O.R. Applications

Response time and vendor–assembler relationship in a supply chain

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Abstract

Relationships between an assembler and a vendor in a supply chain are investigated in two-period models when the assembler wants to reduce response time by incentive systems. The assembler may offer myopic or farsighted incentive contracts to the vendor, under short-term or long-term relationships. Incentive schemes, effort levels, and expected payoffs under different perspectives and relationships are examined. We find that a farsighted assembler provides the vendor with a higher incentive than a myopic assembler in the first period. A long (short)-term relationship is preferred if the value of farsightedness under a long-term relationship is greater (less) than the switching option value under a short-term relationship. We propose several sufficient conditions regarding which perspectives and relationships are preferred.

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1. Introduction

The Japanese model of long-term vendor–assembler relationships in supply chains has had a strong impact on US manufacturers. Even though US manufacturers emulated key features of those relationships, the degree of progress toward them varies across industries. (For example, Richardson (1993) shows that contract periods are much shorter for consumer electronics than for automobile or heavy construction equipments.) In this paper, we investigate relationships between the vendor and the assembler in a supply chain when the assembler wants to reduce response time by contracting. We assume that response time is the only performance measure of the supply chain. Although various types of incentive contracts are offered in practice, we focus only on payment schemes.

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In the early 1980s, some leading companies introduced a new dimension of competition: time-based competition. Time-based competition is a new philosophy that seeks closeness to the customer by delivering high-valued products and services in the least amount of time (Blackburn and Wassenhove, 1993). The goal of the time-based competitor is to eliminate non-value-adding time continuously so that the system response time of the whole supply chain is reduced, thereby bringing the customer closer. In time-based competition, response time is no longer a given parameter, but it is treated as a variable of strategic importance. However, in most mathematical models of production problems, response time has been regarded as a given system parameter. Because of the growing strategic value of response time, researchers have started to examine the relationships between the response time and other factors such as the batch size, set-up time and incentives.

The work of Karmarkar (1987) is one of the first papers that address this issue. He presents the relationship between response time and the batch size, using a single-stage queueing model. Bitran and Tirupati (1984) refine Karmarkar’s model by adding a simple job release mechanism based on the continuous review policy of $(R,Q)$ type. Kim and Tang (1997) provide another extension of Karmarkar’s model, in which they consider the Kanban system as a job release mechanism.

Not much research effort has been dedicated to the use of incentive contracts in reducing response time in a supply chain. Contract theory (for example, Holmström, 1979; Grossman and Hart, 1983; Gibbons, 2005 for review in the supply chain perspectives) has been developed by several researchers and extensively applied to various areas including accounting, finance, industrial organization and marketing (see Ackere, 1993). Recently, this approach has been applied to issues in supply chain management such as flexible supply contracts (Li and Kouvelis, 1999; Tsay, 1999), quantity discounts (Corbett and de Groot, 2000), and quality appraisal activity (Baiman et al., 2000). To our knowledge, Tang (1991) is the first to address the issues of response time reduction in a supply chain. He considers the case in which the vendor offers a contract that includes his order response time as a variable. However, he evaluates a given set of alternative contracts without deriving the optimal contract.

This paper focuses on the incentive contracts based on the response time. We suppose that response time will affect future profits and costs. Therefore, the main concern of the assembler is how to motivate the vendor to reduce the response time to maximize her expected profit. Another key concern of this paper is on the relative advantages of a long-term relationship compared with a short-term relationship between an assembler and a vendor. A long-term relationship enables the assembler to design a better incentive scheme, while a short-term relationship provides the assembler the option to switch vendors. Considering this trade-off, the assembler will determine the time length of the relationship as well as the incentive scheme.

Our work is related to Rhim (1998). He constructs a two-period model to explain the behavior of a farsighted assembler in a long-term relationship. The farsighted assembler considers the effect of a current (first period) contract in the future (second period). He shows two results: (1) a farsighted assembler obtains a higher effort level from the vendor than a myopic assembler in the first period; (2) the expected total cost of the farsighted assembler is smaller in the long run, although it is initially higher than that of the myopic assembler. However, he fails to incorporate the possibilities of vendor-switching at the end of each contract period. As a result, a long-term relationship is always preferred to a short-term relationship in his model. We incorporate the possibility of switching vendors that makes a short-term relationship more attractive. Since vendor-switching will come from the fact that there exists better vendors in the market, we need to assume that multiple types of vendors exist and that the incumbent vendor may be replaced depending on his performance. This setting allows us to compare a long-term relationship with a short-term relationship.

Our analysis is stylized in the following way. We suppose that vendors behave myopically. This assumption seems to be plausible when vendor markets are competitive, or when assemblers may transplant their system into other countries (see Kenney and Florida, 1993). Secondly, we focus on the comparison between a long-term relationship under a farsighted assembler and a short-term relationship under a myopic assembler. This comparison will highlight the differences between a long-term relationship and a short-term relationship.

---

1 According to Stalk (1988), in a traditional manufacturing system, value is added to products for only 0.05–2.5% of the time that they are in the factory.

2 Response time is defined as the time from which an order is placed to the time when the order arrives.
Thirdly, we assume that only one vendor is hired. This approach is commonly used in literature for simplicity (Baiman et al., 2000; Corbett and de Groot, 2000; Tsay, 1999).

The assembler is characterized by her perspectives (farsighted or myopic) and length of relationships (short-term or long-term) as in Table 1. We first construct a single-type-vendor model and investigate the incentive schemes and payoffs based on the perspectives under the long-term relationship. Then, we extend the model into a two-type-vendor model and examine whether we obtain similar results as in the single-type-vendor model. Finally, we compare the long-term relationship of a farsighted assembler with the short-term relationship of a myopic one, and generate sufficient conditions under which one is more beneficial than the other.

The remainder of the paper is structured as follows. In Section 2, we build a single-type-vendor model. In Section 3, a two-type-vendor model is constructed. In Section 4, long-term and short-term relationships are compared. Section 5 concludes.

2. Single-type-vendor model under long-term relationship

We consider a two-period model in which an assembler and a pool of ex-ante homogeneous vendors exist. The assembler enters into a contract with a vendor. We assume that the assembler and the vendor have a long-term relationship. A short-term relationship will be considered in a later section. The vendor will provide supplies to the assembler. The response time is affected by the effort level of the vendor and a random factor. The assembler can observe the realized response time, but cannot observe the effort level of the vendor. The assembler’s problem is to select the incentive scheme to maximize her expected profit.

We distinguish between the myopic assembler and the farsighted assembler. The myopic assembler is only concerned with the profit of the current period, while the farsighted assembler is concerned with the (properly discounted) sum of current and future profits. The vendor is assumed to be myopic.

Time is denoted with subscript $t = 1, 2$ in notations. We denote $e_t$ for the effort level of the vendor. Disutility of effort is denoted by $V(e_t)$ where $V^\prime > 0$, $V^\prime\prime > 0$. Response time (or delay) $R_t$ is determined as $R_t = r(e_t) + \omega_t$ where $r(e_t)$ denotes the expected response time under $e_t$ and $\omega_t$ is a random variable with mean 0. We assume that $r^\prime(e_t) < 0$ and $r^\prime\prime(e_t) > 0$. Let us define $g(R_t|e_t)$ as the probability density function of $R_t$ given $e_t$. Notice that as $e_t$ increases to $e'_t$, the distribution of $R_t$ corresponding to $e'_t$ is first-order-stochastically-dominated by the distribution of $R_t$ corresponding to $e_t$. We assume that the unit cost of $c_t$ per response time is incurred to the assembler.

We denote $M_t$ as the aggregate profit of the assembler before netting the effect of response time. Assume that $M_t$ is known at time $t$. The net profit to the assembler is $X_t = M_t - c_tR_t = M_t - c_t(r(e_t)) - c_t\omega_t \equiv f_t(e_t) + e_t$, where $f_t(e_t) = M_t - c_t(r(e_t))$ is the expectation of $X_t$, and $e_t = -c_t\omega_t$, for a fixed $c_t > 0$. Note that $R_t$ completely determines $X_t$, given $M_t$ and $c_t$. In order to consider the long-run effect of response time, we suppose that $c_2$ and $M_2$ can be affected by $R_1$, thus $f_2(e_2)$ is affected by $R_1$ through $c_2$ and $M_2$. We assume that $c_2$ is weakly increasing and $M_2$ is weakly decreasing in $R_1$. In words, shorter response time in one period results in a higher profit and/or a lower cost structure in the next period.\(^3\) When we need to emphasize this dependence, we will use $f_2(e_2|R_1)$ instead of $f_2(e_2)$. Our assumptions imply that $f_{1,e_1}(e_1) \geq 0$, $f_{2,e_2}(e_2|R_1) \geq 0$, $f_{2,R_1}(e_2|R_1) \leq 0$, where $f_{1,x}(\cdot)$ implies the differential of $f(\cdot)$ with respect to $x$. Finally, we assume that effort is more desirable when the previous response time is longer: $\frac{\partial^2 f_2(e_2|R_1)}{\partial e_2 \partial R_1} > 0$.

Now, let us consider the payment scheme from the assembler to the vendor. Since the assembler cannot observe the effort level, the payment is a function of $R_t$ or equivalently, $X_t$. The payment is denoted by $S_t(X_t)$.

\(^3\) This assumption can be justified because a shorter response time may allow the assembler to obtain a better reputation and more clients and/or to produce products in a more efficient way.
We assume that the reservation utility of the vendor is $m$ in each period. We interpret the reservation utility as the minimum profit for the continuation of operation. Finally, for analytic simplicity, we assume that the assembler is risk neutral and that the vendor is risk neutral in payment, but risk averse in effort (Table 2).

### 2.1. The second period problem

For simplicity, we omit the subscripts for time. Given $R_1$ (thus $M_2$), the problem of the assembler can be stated as follows:

$$
\max_{\{S(X), e\}} E(X - S(X))
$$

s.t. $E(S) - V(e) \geq m$  \hspace{1cm} Participation constraint,

$$
\frac{\partial E(S)}{\partial e} = V'(e) \quad \text{Incentive-compatibility constraint.}
$$

As usual, the first constraint guarantees the participation of the vendor and the second constraint is for the optimal choice of the effort level by the vendor given $S(X)$ (or incentive compatibility). The setting of the model allows us to use a more simplified version of the model. Based on Bhattacharyya and Lafontaine (1995), we can consider $S(X)$ as a linear function of $X$, under the assumptions of zero-mean random variable of $e_i$ and the risk neutrality regarding the payment. From now on, we assume that $S(X) = F + bX$, where $F$ and $b$ will be determined later. It is also easy to see that the participation constraint is binding, since $F$ can be reduced without affecting the incentives. Thus, the above formula can be rewritten as follows:

$$
\max_{\{F, b, e\}} (1 - b)f(e) - F
$$

s.t. $F + bf(e) - V(e) = m,$

$$
bf_e - V'(e) = 0.
$$

The Lagrangian $L$ and the first-order conditions (FOCs) are as follows:

$$
L = (1 - b)f(e) - F + \lambda(F + bf(e) - V(e) - m) + \mu(bf_e - V'(e)),
$$

where $\lambda$ and $\mu$ are the Lagrange multipliers.

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<table>
<thead>
<tr>
<th>Summary of notations and assumptions</th>
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</thead>
<tbody>
<tr>
<td>Subscript $t$: time $t$</td>
</tr>
<tr>
<td>$e_t$: effort level of the vendor</td>
</tr>
<tr>
<td>$V(e_t)$: disutility of effort, $V' &gt; 0$, $V'' &gt; 0$</td>
</tr>
<tr>
<td>$R_t$: response time, $R_t = r_c(e_t) + o_t$, where $r_c(e_t)$ is the average response time for $e_t$ and $o_t$ is a random variable with mean 0</td>
</tr>
<tr>
<td>$r'(e_t) &lt; 0$, $r''(e_t) &gt; 0$</td>
</tr>
<tr>
<td>$g(R_t</td>
</tr>
<tr>
<td>$M_t$: the aggregate profit before netting the effect of response time</td>
</tr>
<tr>
<td>$X_t$: net profit, $X_t = M_t - c_RR_t = M_t - c_r(e_t) - c_o o_t = f_j(e_t) + e_t$, where $e_t = -c_o o_t$</td>
</tr>
<tr>
<td>$\partial c_f/\partial e$</td>
</tr>
<tr>
<td>$S_t(X_t)$: payment scheme to the vendor</td>
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<tr>
<td>$m$: reservation utility of the vendor</td>
</tr>
</tbody>
</table>

$M_t$ and $c_f$ can be affected by $R_t$, thus $f(e_t)$ is affected by $R_t$. We use $f(e_t|R_t)$ instead of $f(e_t)$ when we need to emphasize the dependence $f_2(e_t|e_1) \geq 0$, $f_2, n(e_2|R_t) \leq 0$, $f_2, e_2 (e_2|R_2) \geq 0$, $\partial^2 f_2(e_2 | R_1)/\partial e_2 \partial R_1 \geq 0$

---

The assumption of risk neutrality regarding the payment will greatly simplify the analysis, and thus allow us to focus on the conflicts between farsightedness vs. myopia, and between long-term relationship vs. short-term relationship. Note that we are not concerned with the conflict between risk aversion and incentives. Notice also that firms are often assumed to be risk neutral in economics literature. Justification of this assumption is also found in McMillan (1990).

Based on Bhattacharyya and Lafontaine (1995), we can show that under the assumptions that $e_t$ is a random variable with mean zero and that the risk neutrality regarding payments holds, the optimal payment scheme can be represented by a linear contract. This result also applies to more general cases that we consider later in the first period problem and the two-type-vendor model.
FOCs,
\[
\begin{align*}
L_F &= -1 + \lambda = 0, \\
L_e &= (1 - b)f_e + \lambda(bf_e - V') + \mu(bf_{ee} - V'') = 0, \\
L_b &= -f + \lambda f + \mu f_e = 0.
\end{align*}
\]
Note that \( L_e = (1 - b)f_e + \mu(bf_{ee} - V'') = 0 \) since \( bf_e - V' = 0 \) from the incentive constraint. Optimal solutions are summarized in the following proposition.

**Proposition 1.** The optimal solutions in the second period are characterized as follows:

1. \( \lambda_2^* = 1, \mu_2^* = 0, b_2^* = 1, F_2^* = V(e_2^*) + m - f_2(e_2^*). \)
2. \( e_2^* \) satisfies \( f_{2,2e}(e_2^*) - V'(e_2^*) = 0. \)
3. Payoff to the assembler: \( (1 - b_2^*)f_2(e_2^*) - F_2^* = -F_2^* = f_2(e_2^*) - V(e_2^*) - m. \)

In Proposition 1, \( b_2^* = 1 \) implies that all the risk is borne by the vendor. This result is natural since the vendor who causes the moral hazard is risk neutral regarding payment.

### 2.2. The first period problem

Assume that the subjective discount factor of the assembler is \( \delta \). The assembler now determines a payment scheme to maximize the total expected utility, considering participation and incentive constraints. Then the assembler’s problem becomes:

\[
\begin{align*}
\max_{\{S(X_1), e_1\}} & \quad E(X_1 - S_1(X_1)) + \delta E^{R_1} E(X_2 - S_2(X_2)) \\
\text{s.t.} & \quad E(S_1) - V(e_1) \geq m & \text{Participation constraint,} \\
& \quad \partial E(S_1)/\partial e_1 = V'(e_1) & \text{Incentive-compatibility constraint,}
\end{align*}
\]

\( S_2(X_2) \) is the optimal payment scheme in the second period, as in the previous section. Note that \( X_2 \) is affected by \( R_1 \) and the distribution of \( R_1 \) is determined by \( e_1 \). Thus, \( E^{R_1}(\cdot) \), the expectation over \( R_1 \), is calculated given \( e_1 \). With the linear payment functions, we can restate and solve the problem as follows:

\[
\begin{align*}
\max_{\{F_1, b_1, e_1\}} & \quad (1 - b_1)f_1(e_1) - F_1 + \delta E^{R_1}[f_2(e_2^*|R_1) - V(e_2^*) - m] \\
\text{s.t.} & \quad F_1 + b_1f_1(e_1) - V(e_1) = m, \\
& \quad b_1f_1(e_1) - V'(e_1) = 0.
\end{align*}
\]

Again, note that \( e_2^* \) is the optimal effort level given \( R_1 \), as in the previous section. In Proposition 2, we denote \( E^{R_1}_{e_1} \) for \( \partial E^{R_1}[f_2(e_2^*|R_1) - V(e_2^*) - m]/\partial e_1 \) at \( e_1 = e_1^* \).

**Proposition 2.** The optimal solutions in the first period are characterized as follows:

1. \( \lambda_1^* = 1, \mu_1^* = 0, b_1^* = 1 + \delta E^{R_1}_{e_1} / f_{1,e_1}(e_1^*) \geq 1, F_1^* = m + V(e_1^*) - b_1f_1(e_1^*). \)
2. \( e_1^* \) satisfies \( b_1f_{1,e_1}(e_1^*) - V'(e_1^*) = 0. \)
3. The total expected payoff to the assembler: \( f_1(e_1^*) - m - V(e_1^*) + \delta E^{R_1}[f_2(e_2^*|R_1) - V(e_2^*) - m]. \)

**Proof.** See the appendix. \( \square \)

---

\( ^6 \) From Propositions 1 and 2, it is easy to see that \( \partial e_i/\partial M_l = 0, \partial e_i/\partial c_l \geq 0. \)
From Propositions 1 and 2, we know that the assembler provides the vendor with a stronger incentive in the first period than in the second period: \( b_1^* \geq b_2^* = 1 \). We can also compare between a farsighted assembler and a myopic assembler. A farsighted assembler will provide the vendor with a stronger incentive than a myopic assembler. For a myopic assembler would maximize the expected utility of the first period only (i.e., \( \delta = 0 \)), thus \( b_1^* = 1 \) where double asterisks (**) are used for the myopic case. It follows that \( e_1^* \geq e_1^{**} \). Now, we summarize the results in:

**Corollary 1**

1. The farsighted assembler provides the vendor with a stronger incentive in the first period than in the second period: \( b_1^* \geq b_2^* \).
2. The first-period-effort level is higher under the farsighted assembler than under the myopic assembler: \( e_1^* \geq e_1^{**} \). Note that the first-period-effort has effects on \( M_2 \) and \( c_2 \) in the second period. Considering these effects, the farsighted assembler requires the vendor to make higher effort in the first period. The myopic assembler ignores such effects, thus fails to maximize the total expected utility. Note that the first-period-payoff to the farsighted assembler is lower than that to the myopic assembler. However, the loss is compensated by the higher expected second-period-payoff.

Our next question is when the effort levels under the farsighted assembler and under the myopic assembler coincide: In other words, when is \( b_1^* = 1 \)? Consider the farsighted assembler. From \( b_1 = 1 + \delta E_{c1}^{R1}/f_{1,c1} \), we need \( E_{c1}^{R1} = 0 \), since \( \delta > 0 \) for a farsighted assembler. Recall that

\[
E_{c1}^{R1} = \int [f_2(e_2^* | R_1) - V(e_2^*)] g_{c1}(R_1 | e_1) dR_1.
\]

If \( f_2(e_2^* | R_1) - V(e_2^*) \) is independent of \( R_1 \), then

\[
E_{c1}^{R1} = [f_2(e_2^* | R_1) - V(e_2^*)] \int g_{c1}(R_1 | e_1) dR_1 = 0,
\]

since \( \int g(R_1 | e_1) dR_1 = 1 \) for any \( e_1 \). Let us say “payoff is independent” if \( f_2(e_2^* | R_1) - V(e_2^*) \) is independent of \( R_1 \). Note that whenever \( f_2(e_2^* | R_1) \) is independent of \( R_1 \), \( V(e_2^*) \) is also independent of \( R_1 \). It is obvious that payoff independence holds if \( M_2 \) and \( c_2 \) are independent of \( R_1 \). Let us also say that payoff is dependent if \( f_2(e_2^* | R_1) - V(e_2^*) \) is not independent.

To compare between dependent payoffs, let us consider two payoffs \( f_2(e_2^* | R_1) - V(e_2^*) \) and \( h_2(e_2^* | R_1) - V(e_2^*) \), where \( e_2^* \) denotes the optimal effort level function for \( h_2(\bullet | R_1) \). We define that payoff \( f_2(e_2^* | R_1) - V(e_2^*) \) is more dependent than \( h_2(e_2^* | R_1) - V(e_2^*) \) if the difference \( |f_2(e_2^* | R_1) - V(e_2^*)| - |h_2(e_2^* | R_1) - V(e_2^*)| \) increases as \( R_1 \) decreases. In words, higher dependence implies faster improvement of payoff as the response time decreases. A sufficient condition is that \( M_2 \) and \( c_2 \) for \( f_2(\bullet | R_1) \) are more “responsive” to \( R_1 \) than for \( h_2(\bullet | R_1) : M_2 - \bar{M}_2 \) increases and \( c_2 - \bar{c}_2 \) decreases as \( R_1 \) decreases, where \( \bar{M}_2 \) and \( \bar{c}_2 \) are used for \( h_2(\bullet | R_1) \). When payoff is more dependent, we can show that \( E_{c1}^{R1} \) becomes larger, so that the effort level required by the farsighted assembler becomes higher.

**Corollary 2**

1. When payoff is independent, the effort levels under the farsighted assembler and under the myopic assembler coincide. When payoff is dependent, the first-period-effort level is higher under the farsighted assembler than under the myopic assembler.
2. The first-period-effort level under the farsighted assembler is higher as payoff is more dependent.

\[ \text{Note that we cannot directly compare effort levels } e_1^* \text{ and } e_2^*, \text{ since the gross profits and costs in the second period depend on the realized response time } R_1. \]
Proof. See the appendix.  □

Corollary 2 is intuitive. When payoff independence holds, the first-period-effort by the vendor does not contribute to the profit increase in the second period. Therefore, the farsighted assembler has no need to provide excessive incentives for the vendor to make higher effort than the myopic assembler. On the other hand, as the payoff becomes more dependent, the first-period-effort has a stronger effect on the profit in the second period. Considering this effect, the farsighted assembler requires the vendor to make a higher first-period-effort when the payoff is more dependent.

3. Two-type-vendor model under long-term relationship

In this section, we extend the model in the previous section by considering two types of vendors. We assume that a vendor may have high ability or low ability. We denote types as \( T = H \) for high ability and \( T = L \) for low ability. Type will be denoted by superscripts. Denote the expected response time of the \( T \)-type vendor by \( r_T(e) \). For simplicity, we assume that different ability translates to different expected response time without affecting the distribution shape. Technically, we set \( r_H(e) = r(e) \), and \( r_T(e) = r_H(e) + a \), where \( a \geq 0 \), fixed. Accordingly, \( f_H(e) = M - cr_H(e) \), \( f_T(e) = M - cr_T(e) = f_H(e) - ca \). If \( a = 0 \), the model becomes the single-type-vendor model considered in the previous section.

We assume that the proportion of type \( H \) of the vendor pool in the first period is \( q \). The belief that the vendor is type \( H \) in period \( t \) is denoted by \( p_t \). Since a vendor is randomly chosen in the first period, \( p_1 = q \). However, in the second period, \( p_2 \) may be adjusted for the response time in the first period and possibly other information. We assume that \( \partial p_2 / \partial R_1 \leq 0 \): a longer response time implies a lower probability of high ability. We assume that both assembler and vendor share the same beliefs. 8

Given \( R_1 \) and \( p_2 \), the assembler solves the following problem in the second period:

\[
\begin{align*}
\max & \quad E_T E(X_2 - S_2(X_2)) \\
\text{s.t.} & \quad E_T E(S_2) - V(e_2) = m, \\
& \quad \partial E_T E(S_2)/\partial e_2 = V'(e_2).
\end{align*}
\]

The problem is the same as that in the previous section except that we consider the double expectation over \( T \) and \( X_2 \). Under a linear payment scheme, we replace \( S_2(X_2) \) with \( F_2 + b_2X_2 \), and solve the problem.

\[
\begin{align*}
\max & \quad (1 - b_2)E_T f_T^2(e_2) - F_2 \\
\text{s.t.} & \quad F_2 + b_2E_T f_T^2(e_2) - V(e_2) = m, \\
& \quad b_2E_T f_T^2_{x_2}(e_2) - V'(e_2) = 0.
\end{align*}
\]

Similarly, the first period problem can be stated as follows:

\[
\begin{align*}
\max & \quad (1 - b_1)E_T f_1(e_1) - F_1 + \delta E^{R_1} E^T[f_T^2(e_2|R_1) - V(e_2) - m] \\
\text{s.t.} & \quad F_1 + b_1E_T f_1^T(e_1) - V(e_1) = m, \\
& \quad b_1E_T f_1^T_{x_1} - V'(e_1) = 0.
\end{align*}
\]

Note that \( X_2 \) is affected by \( R_1 \) and the distribution of \( R_1 \) is determined by \( e_1 \). Thus, \( E^{R_1} (\cdot) \), the expectation over \( R_1 \), is calculated given \( e_1 \). The expectation over \( T \) in the second period is also affected by \( R_1 \), since the belief on the vendor type is affected by \( R_1 \). Applying the same procedure as in the single-type-vendor model, we obtain the following results. In Proposition 3, we denote \( E^{R_1} E_{e_1} \) for \( \partial E^{R_1} E^T[f_T^2(e_2|R_1) - V(e_2) - m]/\partial e_1 \) at \( e_1 = e_1^* \).

---

8 The vendor may not know whether or not he has higher ability than his competitors in the beginning of the contract. The vendor will learn about his type as time passes. While information asymmetry is also an interesting topic, it is beyond our concern.
Proposition 3. In the two-type-vendor model:

(1) The optimal solutions in the second period problem are characterized as follows:
   (i) \( \lambda^*_1 = 1, \mu^*_1 = 0, b^*_1 = 1, F^*_1 = V(e^*_2) + m - E^T f^T_2. \)
   (ii) \( e^*_1 \) satisfies \( E^T f^T_2(e^*_2) - V'(e^*_2) = 0. \)
   (iii) The second-period-payoff to the assembler: \( -F^*_2 = E^T f^T_2 - V(e^*_2) - m = f^H_2(e^*_2) - c_2 a(1 - p_2) - V(e^*_2) - m. \)

(2) The optimal solutions in the first period problem are characterized as follows:
   (i) \( \lambda^*_1 = 1, \mu^*_1 = 0, b^*_1 = 1 + \delta E^{R1} E^T / E^T f^T_{e1}(e^*_1) \geq 1, F^*_1 = m + V(e^*_1) - b^*_1 f_1(e^*_1). \)
   (ii) \( e^*_1 \) satisfies \( b^*_1 E^T f^T_{e1} - V'(e^*_1) = 0. \)
   (iii) The first-period-payoff to the assembler: \( (1 - b^*_1) E^T f_1(e^*_1) - F^*_1 = f^H_1(e^*_1) - c_1 a(1 - p_1) - V(e^*_1) - m. \)

(3) The total expected payoff to the assembler: \( E^T f_1(e^*_1) - V(e^*_1) - m + \delta E^{R1} E^T [f^H_2(e^*_2)|R_1] - V(e^*_2) - m = f^H_1(e^*_1) - c_1 a(1 - p_1) - V(e^*_1) - m + \delta E^{R1} [f^H_2(e^*_2) - c_2 a(1 - p_2) - V(e^*_2) - m]. \)

Proof. See the appendix. \( \square \)

Proposition 3 is analogous to the case of the single-type-vendor model (Propositions 1 and 2). It is also easy to see that the results of Corollary 1 still holds under the two-type-vendor model. We restate them in Corollary 3.

Corollary 3. In the two-type-vendor model:

(1) The farsighted assembler provides the vendor with a stronger incentive in the first period than in the second period: \( b^*_1 \geq b^*_2. \)

(2) The first-period-effort level is higher under the farsighted assembler than under the myopic assembler: \( e^*_1 \geq e^*_1. \)

Now, let us check when the effort levels under the farsighted assembler and under the myopic assembler coincide. For \( e^*_1 = e^*_1. \) we need to have \( E^{R1} E^T_{e1} = 0: \)

\[
E^{R1} E^T_{e1} = \int \{ p_2[f^H_2(e_2|R_1) - V(e_2)] + (1 - p_2)[f^L_2(e_2|R_1) - V(e_2)] \} g_{e1}(R_1|e_1) dR_1 = 0.
\]

Analogous to the single-type-model, we can say that payoff is independent given type \( T, \) if \( f^2_2(e_2|R_1) - V(e_2) \) is independent of \( R_1. \) Note that \( f^2_2(e_2|R_1) - V(e_2) = M_2 - c_2 f_2(e_2) - V(e_2) \) is not affected by \( R_1 \) if \( M_2 \) and \( c_2 \) are not affected by \( R_1. \) However, unlike in the single-period-model, the payoff independence does not guarantee \( E^{R1} E^T_{e1} = 0, \) since \( p_2 \) may also be affected by \( R_1. \) Under payoff independence, we have

\[
E^{R1} E^T_{e1} = [f^H_2(e_2|R_1) - V(e_2)] \int \left\{ p_2 g_{e1}(R_1|e_1) dR_1 + [f^L_2(e_2|R_1) - V(e_2)] \int (1 - p_2) g_{e1}(R_1|e_1) dR_1 \right\}.
\]

If \( p_2 \) is also not affected by \( R_1, \) then we have:

\[
E^{R1} E^T_{e1} = [p_2 f^H_2(e_2|R_1) - V(e_2)] + (1 - p_2)[f^L_2(e_2|R_1) - V(e_2)] \int g_{e1}(R_1|e_1) dR_1 = 0.
\]

When \( p_2 \) is not affected by \( R_1, \) let us call it type independence. We say that type is dependent, otherwise. The above calculation shows that payoff independence and type independence together lead to \( E^{R1} E^T_{e1} = 0. \)

On the other hand, there is another extreme case resulting in \( E^{R1} E^T_{e1} = 0. \) Suppose that the ability of the vendor is completely determined based on \( R_1, \) which we call complete type dependence. This case will be observed if, given an effort level, the range of \( R_1 \) of the vendor with high ability does not overlap with the range of \( R_1 \) of the vendor with low ability. In this case,
\[
E_{c_1}^{R1}E_{t}^{H}[f_{2}(e_{2}|R_{1}) - V(e_{2}) - m] = p_{1} \int [f_{2}^{H}(e_{2}|R_{1}) - V(e_{2})]g^{H}(R_{1}|e_{1})dR_{1} \\
+(1 - p_{1}) \int [f_{2}^{L}(e_{2}|R_{1}) - V(e_{2})]g^{L}(R_{1}|e_{1})dR_{1} - m,
\]

where \(g^{T}(R_{1}|e_{1})\) is the conditional probability density function given the vendor’s type \(T\). Thus,

\[
E_{c_1}^{R1}E_{t}^{H} = p_{1} \int [f_{2}^{H}(e_{2}|R_{1}) - V(e_{2})]g^{H}(R_{1}|e_{1})dR_{1} + (1 - p_{1}) \int [f_{2}^{L}(e_{2}|R_{1}) - V(e_{2})]g^{L}(R_{1}|e_{1})dR_{1}.
\]

In addition, if the payoff is also independent, then we have

\[
E_{c_1}^{R1}E_{t}^{H} = 0.
\]

Therefore, under payoff independence and complete type dependence, \(E_{c_1}^{R1}E_{t}^{H} = 0\).

Under type independence, the belief on the vendor’s type is not affected by the response time. Under complete type dependence, the type is known regardless of the effort level of the vendor. In either case, the effort level does not affect the belief on the vendor’s type. This is why the two opposite cases bring the same result. In addition, it is easy to show that the higher payoff dependence results in higher first-period-effort, by applying the same logic as in the single-type-vendor model, ceteris paribus (see Corollary 2). We summarize the above results as follows:

**Corollary 4.** In the two-type-vendor model:

1. The results under the farsighted assembler and under the myopic assembler coincide (i) if both payoff independence and type independence hold, or (ii) if both payoff independence and complete type dependence hold.
2. The first-period-effort level under the farsighted assembler is higher as payoff is more dependent.

### 4. Short-term relationship vs. long-term relationship

Up to now, we have considered the case in which an assembler and a vendor have a long-term relationship. In this section, we investigate the case of a short-term relationship in which an assembler may switch a vendor for another in the second period. Under a short-term relationship, we assume that the assembler also behaves myopically.

Recall that we assumed the proportion of type \(H\) in the vendor pool in the first period was \(q\). Now we slightly change notations as follows: the proportion of type \(H\) in the vendor pool in the period \(t\) is denoted as \(q_{t}\). Thus, if the assembler switches to another vendor in the second period, the belief that the new vendor is \(H\)-type becomes \(q_{2}\). We assume that switching to another vendor changes only the probability of type \(H\) and does not change the profit and cost structures, which implies that \(M_{2}\) and \(c_{2}\) are not changed by switching.

The problem in the first period is the same as the problem of the myopic assembler, which is, in turn, the same as the problem in the second period under the long-term relationship. Thus, we omit the analysis of the first period and focus on the second period.

The assembler will switch a vendor for another if and only if \(p_{2} < q_{2}\). Since, given \(e_{1}\), \(p_{2}\) is decreasing in \(R_{1}\), there is a critical \(R_{1}\), say \(R_{q}\), s.t. \(p_{2} \geq q_{2}\) iff \(R_{1} \leq R_{q}\). Then, in the second period, the assembler will switch vendors if and only if \(R_{1} > R_{q}\). Thus, the payoff to the assembler in the second period is

\[
\int \{p_{2}[f_{2}^{H}(e_{2}|R_{1}) - V(e_{2})] + (1 - p_{2})[f_{2}^{L}(e_{2}|R_{1}) - V(e_{2})]\}g(R_{1}|e_{1})dR_{1} - m, \quad \text{if } R_{1} \leq R_{q}.
\]

\[9\] A farsighted assembler may well enter into a short-term relationship with the vendor. However, focusing on the myopic assembler will highlight the trade-off between a short-term relationship and a long-term relationship. We will show that a short-term relationship may enable even a myopic assembler to enjoy a higher payoff than a farsighted assembler in a long-term relationship.

\[10\] Note that, in the previous section, the proportion of type \(H\) in the vendor pool in the second period is not relevant, since the relationship was long-term.
or
\[
\int \{q_2[f^H_2(e_2|R_1) - V(e_2)] + (1 - q_2)[f^L_2(e_2|R_1) - V(e_2)]\} g(R_1|e_1) dR_1 - m, \text{ if } R_1 > R_q.
\]

Thus, the expected payoff in the second period given \(e_1\) under a short-term relationship is
\[
E^s_2 E^T[f^T_2(e_2|R_1) - V(e_2) - m] = pr(R_1 \leq R_q) \int \{p_2[f^H_2(e_2|R_1) - V(e_2)] + (1 - p_2)[f^L_2(e_2|R_1) - V(e_2)]\} g(R_1|R_1 \leq R_q, e_1) dR_1 + \text{pr}(R_1 > R_q) \int \{q_2[f^H_2(e_2|R_1) - V(e_2)] + (1 - q_2)[f^L_2(e_2|R_1) - V(e_2)]\} g(R_1|R_1 > R_q, e_1) dR_1 - m,
\]
where \(E^s_2 E^T[\bullet]\) denotes the expectation operator for the second period under a short-term relationship; and \(g(R_1|R_1 \leq R_q, e_1)\) and \(g(R_1|R_1 > R_q, e_1)\) are the density functions of \(R_1\), given \(e_1\), conditional on \(R_1 \leq R_q\) and \(R_1 > R_q\), respectively. As a result, the optimal total expected payoff to the assembler is
\[
\Pi_S = E^T[f^T_1(e_1^*) - V(e_1^*) - m] + \delta E^s_2 E^T[f^T_2(e_2^*|R_1) - V(e_2^*) - m],
\]
where \(E^s_2 E^T[f^T_2(e_2^*|R_1) - V(e_2^*) - m]\) is calculated given \(e_1^*\). Recall that double stars (**) were also used for the myopic assembler under the long-term relationship.

From the previous section, we know that the optimal total expected payoff to the farsighted assembler under a long-term relationship is
\[
\Pi_F = E^T[f^T_1(e_1) - V(e_1) - m] + \delta E^s_2 E^T[f^T_2(e_2^*|R_1) - V(e_2^*) - m],
\]
where \(E^s_2 E^T[\bullet]\) denotes the expectation operator for the second period under farsighted assembler:
\[
E^s_2 E^T[f^T_2(e_2^*|R_1) - V(e_2^*) - m] = \int \{p_2[f^H_2(e_2^*|R_1) - V(e_2^*)] + (1 - p_2)[f^L_2(e_2^*|R_1) - V(e_2^*)]\} g(R_1|R_1 \leq R_q, e_1) dR_1 - m.
\]

Similarly, from the previous section, we can calculate the total expected payoff to the myopic assembler under a long-term relationship. The optimal total expected payoff to the myopic assembler under a long-term relationship is
\[
\Pi_M = E^T[f^T_1(e_1^*) - V(e_1^*) - m] + \delta E^s_2 E^T[f^T_2(e_2^*|R_1) - V(e_2^*) - m],
\]
where \(E^s_2 E^T[\bullet]\) denotes the expectation operator for the second period for a myopic assembler under a long-term relationship:
\[
E^s_2 E^T[f^T_2(e_2^*|R_1) - V(e_2^*) - m] = \int \{p_2[f^H_2(e_2^*|R_1) - V(e_2^*)] + (1 - p_2)[f^L_2(e_2^*|R_1) - V(e_2^*)]\} g(R_1|R_1 \leq R_q, e_1^*) dR_1 - m.
\]

We know that \(\Pi_F \geq \Pi_M\) from the previous section. It is also easy to see \(\Pi_S \geq \Pi_M\), since the assembler has an option to switch vendors under a short-term relationship. In order to compare \(\Pi_F\) and \(\Pi_S\), let us define “switching-option value (SV)” as \(\Pi_S - \Pi_M\) and “farsightedness value (FV)” as \(\Pi_F - \Pi_M\).

More concretely, we have the following expressions:
\[
\begin{align*}
\text{FV} &= \Pi_F - \Pi_M = \{f^H_1(e_1) - V(e_1) - \{f^H_1(e_1^*) - V(e_1^*)\}\} + \delta \int \{f^H_2(e_2^*|R_1) - (1 - p_2)c_2a - V(e_2^*)\} \{g(R_1|e_1^*) - g(R_1|e_2^*)\} dR_1, \\
\text{SV} &= \Pi_S - \Pi_M = \delta \text{pr}(R_1 > R_q) \int (q_2 - p_2)c_2ag(R_1|R_1 > R_q, e_1^*) dR_1 = \delta \int_{R_1 > R_q} (q_2 - p_2)c_2ag(R_1|e_1^*) dR_1.
\end{align*}
\]

Now, \(\Pi_F - \Pi_S = (\Pi_F - \Pi_M) - (\Pi_S - \Pi_M) = \text{FV} - \text{SV}\). Thus, \(\Pi_F - \Pi_S \geq 0\) iff \(\text{FV} \geq \text{SV}\).
From the previous section, we know that \( FV = 0 \) under payoff independence and type independence or under payoff independence and complete type dependence. Now let us focus on \( SV \). It is clear that \( SV \geq 0 \).

A sufficient condition for \( SV = 0 \) is that \( q_2 \leq \min\{p_2\} \) or equivalently, \( \max\{R_1\} \leq R_q \), so that \( Pr(R_1 > R_q) = 0 \). This condition implies that the incumbent vendor’s ability is always higher than outside vendors’ average ability. In this case, the assembler has no incentive to change the vendor, thus the switching option has a zero value. This case will hold if the learning effect of the incumbent vendor is significant.

Another sufficient condition for \( SV = 0 \) is that \( p_2 = q_2 \) for all \( R_i \). This case implies that the incumbent vendor’s ability equals outside vendors’ average ability, so that the assembler does not need to change the vendor. This case will hold if the assembler thinks that the incumbent vendor’s ability represents the average ability. We may observe this case if the response time is primarily determined by the events uncontrollable by the vendor, such as change of regulations or change of weather.

Now let us note some observations for \( SV \). First, it is obvious that \( SV \) becomes larger if \( p_2 \) is uniformly lower for each \( R_1 > R_q \), ceteris paribus. In addition, if the lower \( p_2 \) is accompanied with a lower \( R_q \), then \( SV \) will be even larger. As an extreme case, we can show that, given \( Pr(R_1 > R_q) \) fixed, \( SV \) is the highest under complete type dependence. For, \( p_2 = 0 \) for each \( R_1 > R_q \) under complete type dependence, while \( p_2 > 0 \) for some \( R_i \) under other types of dependence. Thus, \( SV \) is higher under complete type dependence than under any other case of type dependence, given \( Pr(R_1 > R_q) \) fixed.

Second, note that a higher \( q_2 \) will induce higher \( SV \), given the response time distribution. For, \( q_2 - p_2 \) becomes larger and, in addition, higher \( q_2 \) is likely to be accompanied with a lower \( R_q \). We will observe a higher \( q_2 \) if the outside vendors’ abilities increase more rapidly than the incumbent’s, or if a group of vendors with innovative technologies enter the market. Note also that \( q_2 \) is not related with \( FV \). Thus, a higher \( q_2 \) will decrease \( \Pi_F - \Pi_S \).

Finally, let us investigate the effect of \( a \) on \( SV \). It is obvious that a higher \( a \) induces higher \( SV \), ceteris paribus. Under Bayesian adjustment of the belief, we show that a higher \( a \) induces higher \( SV \).\(^{11}\) Intuitively, a higher \( a \) implies clearer distinction between vendor’s types based on response time. In this case, the switching option becomes more valuable, since the assembler can switch vendors when he is more likely to be of type \( L \). The technical proof is provided in the appendix. On the other hand, a higher \( a \) may also increase \( FV \). As a result, the effect of a higher \( a \) on \( \Pi_F - \Pi_S \) is ambiguous.\(^{12}\) We summarize the above discussion in the following proposition.

**Proposition 4**

1. \( \Pi_F - \Pi_S \geq 0 \) if \( \min\{p_2\} \geq q_2 \), or if \( p_2 = q_2 \) for all \( R_i \).
2. \( \Pi_F - \Pi_S \leq 0 \) if both payoff independence and type independence hold, or if both payoff independence and complete type dependence hold.
3. \( SV \) is higher under complete type dependence than under any other case of type dependence, unless complete type dependence is accompanied with a lower \( Pr(R_1 > R_q) \).
4. \( SV \) is higher and \( \Pi_F - \Pi_S \) is lower when \( q_2 \) is higher.
5. Under Bayesian adjustment of belief, \( SV \) is higher if \( a \) is higher.

**Proof.** See the appendix. \( \square \)

This proposition has several interesting implications. It provides a distinction between the relative advantages of long-term and short-term relationships. Consider an industry in which rapid technology development is fueled by new entrepreneurs with innovative technologies, like the IT/Internet or electronics industry. In this case, improvements to a product by the incumbent vendor may not be delivered to the aggregate future profit increase (payoff independence). Nor, do they signal the future competitiveness of the incumbent vendor (type

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\(^{11}\) Note that the assumption of \( \hat{p}_{2|o} < R_1 \) restricts the shape of response time distribution given effort, when \( p_2 \) is Bayesian-adjusted. Among others, the probability density function should increase up to some point and decrease afterwards.

\(^{12}\) For example, if \( c_2 \) and \( p_2 (\neq q_2) \) are independent of \( R_i \), then the effect of higher \( a \) on \( FV \) is zero, while it is positive on \( SV \). On the other hand, if \( p_2 = q_2 \), then the effect of higher \( a \) on \( SV \) is zero, while it can be positive on \( FV \).
independence). On the other hand, the chances that the incumbent vendor performs better than the industry average in the future are small (higher $q_2$). In such an industry, SV becomes large relative to FV, so that $P_S$ becomes larger than $P_F$. Thus, a short-term relationship is preferred.

In comparison, consider the automobile or heavy construction equipment industry. Technology development on the vendor side is relatively stable and vendors are often required to provide assembler-specific supplies. Thus, the learning effect of working with one assembler is an important source of efficiency. This implies that the probability of an incumbent vendor being good type is likely higher than the industry average (low $q_2$). As a result, in these industries, SV becomes small relative to FV, so that $P_F$ becomes larger than $P_S$. Thus, a long-term relationship is preferred.

We depict the relative preference of relationship in Fig. 1 based on Proposition 4 and Corollary 2. Preference of relationship is depicted in the $(q_2, \text{type independence})$-plane in Fig. 1. This figure shows how preference of relationship depends on the degree of payoff dependence, the degree of type dependence, and $q_2$.

It is also important to note that a long-term relationship does not always outperform a short-term relationship in the second period, even if $P_F \geq P_S$. This is because the assembler under the short-term relationship has the switching option, while the assembler under the long-term relationship should keep the vendor, even if the vendor is, in fact, of low ability. As a result, poor performance during short periods can be observed even if the long-term relationship is preferred.

5. Summary and conclusions

In this paper, we developed two models: a single-type-vendor model and a two-type-vendor model. In the single-type-vendor model, we showed that the farsighted assembler obtains higher effort level from the vendor than the myopic assembler in the first period and maximizes the total expected utility. Then, we showed that similar results hold for the two-type-vendor model. We also compared long-term and short-term relationships and generated several sufficient conditions under which one relationship is better than the other. Variables which determine proper relationship between a vendor and an assembler in a supply chain are: payoff and type independence; proportion of good vendors in the vendor pool in the second period. Our results imply that a short-term relationship is preferred in industries in which rapid technology development is fueled by new entrepreneurs with innovative technologies, while a long-term relationship is preferred in industries in which technology development is stable and learning effects are important. We depict preference of relationships in Fig. 1. Our results, however, provide us only with a conceptual framework for the optimal supply chain relationship. Interesting future research topics will include empirical analyses of the results.
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**Appendix**

**Proof of Proposition 2.** The Lagrangian and the first-order conditions are as follows:

\[
L = (1 - b_1)f_1(e_1) - F_1 + \delta E_{R_1}^R[f_2(e_2|R_1) - V(e_2) - m] + \lambda_1(F_1 + b_1f_1(e_1) - V(e_1) - m) + \mu_1(b_1f_{1,e_1} - V''(e_1)),
\]

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\[
L_{F_1} = -1 + \partial \{\delta E_{R_1}^R[f_2(e_2|R_1) - V(e_2) - m]/\partial F_1\} + \lambda_1 = 0,
\]

\[
L_{e_1} = (1 - b_1)f_{1,e_1} + \partial \{\delta E_{R_1}^R[f_2(e_2|R_1) - V(e_2) - m]/\partial e_1\} + \lambda_1(b_1f_{1,e_1} - V'(e_1)) + \mu_1(b_1f_{1,e_1} - V''(e_1)) = 0,
\]

\[
L_{b_1} = -f_1(e_1) + \partial \{\delta E_{R_1}^R[f_2(e_2|R_1) - V(e_2) - m]/\partial b_1\} + \lambda_1f_1(e_1) + \mu_1f_{1,e_1} = 0.
\]

In \(L_{F_1} = 0, \partial E_{R_1}^R[f_2(e_2|R_1) - V(e_2) - m]/\partial F_1 = 0\) since \(E_{R_1}^R(\cdot)\) is affected only through \(e_1\). Thus, \(\lambda_1 = 1\). In \(L_{e_1} = 0, b_1f_{1,e_1} - V'(e_1) = 0\) from the incentive constraint. Thus \(L_{e_1} = (1 - b_1)f_{1,e_1} + \partial \{\delta E_{R_1}^R[f_2(e_2|R_1) - V(e_2) - m]/\partial e_1\} + \mu_1(b_1f_{1,e_1} - V'(e_1)) = 0\). On the other hand, \(\partial E_{R_1}^R[f_2(e_2|R_1) - V(e_2) - m]/\partial b_1 = 0\) since \(E_{R_1}^R(\cdot)\) is affected only through \(e_1\). Thus, \(L_{b_1} = \mu_1f_{1,e_1} = 0\). We have \(\mu_1 = 0\). Thus, we have \(L_{e_1} = (1 - b_1)f_{1,e_1} + \partial \{\delta E_{R_1}^R[f_2(e_2|R_1) - V(e_2) - m]/\partial e_1\} = 0\).

Now we show that \(b_1^* \geq 1\). Recall that we denote \(E_{R_1}^R\) for \(\partial E_{R_1}^R[f_2(e_2|R_1) - V(e_2) - m]/\partial e_1\). Then, \(L_{e_1} = (1 - b_1)f_{1,e_1} + \partial E_{R_1}^R = 0\). We have \(b_1^* = 1 + \partial E_{R_1}^R / f_{1,e_1}\). Therefore, \(b_1^* \geq 1\) is equivalent to \(E_{R_1}^R \geq 0\), since \(f_{1,e_1} > 0\). We want to show \(E_{R_1}^R \geq 0\).

Note that

\[
E_{R_1}^R[f_2(e_2|R_1) - V(e_2) - m] = \int [f(e_2|R_1) - V(e_2) - m]g(R_1|e_1)dR_1.
\]

Thus,

\[
E_{R_1} = \int \left[ f_2(e_2 | R_1) - V(e_2) - m \right] g_{e_1}(R_1 | e_1) dR_1 = \int \left[ f_2(e_2 | R_1) - V(e_2) \right] g_{e_1}(R_1 | e_1) dR_1.
\]

First, we show that \(f_2(e_2|R_1) - V(e_2)\) is higher with lower \(R_1\), where \(e_2\) is evaluated at the optimal effort level given \(R_1\). Note that the decrease of \(R_1\) increases \(M_2\) and decreases \(c_2\). Let \(R_1 < R_1'\). Let \(M_2'\) and \(c_2'\) correspond to \(R_1\) and \(M_2'\) and \(c_2'\) correspond to \(R_1'\). Then, we have \(M_2 - c_2r(e) - V_2(e) - m > M_2' - c_2'r(e) - V_2(e) - m\), for all \(e\), thus, \(f_2(e_2 | R_1) - V_2(e) > f_2(e_2 | R_1') - V_2(e)\) for all \(e\). Thus, \(\max_e[f_2(e_2 | R_1) - V_2(e)] > \max_e[f_2(e_2 | R_1') - V_2(e)]\). Note that the optimal \(e_2'\) is chosen to maximize the expected utility of the assembler in the second period, or \(-F = f_2(e_2 | R_1) - V_2(e) - m\). Thus, \(f_2(e_2' | R_1) - V_2(e_2') > f_2(e_2' | R_1') - V_2(e_2')\), where \(e_2'(e_2')\) is the optimal effort level corresponding to \(R_1(R_1')\). Therefore, \(f_2(e_2 | R_1) - V(e_2)\) is higher with lower \(R_1\).

Second, note that the increase of \(e_1\) induces first-order-stochastically-dominated distribution of \(R_1\). As a result, the increase of \(e_1\) puts higher weights to lower \(R_1\) and thus higher \(f(e_2) - V(e_2)\). Thus, \(E_{R_1}^R \geq 0\). Thus, \(b_1^* = 1 + \partial E_{R_1}^R / f_{1,e_1} \geq 1\). Unless \(\partial E_{R_1}^R = 0, b_1^* > 1\).

We have \(F_1^* = m + V(e_1^*) - b_1^*f_1(e_1^*)\). The payoff to the assembler is

\[
(1 - b_1^*)f_1(e_1^*) - F_1^* + \partial E_{R_1}^R[f_2(e_2^* | R_1) - V(e_2^*) - m] = f_1(e_1^*) - m - V(e_1^*) + \partial E_{R_1}^R[f_2(e_2^* | R_1) - V(e_2^*) - m].
\]

**Proof of Corollary 2**

(1) is proved in the text.

(2) Suppose that \(f_2(e_2^* | R_1) - V(e_2^*)\) is more dependent than \(g_{e_2}(e_2^* | R_1) - V(e_2^*)\).
Then, for $R_1^L < R_1^H$, \[ f_2(e_2^L|R_1^L) - V(e_2^L) - g_2(e_2^L|R_1^L) - V(e_2^L) \geq f_2(e_2^H|R_1^H) - V(e_2^L) - g_2(e_2^H|R_1^H) - V(e_2^L). \]

Thus, \[ f_2(e_2^L|R_1^L) - V(e_2^L) - f_2(e_2^L|R_1^L) - V(e_2^L) \geq g_2(e_2^H|R_1^H) - V(e_2^L) - g_2(e_2^H|R_1^H) - V(e_2^L). \] In other words, more-dependent-payoff increases more than less-dependent-payoff as $R_1$ decreases. On the other hand, the increase of $e_1$ induces first-order-stochastically-dominated distribution of $R_1$, which implies that higher $e_1$ puts higher weights on lower $R_1$. Combining these two facts results in higher $E_{e_1}^{R_1}$ for more-dependent-payoff. Recall that the optimal effort level is determined by $b_{f_1,e_1} - V'(e_1) = 0$, or $f_{1,e_1} + \delta E_{e_1}^{R_1} = V'(e_1)$. With higher $\delta E_{e_1}^{R_1}$, the optimal effort level is higher, since $f_{1,e_1}$ is decreasing and $V'(e_1)$ is increasing in $e_1$. \[ \square \]

**Proof of Proposition 3.** Except for $b_1^* \geq 1$, the proof is the same as those in Propositions 1 and 2, if we apply expectations over $T$, where necessary. In proving $b_1^* > 1$, the only additional change is that we apply our assumption that $\partial p_2/\partial R_1 \leq 0$ to show $E_{e_1}^{R_1} f_1^2(e_2) - V(e_2)$ is decreasing in $R_1$. \[ \square \]

**Proof of Proposition 4.** Proofs of (1)-(4) are provided in the text above Proposition 4. Under Bayesian adjustment, $p_2$ given $R_1$ is determined by the relative size between $g^H(R_1|e_1^*)$ and $g^L(R_1|e_1^*)$: $p_2 = p_1 g^H(R_1|e_1^*)/g(R_1|e_1^*)$, where $g(R_1|e_1^*) = p_1 g^H(R_1|e_1^*) + (1 - p_1) g^L(R_1|e_1^*)$. Suppose that $a \leq a'$. Note that $g^L(R_1|e_1^*, a) = g^L(R_1 + (a' - a)|e_1^*, a')$, where $a$ and $a'$ denote for corresponding distributions, since the change of $a$ moves only the location the distribution. Let us denote SV(a) for $SV(a)$ associated with $a$.}

\[
SV(a) = \int_{\{R_1 \geq R_0\}} (q_2 - p_2) c_2 a g(R_1|e_1^*) dR_1
= \int_{\{R_1 \geq R_0\}} c_2 a \{q_2 g(R_1|e_1^*) - p_1 g^H(R_1|e_1^*)\} dR_1 \quad \text{(since $p_2 = p_1 g^H(R_1|e_1^*)/g(R_1|e_1^*)$)}
= \int_{\{R_1 \geq R_0\}} c_2 a \{(q_2 - 1)p_1 g^H(R_1|e_1^*) + q_2 (1 - p_1) g^L(R_1|e_1^*, a)\} dR_1 \quad \text{(since $g(R_1|e_1^*) = p_1 g^H(R_1|e_1^*) + (1 - p_1) g^L(R_1|e_1^*)$)}
= \int_{\{R_1 \geq R_0\}} c_2 a \{(q_2 - 1)p_1 g^H(R_1 + (a' - a)|e_1^*) + q_2 (1 - p_1) g^L(R_1 + (a' - a)|e_1^*, a')\} dR_1
\leq \int_{\{R_1 \geq R_0\}} c_2 a \{(q_2 - 1)p_1 g^H(R_1 + (a' - a)|e_1^*)\} dR_1,
\]

where $c_2$ denotes for the cost corresponding to $R_1 + (a' - a)$.

\[
= \int_{\{R_1 \geq R_0\}} c_2 a \{(q_2 - 1)p_1 g^H(R_1 + (a' - a)|e_1^*)\} dR_1
= \int_{\{R_1 \geq R_0\}} c_2 a \{(q_2 - 1)p_1 g^H(R_1 + (a' - a)|e_1^*)\}
= \int_{\{R_1 \geq R_0\}} c_2 a \{(q_2 - 1)p_1 g^H(R_1 + (a' - a)|e_1^*)\} dR_1
\leq \int_{\{R_1' \geq R_0\}} c_2 a \{(q_2 - 1)p_1 g^H(R_1'|e_1^*)\} dR_1' = SV(a'),
\]

where $R_1'$ is the critical response time for $a'$.

The last inequality holds since $R_1' \leq R_0 + (a' - a)$ which follows from the assumption of $\partial p_2/\partial R_1 \leq 0$. \[ \square \]
References


