral $R(\omega)$ to a simple integral and taking its inverse Fourier transform, we obtain the output field of the fiber in the time domain $r(t)$ given by (6)-(8) on the next page. The appendix contains the definitions of the simplifying functions $A_0, A_1, A_2, B_0, B_1$ and $C$ used in (8).

A desirable fiber model for the development of physical impairment mitigation techniques needs to characterize various physical impairments individually. To quantify the impact of these impairments on the fiber output, we introduce the following impairment characteristic coefficients:

1. **ISI coefficient**:\[ \rho_{k,k}^{ISI} = \exp\left[-\frac{(k-k)^2T^2_f}{2T_f^4}\right].\] (9)

2. **SPM, IXPM, and IFWM coefficients**:\[ \rho_{j,k,l,m,n}^{\text{intra}} = i\gamma E_{l,m,n}^{\text{intra}}\int_0^L \exp(-\alpha z)j_{j,l,m,n}^{\text{intra}}dz, \] (10)

where

$$E_{l,m,n}^{\text{intra}} = \exp\left[-\frac{3(f\Delta)^2T_f^2}{2}\right],$$ (11)

$$j_{j,l,m,n}^{\text{intra}} = \sqrt{\frac{(3f\Delta T_f^2 + A_1 + B_1)^2}{2(3T_f^2 + 2A_2)}} + A_0 + B_0 + C.$$ (12)
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2011 proceedings.

\[ r(t) = \sum_{f=0}^{F-1} \sum_{k=0}^{K-1} a_{fk} A_f \exp \left[ -\frac{(t - kT_s)^2}{2T_f^2} + \text{i} f \Delta t + \text{i} \Phi_{fk} \right] + N \gamma \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} a_{ul} a_{vm} A_u A_v \tilde{T}_u \tilde{T}_v \exp [\text{i} (\Phi_{ul} - \Phi_{vm}) + \text{i} \Delta T (ul - vm + wn)] \]

\[ E(t) = \exp \left[ -\frac{(u\Delta)^2}{2} - \frac{(v\Delta)^2}{2} - \frac{(w\Delta)^2}{2} \right] \]

where

\[ J(t, z) = \frac{F}{2} \sum_{u, v, w=0}^{F} \exp \left[ \frac{(u\Delta)^2}{2} + \frac{(v\Delta)^2}{2} + \frac{(w\Delta)^2}{2} \right] \]

\[ \rho_{f,k,l,m,n} = \exp \left[ \frac{(u\Delta)^2 + (v\Delta)^2 + (w\Delta)^2 + A_1 + B_1 + \text{i} [t - (l - m + n)T_s] + A_0 + B_0 + C} {2(T_u^2 + T_v^2 + T_w^2 + 2A_2)} \right] \]

\[ J_{\text{inter}} = \frac{E_{\text{inter}}(u,v,w)}{\sqrt{T_u^2 + T_v^2 + T_w^2 + 2A_2}} \]

XPM and FWM coefficients:

\[ \rho_{f,k,l,m,n} = \exp \left[ \frac{(u\Delta)^2 + (v\Delta)^2 + (w\Delta)^2 + A_1 + B_1 + \text{i} [t - (l - m + n)T_s] + A_0 + B_0 + C} {2(T_u^2 + T_v^2 + T_w^2 + 2A_2)} \right] \]

\[ J_{\text{inter}} = \frac{E_{\text{inter}}(u,v,w)}{\sqrt{T_u^2 + T_v^2 + T_w^2 + 2A_2}} \]

Note that normally a WDM demultiplexer is required to obtain the output field of the \( f \)th channel \( r_f(t) \), but it is not necessary for characterizing physical impairments. Without loss of generality, the WDM demultiplexer is not considered in this paper for notational simplicity. With the above coefficients, the output field on the \( f \)th channel at discrete times \( kT_s \) is given in (16). This is our 2D discrete-time input-output model of multichannel multipulse multispans with periodic dispersion compensation and amplification. This model establishes a mapping from the fiber input \( a_{fk} \) to the fiber output \( r_f(kT_s) \). This model applies to any modulation format. For \( F = 1 \), this model reduces to the single channel multipulse multispans model in [6]. If both the frequency chirp and the attenuation are ignored, and a single channel single span case is considered, this model reduces to the form obtained by the perturbation approach [7] [8]. Further, this model has been extended to take the WDM demultiplexer and the photodetector into account and its accuracy and computational complexity are presented in [9].

The last term in (16) is due to the overlap of other channels on the channel of interest, \( f \). Once optical filtering is added to the model, the linear adjacent channel interference experienced by the signal could be computed from the last term. Without optical filtering, this interference cannot be isolated.

The key to use this model is to find the index combinations that meet the requirements for various nonlinear physical impairments, i.e., SPM, XPM, IFWM, XPM, and FWM, and to calculate the impairment coefficients corresponding to these index combinations. The index combinations must satisfy the following conditions: for SPM, \( u = v = w = f \) and \( l = m = n = k \); for XPM, \( l = m \neq n \) or \( l \neq m = n \).
and $u = v = w = f$; for IFWM, $l \neq m \neq n$ or $l = n \neq m$ and $u = v = w = f$; for XPM, $u = v \neq w$ or $u = v = w$ and $l = m = n = k$; for FWM, $u \neq v \neq w$ or $u = w \neq v$ and $l = m = n = k$. In addition, for interchannel effects, i.e., XPM and FWM, the frequency location of the nonlinear interaction satisfies the phase-matching condition: $(u - v + w)\Delta = f\Delta$; for intrachannel effects, i.e., IXPM and IFWM, the time location of the nonlinear interaction satisfies $(l - m + n)T_s = kT_s$.

For example, consider a 3-pulse case ($K = 3$). There are $3^3 = 27$ index triplets $[lmn]$ and 7 possible time locations because $-2 \leq (l - m + n) \leq 4$, e.g., the output field at $t = T_s$ ($k = 1$) is affected by 7 different triplets. The triplet [111] contributes to the SPM impairment. The IXPM impairment is caused by [001], [221], [100], and [122], whereas the triplets for IFWM impairment are [012], and [210]. The type and location of the intrachannel nonlinear impairments generated by a pulse triple located at $t = 0$, $T_s$ and $2T_s$ are summarized in Table I. When the following parameters are used: $\alpha = 0.2$ dB/km, $\beta_2 = -20$ ps$^2$/km, $\gamma = 2$ W$^{-1}$km$^{-1}$, $L = 100$ km, $T_0 = 7.51$ ps, $T_s = 25$ ps and $C = 0$, the absolute values of the corresponding intrachannel impairment coefficients are summarized in Table II. Due to space limitation, only 5 valid time locations are presented.

Similarly, consider a 3-channel case ($F = 3$). The type and location of the interchannel nonlinear impairments generated by a channel triple located at $f = 0$, 1, and 2 are summarized in Table III. When $\Delta = 50$ GHz and other parameters take the same values as above, the absolute values of the corresponding interchannel impairment coefficients are summarized in Table IV. Due to space limitation, only 5 valid frequency locations are presented.

### TABLE I
<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Time Location $l - m + n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPM</td>
<td>000 111 222</td>
</tr>
<tr>
<td>IXPM</td>
<td>011 001 002</td>
</tr>
<tr>
<td>IFWM</td>
<td>121 012 210</td>
</tr>
</tbody>
</table>

### TABLE II
<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Time Location $l - m + n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPM</td>
<td>0.0165 0.0165 0.0165</td>
</tr>
<tr>
<td>IXPM</td>
<td>0.0070 0.0070 0.0036</td>
</tr>
<tr>
<td>IFWM</td>
<td>0.0024 0.0024 0.0024</td>
</tr>
</tbody>
</table>

### TABLE III
<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Frequency Location $u - v + w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>XPM</td>
<td>011 001 002 221 112</td>
</tr>
<tr>
<td>FWM</td>
<td>121 012 210 101 201 212</td>
</tr>
</tbody>
</table>

### TABLE IV
<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Frequency Location $u - v + w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>XPM</td>
<td>0.0159 0.0159 0.0159 0.0144</td>
</tr>
<tr>
<td>FWM</td>
<td>0.0158 0.0150 0.0158 0.0146</td>
</tr>
</tbody>
</table>

### III. RANGE OF INFLUENCE OF PHYSICAL IMPAIRMENTS

This section presents the RoIs of various physical impairments. The definition of the RoI differs for different physical impairments. For ISI, IXPM, and IFWM, the RoI is defined as the number of adjacent symbols causing a significant effect. For XPM and FWM, the RoI is defined as the number of channels causing notable degradation. In this paper, for ISI, the RoI is defined as $k$ if $\hat{k}$ is the smallest integer such that the worst-case cumulative ISI degradation $D^{ISI}(\hat{k}) = \sum_{k=\hat{k}}^{\infty} |\rho^{ISI}_{k,k}|$ is practically unchanged. For IXPM (IFWM), the RoI is $\hat{k}$ if $\hat{k}$ is the smallest integer such that the worst-case cumulative IXPM (IFWM) degradation $D^{IXPM(IFWM)}(\hat{k}) = \sum_{k=\hat{k}}^{\infty} \left| \rho^{IXPM(IFWM)}_{k,k,l,m,n} \right|$ is practically unchanged; interchannel effects depend on the symbol rate $R_s$ but are independent of channel spacing $\Delta$. For FWM (PWM), the RoI is $\hat{f}$ if $\hat{f}$ is the smallest integer such that the worst-case cumulative XPM (PWM) degradation $D^{XPM(FWM)}(\hat{f}) = \sum_{s=\hat{f}}^{\infty} \left| \rho^{XPM(FWM)}_{s,s,v,w} \right|$ is practically unchanged; interchannel effects depend on the channel spacing but not the symbol rate. The SPM coefficient is a constant, unaffected by the symbol rate or the channel spacing. There is no RoI for SPM. For the above parameters, $|\rho^{SPM}| = 0.0165$ mW$^{-1}$ps$^{-3}$.

#### A. RoI of ISI

The definition of the ISI coefficient $\rho^{ISI}$ in (9) suggests that the severity of ISI is determined by $|k - \hat{k}|$. The cumulative ISI degradation $D^{ISI}(\hat{k})$ as the number of adjacent symbols $\hat{k}$ varies is given in Fig. 2. $D^{ISI}(\hat{k})$ remains effectively unchanged when $\hat{k} \geq 1$; therefore, the RoI of ISI is 1 for both 40 Gs/s and 100 Gs/s. This result has been verified using the split-step Fourier method by varying the number of pulses and ignoring nonlinearity.
B. RoI of Intrachannel Nonlinear Impairments

The changes in $D^{IXPM}(\bar{k})$ and $D^{IFWM}(\bar{k})$ with varying $\bar{k}$ are given in Fig. 3 and 4, respectively. From Fig. 3, at 40 Gs/s, $D^{IXPM}(\bar{k})$ remains unchanged when $\bar{k} \geq 19$; at 100 Gs/s, $D^{IXPM}(\bar{k})$ remains unchanged when $\bar{k} \geq 185$. Therefore, the RoIs of IXPM are 19 and 185, respectively, for 40 Gs/s and 100 Gs/s. Similarly, from Fig. 4, we can easily conclude that the RoIs of IFWM are 65 and around 300, respectively, for 40 Gs/s and 100 Gs/s. When the symbol rate increases, the symbol interval reduces and the RoIs of IXPM and IFWM increase. Note that the phase term $e^{i[\Phi_{f_i} - \Phi_{f_m} + \Phi_{f_n}]}$ is not included in our definition of $\rho_{f,k,l,m,n}^{IFWM}$ in (11). When we take the phase information into account, the IFWM impairment is significantly reduced on average. The IFWM impairment can be reduced by manipulating the input phases [10].

C. RoI of Interchannel Nonlinear Impairments

The changes in $D^{XPM}(\bar{f})$ and $D^{FWM}(\bar{f})$ with varying number of channels $\bar{f}$ are given in Fig. 5 and 6, respectively. At 50 GHz, $D^{XPM}(\bar{f})$ remains unchanged when $\bar{f} \geq 3$; at 100 GHz, $D^{XPM}(\bar{f})$ remains unchanged when $\bar{f} \geq 2$. Therefore, the RoIs of XPM are 3 and 2, respectively, for 50 GHz and 100 GHz. Similarly, from Fig. 6, the RoIs of FWM are 4 and 3, respectively, for 50 GHz and 100 GHz. When channel spacing increases, the RoIs of XPM and FWM decrease.

IV. CONCLUSION

In this paper, we develop a two-dimensional discrete-time input-output model of physical impairments in WDM systems based on the VSTF method and determine the RoIs for different physical impairments. These RoIs are of great value to the development of physical impairment mitigation techniques and other signal processing techniques for optical communications. We are currently extending this model to the general case,
including predetection optical filtering, photodetection, ASE (amplified spontaneous emission) noise, polarization mode dispersion, and postdetection electrical filtering. At the same time, we are applying the concept of RoI in the development of physical impairment mitigation techniques.

APPENDIX

SIMPLIFYING FUNCTIONS IN TIME DOMAIN OUTPUT

This appendix defines the simplifying functions $A_0$, $A_1$, $A_2$, $B_0$, $B_1$, and $C$ used in (8).

$$A_0 = - \frac{(T^2_u u \Delta + \overline{T}^2_u u \Delta)(\overline{T}^2_w u \Delta - i\beta z)}{(T^2_u + \overline{T}^2_u)(T^2_v + \overline{T}^2_v) - (T^2_v + i\beta z)} + \frac{2 (\overline{T}^2_u + \overline{T}^2_v)(\overline{T}^2_v + T^2_w) - (T^2_v + i\beta z)^2}{(\overline{T}^2_v + \overline{T}^2_u)(\overline{T}^2_u + T^2_w) - (T^2_v + i\beta z)^2}.$$  

$$A_1 = \frac{(T^2_u + \overline{T}^2_u)(\overline{T}^2_v + T^2_w)(T^2_w u \Delta + 2\overline{T}^2_w u \Delta - (T^2_w + i\beta z))}{(T^2_u + \overline{T}^2_u)(T^2_v + \overline{T}^2_v) - (T^2_v + i\beta z)^2} + \frac{(\overline{T}^2_u + T^2_u)(\overline{T}^2_u + T^2_w) - (T^2_u + i\beta z)^2}{(T^2_u + \overline{T}^2_u)(T^2_v + \overline{T}^2_v) - (T^2_v + i\beta z)^2} + \frac{(\overline{T}^2_u + \overline{T}^2_v)(\overline{T}^2_v + T^2_w)(\overline{T}^2_w u \Delta - (T^2_w + i\beta z)^2}{(T^2_u + \overline{T}^2_u)(T^2_v + \overline{T}^2_v) - (T^2_v + i\beta z)^2}.$$  

$$B_0 = -i(l - m)T_s \frac{(T^2_u u \Delta + \overline{T}^2_u u \Delta)(\overline{T}^2_w u \Delta - i\beta z)}{(T^2_u + \overline{T}^2_u)(T^2_v + \overline{T}^2_v) - (T^2_v + i\beta z)^2} + i(n - m)T_s \frac{(\overline{T}^2_v u \Delta + \overline{T}^2_w u \Delta)(\overline{T}^2_u u \Delta - i\beta z)}{(T^2_u + \overline{T}^2_u)(T^2_v + \overline{T}^2_v) - (T^2_v + i\beta z)^2}.$$  

$$B_1 = i(l - m)T_s \frac{(T^2_u + \overline{T}^2_u)(T^2_v - i\beta z)}{(T^2_u + \overline{T}^2_u)(T^2_v + \overline{T}^2_v) - (T^2_v + i\beta z)^2} + i(n - m)T_s \frac{(T^2_v + \overline{T}^2_v)(T^2_u - i\beta z)}{(T^2_u + \overline{T}^2_u)(T^2_v + \overline{T}^2_v) - (T^2_v + i\beta z)^2}.$$  

$$C = -2 \frac{(l - n)^2 T^2_s (T^2_w + \overline{T}^2_u)}{(T^2_u + \overline{T}^2_u)(T^2_v + \overline{T}^2_v) - (T^2_v + i\beta z)^2} - \frac{(l - m)(n - m)T^2_s (\overline{T}^2_u - i\beta z)}{(T^2_u + \overline{T}^2_u)(T^2_v + \overline{T}^2_v) - (T^2_v + i\beta z)^2}.$$  

ACKNOWLEDGMENT

This work was supported by the U.S. National Science Foundation (NSF) under grant CCF-0916880.

REFERENCES