Analysis and Optimization of Distributed Random Sensing Order in Cognitive Radio Networks

Hossein Shokri-Ghadikolaei and Carlo Fischione

Abstract—Developing an efficient spectrum access policy enables cognitive radios to dramatically increase spectrum utilization while ensuring predetermined quality of service levels for the primary users. In this paper, modeling, performance analysis, and optimization of a distributed secondary network with random sensing order policy are studied. Specifically, the secondary users create a random order of the available channels upon primary users return, and then find optimal transmission and handoff opportunities in a distributed manner. By a Markov chain analysis, the average throughputs of the secondary users and average interference level among the secondary and primary users are evaluated. A maximization of the secondary network performance in terms of throughput while keeping under control the average interference is proposed. It is shown that despite of traditional view, non-zero false alarm in the channel sensing can increase channel utilization. Then, two simple and practical adaptive algorithms are established to optimize the network. The second algorithm follows the variations of the wireless channels in non-stationary conditions and outperforms even static brute force optimization, while demanding few computations. Finally, numerical results validate the analytical derivations and demonstrate the efficiency of the proposed schemes. It is concluded that fully distributed algorithms can achieve substantial performance improvements in cognitive radio networks without the need of centralized management or message passing among the users.

Index Terms—Cognitive radio networks, sequential channel sensing, Markov chain analysis, average throughput, average interference, distributed optimization.

I. INTRODUCTION

EMERGING new wireless applications and ever-growing demands for a higher data rate challenges the limited spectrum resources and consequently the current fixed spectrum allocation policies. To effectively mitigate the problems associated with the fixed spectrum allocation, the promising concept of cognitive radio networks (CRNs) has been the focus of intense research in both academic and regulatory bodies [1], [2].

CRN promotes spectrum utilization by allowing low priority secondary users (SUs) to opportunistically exploit the underutilized licensed channels of high priority primary users (PUs) in an intelligent manner [3]. Meanwhile, due to preemptive priority of the PUs to access the channels, the SUs must vacate the channel whenever the corresponding PUs appear. In this case, a set of procedures called spectrum handoff (SHO) is initiated to help the SUs in finding new transmission opportunities, through reliable spectrum sensing, and resuming their unfinished transmissions [4]. Clearly, the performance of an SHO framework depends heavily on the performance of spectrum sensing. The noise and the channel impairments such as shadowing and fading, however, lead to decision errors, quantified in terms of false alarm and miss detection probabilities. A false alarm occurs when a free channel is mistakenly sensed busy, while a miss detection happens whenever an occupied channel is sensed free. With each false alarm, a transmission opportunity is lost, and after each miss detection, an SU starts to transmit on the channel and consequently interferes with signal present in the channel.

A. Spectrum Handoff Models

Broadly speaking, SHO procedures can be modeled by connection-based and slot-based modeling techniques [5]. The connection-based model defines the spectrum handoff upon appearance of the PUs (event-driven manner), while in the slot-based methods, the spectrum handoff process can be performed in each time slot (time-driven manner). In [5]–[8], performance of connection-based SHO in terms of extended data delivery time and handoff delay is extensively evaluated, and several optimization framework for SHO are proposed. In this paper, we focus on the slot-based SHO model.

An SU can conduct wideband or narrowband spectrum sensing at the beginning of each time slot, depending on the power budget and affordable computational complexity. In the wideband spectrum sensing, an SU senses multiple channels simultaneously, while only one channel can be sensed in the narrowband spectrum sensing [9]. Easier implementation, lower power consumption, and less computational complexity lead to great interest in narrowband spectrum sensing. Here, we assume that the SUs are able to sense and possibly transmit on one channel at a time. In this case, an SU sorts the channels in an order, called sensing order, and transmits on first channel that is sensed free in the established order. If the channel is sensed busy, the SU initiates the SHO procedure and then senses the second channel of the sensing order, and so on. Such a sensing-access is called sequential channel sensing [4].

B. Related Work

Recently, the problem of designing a proper framework for sequential channel sensing has gained much interests. In [10], the optimal sensing order design has been investigated in order for an SU to achieve the maximum energy efficiency by applying a dynamic programming solution. The tradeoff between sensing accuracy and consumed energy in sequential channel sensing is investigated in [11], wherein optimal solution along with two suboptimal heuristic algorithms are proposed for determining proper sensing time and order that maximize the energy efficiency. In [12], the authors find the optimal
sensing order to minimize the probability of not finding a free channel upon triggering of SHO. Besides energy efficiency, throughput maximization is also extensively studied. Optimal and suboptimal sensing orders of a CRN containing only one SU are developed in [15]–[17], which maximize the average achievable throughput of the SU in a time slot. These results have been further extended for a CRN with two [18] and multiple SUs [19]–[22]. A closed-form optimal solution as well as three suboptimal solutions for maximizing the average throughput by setting proper sensing orders have been proposed in [21], and an intelligent sensing order setting scheme for a centralized CRN has been introduced in [22].

In [23], a dynamic programming-based framework for sequential channel sensing is proposed to minimize the SHO delay for a heterogeneous network. Although maximizing the average throughput is of critical importance in secondary communications design, the final framework might be of difficult applicability, since throughput maximization may result in a large interference with the primary network, which violates the prerequisites of a CRN, being transparent to the primary network. Therefore, a general framework considering both throughput and interference is desirable. In [24], the authors investigate the optimal sensing time and order for maximizing the expected throughput of a CRN with one SU, and for penalizing interferences that disrupt the primary communications. The same problem is investigated in [25], wherein optimal parameters for spectrum sensing, i.e., the sensing time and decision threshold, are found.

C. Motivation

Most of the literatures in the slot-based SHO focus on single SU or centralized CRNs [10], [16]–[25], where the existence of a coordinator is an inseparable part of these centralized algorithms. The coordinator computes the best parameters for the optimum networks operation, and then let the SUs know the parameters. The main problem is that not only a centralized network coordinator cannot be assumed in many CRNs applications, but it imposes a massive computational burden on the network as well. As shown in [21], for instance, the computational complexity of finding the optimal sensing orders exponentially increases with number of PUs and number of SUs. Actually, this holds even in single SU case [14], which motivates the authors to develop a suboptimal SHO algorithm.

An SHO framework for a distributed CRN without a common control channel is proposed in [26]. However, a wideband and perfect spectrum sensing is considered in the paper, which are hard to be applicable in many practical systems. In [27], an autonomous weighting policy is developed with the aim of minimizing the likelihood of collisions with other SUs in a distributed manner. The authors show that their algorithm might achieve collision-free sensing orders, i.e., the SUs never collide among them. However, the miss detection probability is assumed zero, meaning that the SUs would not make interference for the PUs as well as other SUs. Therefore, quality of service (QoS) provisioning for the PUs is not addressed. In [28], the authors exploit a modified p-persistent MAC protocol to set the sensing orders of the SUs in a distributed manner. However, it is assumed that the SUs can successfully transmit on the channel even if the PU presets on the channel. In fact, they focused on maximum achievable throughput and did not study the interference (inter or intra) in the network. As a result, there is no QoS guaranteeing mechanism for the PUs. Also, the authors assumed that the spectrum sensing performance does not change, even though the level of signals present in the channel changes.

D. Contribution

In this paper, we substantially extend our previous study [28] and extend the investigations of [25] to a multiuser distributed CRN, where not only the interference between the SUs and the PUs, hereafter called inter-CRN interference, is important, but also the interference between the secondary users, hereafter called intra-CRN interference, is highly important, since it affects the QoS of the secondary connections. We investigate the performance of a CRN adopting random sensing order policy. That is, once an SHO is triggered, all the SU create a set of random channels to be sensed, and sequential channel sensing process is initiated. We propose a finite state novel Markov chain to effectively model the SUs, which allow to evaluate the average throughputs of the SUs and average inter and intra-CRN interferences. Then, an optimization problem is formulated to maximize the average throughput while keeping the average interferences bounded.

However, this optimization poses a high computational burden, and it is not always consistent with real non-stationary wireless channels. Therefore, we propose novel cross-layer adaptive algorithms of light computational complexity that use no analytical models for the link statistics, where the analytical optimization problem is used as a benchmark to check the performance of the distributed algorithms. By these algorithms, each SU just needs to receive ACKs from its receiver to iteratively maximize the average throughput for a given maximum allowable inference. Furthermore, as the proposed algorithms can be implemented in a fully distributed fashion without any need of message passing among the SUs, they decrease the problems associated with control channel establishment in CRN terminology [29], [30].

In addition, we show that the traditional view of false alarm, i.e., smaller false alarm higher average throughput, is no longer valid in distributed CRN, where the contention among the users plays an important role. In fact, higher false alarm can substantially increase channel utilization.

Compared to the literature mentioned above, this is the first paper to 1) consider the problem of SHO for sequential channel sensing in a distributed set-up with more realistic assumptions including miss detection and false alarm probabilities, 2) investigate the inter and intra interferences and keep both of them under control, 3) investigate the impact of the SUs’ transmissions on the channel occupation probabilities and spectrum sensing performance, 4) propose simple and practical algorithms to keep the overall system performance at the optimum level while maintaining the QoS guarantees in non-stationary conditions.

The rest of this paper is organized as follows. In Section [II] we describe the considered CR network. In Section [III] we...
the structure of the random sensing order policy is described, and its performance is evaluated. Moreover, two efficient algorithms are proposed in Section IV to optimize the performance of the network. Numerical results are then provided in Section V followed by concluding remarks provided in Section VI.

II. SYSTEM MODEL

A time slotted CRN with $N_s$ SUs is assumed. The SUs attempt to opportunistically transmit on the channels exclusively dedicated to the $N_p$ PUs, each having one channel. As assumed in [16]–[18], the SUs are synchronous in time-slots with other SUs as well as the PUs. In the sequential channel sensing methodology, once a handoff is requested, each SU’s time slot divides into sensing and transmission modes. In the sensing mode, the SUs sequentially sense the channels based on their sensing orders [16]–[18]. Suppose that the sensing order of SU $n$, for $n = 1, \ldots, N_s$ (see Table I for a summary of the frequently used notations), is

$$c_n = [c_{1n}, c_{2n}, \ldots, c_{jn}] ,$$

where $c_{1n}$ and $c_{jn}$ denote the first and the last channels to be sensed. $\delta$ is the maximum number of channels that an SU can sense in a time slot. The SUs sense the first channel of their sensing orders, $c_{1,j}$ for $1 \leq j \leq N_p$, and start their communications on the channels sensed free. Other SUs initiate the handoff process, which takes $\tau_h$ seconds, and then sense the second channel of their sensing orders. The procedure continues until [31]: a) all the SUs find transmission opportunities, b) no time remains for sensing new channels in the time slot, or c) no non-sensed channels remains. It holds [31] that

$$\delta = 1 + \min \left( \frac{T - \tau}{\tau + \tau_h}, N_p - 1 \right) ,$$

where $T$ is a time slot duration, and $\tau$ is the channel sensing time. After sensing $(n-1)$ occupied channels, if an SU finds $n$-th channel of its sensing order free, the user will transmit data on that channel for the rest of the slot. In the case, the time length left in the slot for the transmission is

$$RT_n = T - \tau - (n - 1)(\tau + \tau_h) .$$

Fig. I demonstrates the timing structure of each SU $j$.

III. MODELING AND PERFORMANCE EVALUATION

In this section, the random sensing order policy (RSOP) is modeled, and the performance of a CRN with RSOP is derived using a Markov chain analysis. Then, an adaptive protocol is proposed to optimize the performance of the CRN. Recall that each SU sequentially senses the channels based on an order. It has been shown that regardless the computational complexity, the sensing orders can be optimally determined in a single user [16] or centralized multiple user [21] CRNs. However, we cannot directly apply those proposals to distributed CRNs.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$p$</td>
<td>Channel sensing probability</td>
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<tr>
<td>$\tau$</td>
<td>Average throughput of each SU</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Average interference time in the network</td>
</tr>
<tr>
<td>$C_{p,m}$</td>
<td>Transmission rate</td>
</tr>
<tr>
<td>$L^{(n)}$</td>
<td>Number of SUs sensing channel $m$ at stage $n$</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Number of PUs</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Number of SUs</td>
</tr>
<tr>
<td>$N_{sp}$</td>
<td>Number of slots in a frame estimation period</td>
</tr>
<tr>
<td>$P_{fa,m}$</td>
<td>False alarm probability of sensing channel $m$ in sensing stage $n$</td>
</tr>
<tr>
<td>$P_{md,m}$</td>
<td>Detection probability of sensing channel $m$ in sensing stage $n$</td>
</tr>
<tr>
<td>$P_{md,1}$</td>
<td>Presence probability of the PU $m$</td>
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<tr>
<td>$P_{md,m}^{(n)}$</td>
<td>Probability that channel $m$ is busy at the beginning of stage $n$</td>
</tr>
<tr>
<td>$P_{md,m}^{(n)}$</td>
<td>Maximum allowable miss detection probability</td>
</tr>
<tr>
<td>$P_{md,m}^{(n)}$</td>
<td>Maximum allowable false alarm probability</td>
</tr>
<tr>
<td>$RT_n$</td>
<td>Transmission time if $n$-th channel of sensing order is sensed free</td>
</tr>
<tr>
<td>$T$</td>
<td>Time slot duration</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Maximum number of channels can be sensed in a time slot</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Sensing time</td>
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For such networks, simple sensing orders are proposed in [28], wherein the channels are arranged by their indices. For a simple order, we have [28]:

$$c_{1j} = 1 \quad c_{2j} = 2 \quad \ldots \quad c_{\delta_j} = \delta , 1 \leq j \leq N_s .$$

While this order facilitates the network modeling and performance evaluation, it causes a high level of contention to access the same channels, which significantly degrades the average throughput of the CRN. Also, more efficient sensing orders, as proposed in [29], [27], are highly sensitive to false alarm and miss detection probabilities. In fact, they are originally developed for perfect spectrum sensing, which is not achievable in real world.

In order to mitigate the aforementioned problem, we propose to use optimal RSOP. In this scheme, an SU chooses by a random distribution a target channel in each sensing interval between 1 and $N_p$ for each $c_{ij}$. Therefore, the requests of the SUs are uniformly distributed among all available channels, and thereby the CRN throughput increases by the reduction of the contention for accessing the same channels. Moreover, adopting RSOP desirably bypasses message passing or other signaling overheads required for designing optimal sensing orders in a distributed CRN. Altogether, RSOP is highly...
more channels will be sensed in the conventional p-persistent multiple access algorithm compared to the modified one. Further comparisons between their average throughput and energy consumptions (for spectrum sensing) will present in Section V.

Fig. 2 models by a Markov chain the channel sensing-access policy of the RSOP used by each SU. The state \(m^{(n)}\) refers to the case that the SU starts to sense, which takes \(\tau\) time units, and possibly transmits on the channel \(m\) at the \(n\)-th sensing stage. Let \(P_{m,0}^{(n)}\) and \(P_{m,1}^{(n)}\) respectively be the probability that the channel \(m\) is free and occupied at the beginning of the \(n\)-th sensing stage. Let \(P_{fa,m}^{(n)}\) and \(P_{md,m}^{(n)} = 1 - P_{fa,m}^{(n)}\) denote the false alarm and miss detection probabilities of sensing the channel \(m\) in the \(n\)-th sensing stage. An SU can successfully transmit on the channel \(m\) if it is free, and the false alarm does not occur. Once this event happens, with the probability \(P_{m,0}^{(n)} \left(1 - P_{fa,m}^{(n)}\right)\), the SU’s state changes to the transmitter nodes \(T_n\), and it transmits on the channel for the rest of the time slot, \(R_{T_n}\), with the constant rate of \(C_R\). Even though we consider constant transmission rate, we can easily extend the formulations present in the paper to cover heterogeneous SUs. This assumption is done also in [18], [20]. The interference experienced at node \(n\) is denoted by \(I_n\) and happens whenever the channel is busy, and the SU mistakenly senses it free, with probability \(P_{m,1}^{(n)} \left(1 - P_{fa,m}^{(n)}\right)\). Different handoffs are modeled using nodes \(HO_i\), \(i = 1, 2, \ldots, \delta\), where recall that \(\delta\) is defined in (2). Note that the first handoff node does not exist in the search process, and we use it for easing the analysis without loss of generality.

At the beginning of each time slot, an SU gives the state \(HO_1\) (in Fig. 2), and immediately transits to one of the first sensing nodes, \(1^{(1)}, 2^{(n)}, \) or \(N_{p}^{(1)}\) with the identical probabilities of \(p/N_p\), or to the synchronizer state \(SYN_1\). After \(\tau\) time units, the SU’s state changes to the transmitter state (node \(T_1\)), interference state (node \(I_1\)), or to the second handoff node \((HO_2)\) with the probability \(q_{m}^{(1)}\). From Fig. 2

\[
q_{m}^{(1)} = P_{m,0}^{(1)} P_{fa,m}^{(1)} + P_{m,1}^{(1)} P_{md,m}^{(1)}.
\]

This procedure continues until the maximum number of admissible handoff is reached. Let us define the \(i\)-th stage of the sensing-access process, shown in Fig. 2 as the set of nodes of the Markov chain \(HO_i, SYN_i, m^{(i)}, T_i,\) and \(I_i\). After the stage \(\delta\), the SU’s state transits to the terminate node \(TE\), meaning that the SU sleeps for the rest of the time slot, \(T - \tau - \delta (\tau + \tau_h)\). Then, it repeats the search-access process at the beginning of the next slot [16]. In the RSOP, a busy channel can be occupied either by the corresponding PU or other SUs; other SUs might detect the channel as a transmission opportunity at the previous stages. This event allows us to establish two results. First, the channel occupancies status changes in successive stages. Second, the average signal level that is present in the wireless media

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3Note that the SUs that are not routed to the sensing stages, with probability \((1 - p)\), enter standby mode (at node \(SYN_1\)) and wait for \(\tau\) time units (sensing period). Then, they are directed to the state \(HO_2\). With the help of the synchronizer nodes, all SUs will enter the \(i\)-th stage node at the same time.
changes by each sensing stage. In other words, as it is possible that some SUs transmit on occupied channels in each stage $n$, the remained SUs face a higher received signal levels if they sense those channels at the stage $n + 1$.

Proposition I: Consider the Markov chain of Fig. 2. The occupation probability of channel $m$ at the beginning of stage $n$ is

$$P_{m,1}^{(n)} = P_{m,0}^{(n-1)} + P_{m,0} \left( P_{fa,m}^{(1)} \right) \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots + \mathcal{L}^{(n-2)} T_{m}^{(n-1)}, \quad (6)$$

for $1 \leq m \leq N_p$ and $1 \leq n \leq \delta$, where $\mathcal{L}^{(n)}$ denotes the number of SUs sensing channel $m$ at stage $n$, $U_{m}^{(n)}$ is the probability of transmission on channel $m$ at stage $n$ by at least one SU conditioned on the absence of the corresponding PU, and $P_{m,0}$ denote the absence probability of the PU $m$.

Proof: A proof is given in Appendix A.

The received SNR affects the performance of spectrum sensing and sequential channel sensing. To increase the accuracy of the RSOP model, the different detection and false alarm probabilities have to be considered in various sensing stages. The performance analysis of various spectrum sensing techniques are out of the scope of this paper. However, we derive the formulations for energy detector-based spectrum sensing and use it for optimization purposes and numerical results.

Proposition II: Consider the Markov chain of Fig. 2. For the energy detector-based spectrum sensing, it holds

$$P_{fa,m}^{(n)} = P_{fa,m}^{(1)} \mathcal{L}^{(1)} \mathcal{L}^{(2)} \cdots \mathcal{L}^{(n-2)} T_{m}^{(n-1)}, \quad (7)$$

and

$$P_{d,m}^{(n)} \approx P_{d,m}^{(2)}, \quad 1 \leq m \leq N_p, \quad 3 \leq n \leq \delta, \quad (8)$$

where $P_{fa,m}^{(1)}$, $P_{d,m}^{(1)}$, and $P_{d,m}^{(2)}$ are given in (26)-(29).

Proof: A proof is given in Appendix B.

Proposition III: Consider the Markov chain of Fig. 2. Let $T$ be the slot duration, $Q_{T_n,m}$ be the probability of successful transmissions of each SU at channel $m$ from node $T_n$, $RT_n$ be the remained time of the current slot, and $CR$ be the constant transmission rate of the SUs. Let $B_{T_n,m}$ be the probability that no SU cause interference on the channel $m$ at the stage $n$. Then, the average throughput of each SU is

$$r(\tau, p) = \frac{1}{T} \sum_{m=1}^{N_p} \sum_{n=1}^{\delta} Q_{T_n,m} RT_n CR. \quad (9)$$

The average interference time due to the each SU’s transmissions is

$$t_I(\tau, p) = \frac{1}{T N_p} \sum_{m=1}^{N_p} \sum_{n=1}^{\delta} \left(1 - B_{T_n,m}\right) RT_n. \quad (10)$$

Proof: A proof is given in Appendix C.

Now that we have derived the key performance indicators, we can turn our attention to the optimal selection of the parameters that maximizes the throughput.

IV. DISTRIBUTED CHANNEL SENSING OPTIMIZATION

In this section, first we investigate the optimal theoretical value that the sensing time and channel sensing probability should assume. Then, we present practical distributed algorithms of light computational requirements to achieve such an optimum with an adequate accuracy.

A. Theoretical Optimal Parameter Selection

As can be observed from the propositions established in the previous section, performance measures given by (9) and (10) depend on $\tau$ and $p$, where recall that $\tau$ is the channel sensing time and $p$ is the channel sensing probability. Hence, the performance of the CRN can be maximized by optimally choosing the values of $p$ and $\tau$ that maximize the average throughput, as a QoS metric for the SU, and bounding the interference time, as a QoS metric for the SUs as well as the SUs. That is,

$$\argmax_{\tau, p} r(\tau, p) \quad (11)$$

s.t. $t_I(\tau, p) \leq t_I^{\text{max}}, \quad (11.a)$

$p_{md,m}^{(n)} \leq p_{md}^{\text{max}}, \quad 1 \leq m \leq N_p, \quad 1 \leq n \leq \delta, \quad (11.b)$

$0 \leq \tau \leq T, \quad (11.c)$

$0 \leq p \leq 1, \quad (11.d)$

where $p_{md}^{\text{max}}$ represents the maximum tolerable value of the interference time, which depends on the QoS level guaranteed for the SUs as well as the SU. $p_{md}^{\text{max}}$ is the maximum tolerable miss detection probability imposed by the standard [33]. Constraint (11.a) guarantees a QoS level for both the PUs and SUs. Constraint (11.b) further provides QoS for just PUs, and (11.c) and (11.d) sets the admissible values for decision variables.

By direct inspection, we see that the optimization problem is generally non-convex, making it difficult to be efficiently solved. Such a complexity is exacerbated by that the parameter $\delta$ in the cost and constraint functions makes them not differentiable (see (2)). In order to find an approximate optimal solution to (11), we could search for a convex approximation of the problem [34]. However, this is difficult, because it is no obvious how to find a good convex approximation and, moreover, there is no guarantee on the distance between the optimal solution and the approximated one [34].

From another perspective, the dimension of decision variables is 2 ($\tau$ and $p$), regardless the size of the networks, i.e., number of PUs and SUs. Feasible solution(s) is in a bounded box due to (11.c) and (11.d). Altogether, running a centralized brute force optimization is more reasonable than finding an approximate sub-optimal solution. This is particularly motivated by that the availability of the optimal solution is herein interesting as a benchmark for an approximate solution provided by distributed algorithms. Indeed, one of the core contribution of this paper is to develop a distributed solution algorithm of low computational complexity that allows us to reach with an approximation of adequate accuracy the optimal solution of (11). Therefore, it is not essential to establish a centralized solution method of (11). However, note that
the results of the propositions and the formulation of the optimization problem are of paramount importance to establish the optimally of the distributed solution method, as we propose in the following.

Note that one assumption we adopted, as widely done in the literature [20]–[23], [26]–[28], is that the SUs use the same channel sensing time and probability. In real world scenarios, this may not be the case because every SU may experience different wireless channel conditions. To mitigate this issue, we develop novel fully adaptive and distributed algorithms to let the SUs follow the variation of the environment and keep the performance of the network at a near-optimal point. In the following, we characterize such a distributed algorithm that gives a near-optimal solution of (11), when (a) each SU is able to adaptively change its channels sensing time and probability in each sensing stage and (b) the parameters describing the channels (e.g., PUs’ traffic pattern and fading properties) change.

An interesting aspect of the cost and constraint functions that appear in (11) is that we can estimate (or calculate) them locally at each SU by taking local measurements. This allows developing a distributed solution for scenario of equal channel sensing and time probabilities and the general scenario of unequal channel sensing and time probabilities. In the general scenario, the SUs may use different sensing-access parameters, depending on their own preferences. In the general scenario, an optimization problem as (11) can be formulated. We do not characterize analytically the cost and constraints of this general case due the analytical intractability. However, inspired by problem (11), we develop a sub-optimal distributed solution for the general scenario, which can be applied also to the scenario of problem (11) as a particular case. We show by numerical simulations, that the algorithm works well in both scenarios.

**B. Adaptive Sequential Channel Sensing Algorithms**

As noted in [35], each SU is able to estimate the average throughput and interference time for a given \( \tau \) and \( p \). Also, \( P_{\text{ned},m} \) can be calculated (see Appendix B) by individual SU. Coherently, we can develop an algorithm, considering the impact of decision variables on the cost and constraint functions. The starting point of the algorithm that we develop is the following observation on the optimization decision variables \( p \) and \( \tau \): increasing \( p \) leads to higher demands for transmission (pros) and contention level in the network (cons). Reducing \( \tau \), from another perspective, increases transmission time (pros) at the expense of higher false alarm and miss detection probabilities (cons). Therefore, an SU decides for increasing/reducing \( p \) and \( \tau \) in each estimation period (\( N_{\text{ep}} \) consecutive slots defined as a frame) so that the average throughput increases while (11.a)–(11.d) are met. Otherwise, it adjusts decision variables in the reverse direction. We are now ready to present the details of the algorithm next.

Let \( p_m \) and \( \tau_m \) denote the channel sensing probability and sensing time of SU \( m \), and \( p_{\text{min}}, p_{\text{max}}, \tau_{\text{min}}, \) and \( \tau_{\text{max}} \) are their minimum and maximum values. Assume that NEWTHR and INT denote the estimated average throughput and interference in the current frame, and OLDTHR is the estimated average throughput in previous frame. We implement an additive-increase/additive-decrease (AIAD) policy for adjusting the sensing time and an additive-increase/multiplicative-decrease (AIMD) policy for adjusting the transmission rate through reducing \( p_{\text{min}} \) and \( p_{\text{max}} \) and also increasing \( \tau_{\text{max}} \). Otherwise, the SU increases \( p_{\text{min}} \) and \( p_{\text{max}} \) and reduces \( \tau_{\text{max}} \) in order to enhance the transmission rate. The adjusted values of \( \tau_m \) and \( p_m \) are then projected onto \([0, T]\) and \([0, 1]\) to met (11.c) and (11.d), respectively. Meanwhile, INT will be checked with \( \tau_{\text{max}} \), and the sign of this comparison determines whether the decision variables should be increased or decreased. Note that (11.e) would be met by properly selecting a minimum value for sensing time, which discussed later. The proposed distributed algorithm is summarized in Algorithm 1.

**Algorithm 1 Adaptive sequential channel sensing algorithm for SU \( m \)**

1: **Initialization:** Choose \( \tau_{\text{min}} \) and initial values for \( \tau_m \) and \( p_m \).
   - Set counter \( \leftarrow 0 \) and OLDTHR \( \leftarrow 0 \).
2: **for each slot do**
3:   **if** counter \( \leftarrow N_{\text{ep}} \) **then**
4:     **Calculate and then report NEWTHR and INT.**
5:     **if** NEWTHR \( > \) OLDTHR and INT \( < \tau_{\text{max}} \) **then**
6:       \( \tau_m \leftarrow \min \{ \tau_{\text{min}}, \tau_{\text{max}} - \Delta \tau \} \),
7:       \( p_m \leftarrow \min \{ 1, p_{\text{min}} + \Delta p \} \).
8:     **else**
9:       \( \tau_m \leftarrow \min \{ T, \tau_{\text{max}} + \Delta \tau \} \),
10:      \( p_m \leftarrow \alpha_m p_{\text{max}}, \quad 0 < \alpha_m < 1 \).
11:     **end if**
12:   **end if**
13: **end if**
14:  OLDTHR \( \leftarrow \) NEWTHR.
15:  counter \( \leftarrow \) counter + 1.
16: **end for**
allow the SUs to adjust the channel sensing probability as well as sensing time in each stage of a slot. In each slot, an SU increases its sensing probabilities from a minimum value \( p_{\text{min}} \) to a maximum value \( p_{\text{max}} \) to increase the chance of participating in sensing-access procedures. Similarly, sensing time will be decreased from a maximum value \( \tau_{\text{max}} \) to a minimum value \( \tau_{\text{min}} \) to increase the time left for the transmission. The SU \( m \) starts with \( p_{m}[1] = p_{\text{min}} \) and \( \tau_{m}[1] = \tau_{\text{max}} \). Then, it linearly increases (decreases) the channel sensing probability (sensing time) in every stage. Meanwhile, the estimation and decision processes are periodically performed in each frame. Algorithm 2 summarizes the proposed procedures. It is clear that the nonadaptive protocol is a special case of the adaptive one for \( p_{m}[n] = p \), \( \tau_{m}[n] = \tau \), and a constant scaling factor \( \alpha_{m} = 1 \) for all \( m \) and \( n \). Also, it reduces to Algorithm 1 by

\[
\tau_{m}[n] = \tau_{\text{max}}, \quad 1 \leq n \leq \delta, \\
p_{m}[n] = p_{\text{min}}, \quad 1 \leq n \leq \delta.
\]

In a nutshell, Algorithm 1 adjusts decision variables in each frame (coarse tuning), allowing the SUs to follow the variations of the environment, whereas further adjusting of the decision variables in each slot (fine tuning) enables Algorithm 2 to optimize the performance of the SUs in each frame as well.

In Algorithm 2 the initial values are set as follows. Roughly speaking, \( \Delta\tau_{1} \) and \( \Delta\tau_{2} \) should be small fractions of \( T \), since the sensing time of each SU should be finely tuned by the algorithms considering the fact that a slight variation in the sensing time can change the performance of the spectrum sensing and thereby the average throughput and interference time substantially. Estimation period \( N_{ep} \) follows a tradeoff between the estimation accuracy of the performance measures and agility of the algorithms. Larger value of \( N_{ep} \), for instance, provides a more accurate estimation of the average throughput, meanwhile makes the algorithms lazy. That is, an SU cannot follow the dynamic of the environment very fast and reach to the optimal point slowly. \( \Delta\tau_{1}, \Delta\tau_{2}, \) and \( \delta \), from another perspective, regulate the contention level in the networks, and their proper values depend heavily on the size of the primary and secondary networks, i.e., the number of PUs and SUs. Smaller value of \( \alpha \) makes some sudden reductions in the value of \( p \) and consequently transmission rate, leading to a faster response to the congestion at the expense of higher fluctuations in the average throughput. Higher values for \( \alpha \) decrease sensitivity of the algorithms to the congestion, and therefore though the average throughput experiences lower fluctuations, the optimal value of \( p \) might not be found (see [37], [38] for further details). Also, in order to find a proper value for \( \tau_{\text{min}} \), we should firstly recall that energy detector is used for spectrum sensing in the numerical results section. For such a spectrum sensing scheme, \( \tau_{\text{min}} \) exists so that false alarm and miss detection probabilities become lower than a certain threshold [36]. It holds

\[
\tau_{\text{min}} = \frac{1}{\gamma^{2}f_{s}}\left(\frac{Q^{-1}(P_{\text{fa}}^{\text{max}})}{Q^{-1}(P_{\text{d}}^{\text{min}})}\right)^{2},
\]

where \( f_{s} \) is the sampling frequency, \( \gamma \) is the SNR at the SU receiver. For simplicity of presentation, we use [12] for initializing the proposed algorithms, i.e., \( \tau_{\text{min}} = \tau_{\text{min}} \). Due to space limitation, a detailed discussion on the impact of the other parameters is left for the interested readers.

V. NUMERICAL RESULTS

In this section, we investigate the performance of the RSOP as well as the efficiencies of the proposed adaptive protocols by simulating a network of SUs performing sequential channel sensing.

A. Simulation Set-up

To set up a simulation environment, the values of \( P_{\text{fa}}^{\text{min}}, P_{\text{d}}^{\text{max}} \), time slot duration \( T \), and the value of sampling frequency used by the energy detector, are chosen according to IEEE 802.22 standard [33]. Table II summarizes the descriptions and values of the parameters considered for the simulations. Using a Monte Carlo simulation, the average throughput and the average interference time are computed after simulating the scenarios for \( 10^{4} \) times. To simulate the proposed algorithms, we use \( N_{ep} = 100, \tau_{\text{max}} = 0.1T, \Delta\tau_{1} = \Delta\tau_{2} = 0.01T, p_{\text{min}} = 0.5, p_{\text{max}} = 1, \Delta\tau_{1} = \Delta\tau_{2} = 0.025, \) and \( \alpha = 0.5 \). In fact, the utilized initial values are just an example to illustrate the effectiveness of the proposed algorithms, and one can easily investigate the impact of the aforementioned parameters on the performance of the algorithm such as convergence rate and throughput enhancement rate (see Section IV), and then tries to adopt optimal initial values. Using those parameters, the time behaviors of the algorithms are depicted in Fig. 4.

B. Effects of Simulation Parameters

Figs. 4 and 5 depict the average throughput of the secondary network and the normalized interference time, versus channel sensing probability \( p \) and normalized sensing time
C. False Alarm Paradox

After a false alarm, not only the SU misses a transmission opportunity, but also less time remains for possible transmission due to the time wasted for sensing the current channel. Therefore, false alarm reduces the average transmission rate (and consequently throughput) in traditional view. In a network with several uncoordinated users, however, increasing transmission rate of each individual user does not necessarily lead to higher average throughput. For the same reason, the average interference time between the SUs and PUs is reduced. Moreover, the well known sensing-throughput tradeoff is verified. That is, after an optimum point, wherein the false alarm and miss detection probabilities are in acceptable levels, the average throughput starts decreasing due to the reduction of the time left for the transmission.

D. Effects of Network Size

Fig. 7 shows the maximum throughput of the network with respect to the number of primary channels. Three points can be made from the figure. First, mathematical derivations coincide numerical simulations, which further verifies our theoretical analysis. Second, the maximum throughput raises with the number of primary channels in a saturating manner. This is due to that more channels are sensed, and therefore more transmission opportunities are found. Also, with extreme high number of primary channels, almost no collision happens among the SUs, and consequently a CRN with $N_s$ SUs can...
be modeled by $N_s$ distinct CRNs, each having one SU. For example, by looking at the maximum throughput for $N_p = 100$, both curves ($N_s = 2$ and $N_s = 5$) reach their saturating regions, and the maximum throughput when 5 SUs exist in the CRN is around 2.5 times of one achieved in the CRN with 2 SUs. Third, the proposed algorithms well mimics static optimal solution of (11). More interestingly, Algorithm 2 outperforms the static optimal throughput. The main reason is that all the SUs adopt similar values sensing time and also sensing probabilities, whereas Algorithm 2 enables the SUs to adaptively adjust their sensing-access parameters in each sensing stage. In fact, we have more degrees of freedom compared to static optimal design.

Fig. 8 investigates the impact of secondary network size on the performance. The maximum throughput increases by the number of the SUs, but the contention level is raised as well. Therefore, with more SUs in the network, each of them adopts lower value for $p$ to avoid collision with other SUs. This adjustment leads to less demand for sensing the channels and a high interest in being slept, which in turn results in wasting the transmission time. Clearly, this problem can be mitigated by increasing the number of primary channels, as can be observed in the figure. Again, the proposed algorithms perform well. To more elaborate, Table III demonstrates the performance enhancement due to optimal $p$ and $\tau$ derived in (11), and compares the average throughput and interference for three different scenarios: (1) static optimal values, which are obtained by a brute force numerical optimization search and (2) adaptive values as achieved by the proposed Algorithm 1 (3) adaptive values as achieved by the proposed Algorithm 2. As expected, adopting the static optimal and adaptive values for $p$ and $\tau$ increases the average throughput while the interference meets the constraint. Specifically, for the case $N_s = 3, N_p = 7$, the average throughput of the SUs achieved by the static optimal design respectively is about 24% and 2.1% more than the ones achieved in $p = 0.8, \tau = 0.1T$ (see Fig. 5) and adaptive algorithm. Also, the average interference does not violate $t_{max}$.

Fig. 6. Impact of false alarm on the average throughput ($p = 0.8$).

Fig. 7. Average throughput against number of primary users. In the figure, A stands for analysis and -S stands for simulations.

Fig. 8. Average throughput against number of secondary users.

### Table III

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Ns = 3, Np = 7</th>
<th>Optimal</th>
<th>Algorithm 1</th>
<th>Algorithm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thr.</td>
<td>1.441</td>
<td>0.047</td>
<td>1.411</td>
<td>0.042</td>
</tr>
<tr>
<td>Int.</td>
<td>1.920</td>
<td>0.050</td>
<td>1.908</td>
<td>0.049</td>
</tr>
<tr>
<td>Ns = 5, Np = 5</td>
<td>1.371</td>
<td>0.050</td>
<td>1.340</td>
<td>0.047</td>
</tr>
<tr>
<td>Ns = 7, Np = 5</td>
<td>1.689</td>
<td>0.043</td>
<td>1.677</td>
<td>0.048</td>
</tr>
</tbody>
</table>

E. Energy Efficiency of Modified p-persistent Protocol

The advantages of our p-persistent protocol compared to the conventional one is illustrated in Table IV. In the table, sensing overhead represents the average number of channels sensed by the SUs in a time slot. Considering a fixed energy consumption for sensing each channel (10), the sensing overhead can be easily converted to energy consumption. As stated, the SUs always perform spectrum sensing in each stage, if conventional p-persistent random access algorithm is utilized. From the table, although both schemes can support multiple accesses among the SUs, but the modified scheme considered in this paper achieves the same average throughputs with considerable less consumed energies. For $N_s = 7, N_p = 5$, as an example, conventional protocol achieves 0.017% higher throughput at the expense of 17.47% more energy consumption compared to the modified protocol.
TABLE IV
THE PERFORMANCE COMPARISON OF THE CONVENTIONAL AND MODIFIED P-PERSISTENT SCHEMES.

<table>
<thead>
<tr>
<th></th>
<th>Average throughput</th>
<th>Sensing overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conventional</td>
<td>Modified</td>
</tr>
<tr>
<td>( N_s = 3, N_p = 7 )</td>
<td>1.6742</td>
<td>1.6746</td>
</tr>
<tr>
<td>( N_s = 5, N_p = 7 )</td>
<td>1.9349</td>
<td>1.9350</td>
</tr>
<tr>
<td>( N_s = 7, N_p = 3 )</td>
<td>1.2986</td>
<td>1.2981</td>
</tr>
<tr>
<td>( N_s = 7, N_p = 5 )</td>
<td>1.7655</td>
<td>1.7652</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

Modeling and performance evaluation of random sensing order policy (RSOP) in a distributed cognitive radio network (CRN) were investigated in this paper. The behaviors of the secondary users (SUs) were modeled through a novel Markov process. The performance of the RSOP in terms of the average throughput of the CRN and average interference levels in the network was evaluated. Then, an optimization problem was formulated to maximize the average throughput while the interference level is kept bounded. Finally, to enhance the RSOP performance, two simple but efficient algorithms were proposed to adaptively adjust the sensing-access parameters. The algorithms enhance the performance of the CRN without high computational burden, as demonstrated through exhaustive numerical performance evaluations.

APPENDIX A

Let \( P_{m,1} \) denote the presence probability of the PU \( m \). Also let \( P_{m}^{(n)} \) be the occupation probability of the \( m \)-th channel at the beginning of the \( n \)-th stage. At the beginning of the first stage, the SUs have not sensed any channels yet, and therefore the occupation probability of each channel is equal to the corresponding PU’s presence probability. Thus, we have

\[
P_{m,1}^{(1)} = P_{m,1}, \quad 1 \leq m \leq N_p.
\]

Let \( N_s \) be the number of the SUs that have requests at the node \( x \). So, from Fig. 2 we have, \( \mathcal{N}_{HO_1} = N_s \). The average number of the SUs that sense the \( m \)-th channel at the first stage, represented by \( \mathcal{L}^{(1)} \), can be computed as

\[
\mathcal{L}^{(1)} = \mathcal{N}_{m}^{(1)} = \frac{p}{N_p} \mathcal{N}_{HO_1} = \frac{N_s}{N_p}.
\]

Each channel \( m \) is sensed by \( \mathcal{L}^{(1)} \) SUs at the first stage. Each of these SUs might sense the corresponding channel free. In this case, the user starts its transmission on the channel, and therefore contributes to this channel’s occupation probability. The probability of transmission on the \( m \)-th channel by at least one SU conditioned on the absence of the PU is

\[
U_{m}^{(1)} = 1 - \left( \frac{P_{m}}{P_{fa,m}} \right)^{\mathcal{L}^{(1)}}.
\]

\( U_{m}^{(n)} \) is the probability that at least one SU transmits on the \( m \)-th channel (or equivalently one SU senses the channel free) at the end of the \( n \)-th stage conditioned on the absence of the PU. Considering (13), (14), and (15), we have

\[
P_{m,1}^{(2)} = P_{m,1}^{(1)} + P_{m,0} U_{m}^{(1)},
\]

and

\[
\mathcal{N}_{HO_2} = (1 - p) \mathcal{N}_{HO_1} + \sum_{m=1}^{N_p} q_m^{(1)} \mathcal{N}_{m}^{(1)}
\]

\[
= \left( 1 - p \right) + \frac{p}{N_p} \sum_{m=1}^{N_p} q_m^{(1)} \mathcal{N}_{HO_1},
\]

where \( P_{m,0} = 1 - P_{m,1} \).

In Appendix B, it is proved that \( P_{fa,m}^{(n)} = P_{fa,m}^{(1)} \) for \( 1 \leq m \leq N_p \) and \( 1 \leq n \leq \delta \). At the second stage, the number of SUs whose requests enter the node HO2 is calculated in (17), where \( q_m^{(n)} \) is defined in (5). We have

\[
U_{m}^{(2)} = 1 - \left( \frac{P_{fa,m}}{P_{fa,m}} \right)^{L^{(2)}} = 1 - \left( \frac{P_{fa,m}}{P_{fa,m}} \right)^{L^{(2)}},
\]

where \( L^{(2)} = (p/N_p) \mathcal{N}_{HO_2} \). Therefore, the \( m \)-th channel occupation probability at the beginning of the third stage can be computed as

\[
P_{m,1}^{(3)} = P_{m,1} + P_{m,0} U_{m}^{(1)} + P_{m,0} P_{fa,m} U_{m}^{(2)} = P_{m,1}^{(2)} + P_{m,0} P_{fa,m} U_{m}^{(2)},
\]

Following the same steps, at the \( n \)-th stage we have,

\[
P_{m,1}^{(n)} = P_{m,1}^{(n-1)} + P_{m,0} \left( \frac{P_{fa,m}}{P_{fa,m}} \right)^{\mathcal{L}^{(n)}} U_{m}^{(n-1)}
\]

where

\[
U_{m}^{(n)} = 1 - \left( \frac{P_{fa,m}}{P_{fa,m}} \right)^{\mathcal{L}^{(n)}}
\]

\[
\mathcal{L}^{(n)} = \frac{p}{N_p} \mathcal{N}_{HO_n},
\]

and

\[
\mathcal{N}_{HO_n} = \left[ 1 - p + \frac{p}{N_p} \sum_{m=1}^{N_p} q_m^{(n-1)} \right] \mathcal{N}_{HO_{n-1}}.
\]

APPENDIX B

The state of channel \( k \), which is dedicated to \( k \)-th PU, is represented by

\[
s_k(t) = \begin{cases} 1 : & \mathcal{H}_1 \\ 0 : & \mathcal{H}_0 \end{cases}
\]

where \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) respectively denote the occupancy and idleness hypotheses of the channel \( k \). Recall that each channel may be occupied by the corresponding PU or land some SUs. Here, for simplicity of presentation, we only formulate false alarm and miss detection probabilities for an AWGN channel. The formulations, however, can easily be extended to consider more realistic channels [40]. For an AWGN channel, the spectrum sensing process is modeled as a binary hypotheses testing problem [36]

\[
\{ \mathcal{H}_0 : y(k) = z(k) \}
\]

\[
\{ \mathcal{H}_1 : y(k) = u_m(k) + z(k) \}
\]
where $z(k)$ is $k$-th sample of zero mean complex-valued Gaussian noise with independent and identical distribution (i.i.d). $u_m(k)$ and $y(k)$ denote the $k$-th sample of the accumulated signal (exlude noise) that presents in the channel $m$, which is independent of $z(k)$, and the $k$-th sample of the received signal.

There are various spectrum sensing proposals [39]. Among them, we derive the formulations for energy detector; because it is the most prevalent spectrum sensing scheme in the literature. Also, it is the optimal detector for unknown signals [39]. To decide about occupation status of a channel, in the energy detector scheme, energy of a received signal is accumulated during a sensing time $\tau$, and then it is compared to a threshold $\lambda$. Let $w = \tau f_s$ represent the number of samples taken from the received signal, where $f_s$ is the sampling frequency. Defining $X$ as the accumulated energy of $w$ consecutive samples, the decision criteria is defined as

$$X = \sum_{k=1}^{w} |y(k)|^2 = \begin{cases} < \lambda : & H_0 \\ \geq \lambda : & H_1 \end{cases}.$$ (25)

Considering a gaussian distribution for $X$ (which is meaningful for large $w$ [36]), we have [41]

$$P_{fa,m}^{(n)} = Q\left(\left(\frac{\lambda}{\sigma_z^2} - 1\right) \sqrt{\frac{\tau f_s}{w}}\right),$$ (26)

and

$$P_{md,m}^{(n)} = 1 - Q\left(\left(\frac{\lambda}{\sigma_z^2} - 1 - \frac{\gamma_m^{(n)}}{\gamma_m^{(n)}}\right) \sqrt{\frac{\tau f_s}{w}}\right),$$ (27)

where $P_{fa,m}^{(n)}$ and $P_{md,m}^{(n)}$ respectively are the false alarm and miss detection probabilities when an SU senses the channel $m$ at stage $n$. $\sigma_z^2$ is the noise variance, and $\gamma_m^{(n)}$ is the received signal to the noise ratio of the channel $m$ at the stage $n$. Suppose that $\sigma_m^{(2)} = \sigma_s^{(2)}$ is the power of the PU $m$ at the secondary receiver. Let $\sigma_m^{(2)}$ be the power of each SU. At the beginning of each time slot, the occupancy status of the channels only depends the PUs activities. Hence,

$$\gamma_m^{(1)} = \frac{\sigma_m^{(2)}}{\sigma_z^2}. \tag{28}$$

Also, considering the definition of $L^{(1)}$, introduced in [14], $pN_s/N_p \left(1 - q_m^{(1)}\right)$ SUs transmit on the channel $m$ at the first stage. Therefore, the remained SUs take samples from a received signal with an SNR

$$\gamma_m^{(2)} = \frac{P_{m,1}\sigma_m^{(2)} + N_s p/N_p \left(1 - q_m^{(1)}\right) \sigma_s^{(2)}}{\sigma_z^2}, \tag{29}$$

when they intend to sense the channel $m$ at the second stage. $P_{m,1}$ is as defined in [13].

A false alarm occurs when a free channel is mistakenly sensed busy. Consequently, there are no PU or SUs signals on the channel when a false alarm happens. Therefore, the possible changes in the level of the signals in successive stages do not affect the false alarm probability, as can be concluded from (26). For each channel $m$ and each stage $n$, $P_{fa,m}^{(n)} = P_{fa,m}^{(1)}$ for $1 \leq m \leq N_p$. On the other hand, the miss detection probability directly relates to the received SNR and may change in different stages, depending on the number of SUs transmit on the corresponding channel. $P_{d,m}^{(1)}$ and $P_{d,m}^{(2)}$ can be computed by substituting (28) and (29) into (27).

The increment of the detection probability has a saturating pattern regarding the SNR value [42]. That is, a small enhancement in the SNR manifests itself as a significant enhancement in the detection performance, and then further increment of the SNR value does not improve the detection performance substantially. From (29), the SNR of the channel $m$ is significantly increased in the second stage in the average sense, leading to a meaningful improvement in the detection probability. However, we expect that the detection performance will not be dramatically enhanced in the next sensing stages due to the aforementioned saturating pattern. To more illustrate this phenomenon, in the following we consider a numerical example. Table V shows the detection probability for consecutive sensing stages, assuming $T = 10$ ms, $\tau = 0.1T$, $\tau_n = 0.1\mu$s, $\sigma_p^{(2)} = \sigma_s^{(2)}$, $N_s = 20$, $N_p = 10$, and $p = 0.8$. The simulation setup procedure is described in Section V. From the table, the detection probability is not substantially improved after the second sensing stage. Clearly, the enhancement ratio highly depends on the number of SUs and PUs, and also the channel sensing probability $p$. Although the SNR in the second stage is higher than the SNR of the first stage regardless of the exact values of the above parameters, the small enhancement in the SNR can drive the detection probability to its saturation region. Here, we assume that

$$P_{md,m}^{(n)} \approx P_{md,m}^{(2)} \quad 3 \leq n \leq \delta. \tag{30}$$

APPENDIX C

Let $P_x$ denote the steady state probability of being at the state $x$. From Fig. 2 we have

$$
\Pi_{HO_n} = \left(1 - p\right) + \frac{p}{N_p} \sum_{m=1}^{N_p} q_m^{(n-1)} \Pi_{HO_{n-1}}, \tag{30}
$$

$$
\Pi_{m^{(n)}} = \frac{p}{N_p} \Pi_{HO_{n-1}}, \tag{31}
$$

$$
\Pi_{I_{n}} = \sum_{m=1}^{N_p} P_{m,1}^{(n)} \left(1 - P_{d,m}^{(n)}\right) \Pi_{m^{(n)}}, \tag{32}
$$

and

$$
\Pi_{T_{n}} = \sum_{m=1}^{N_p} P_{m,0}^{(n)} \left(1 - P_{fa,m}^{(n)}\right) \Pi_{m^{(n)}}. \tag{33}
$$

Note that considering the channel search and access policy described in Section III, the procedure always initiates from the node HO, and thus

$$\Pi_{HO_1} = 1. \tag{34}$$
Then, from Eqs. 30–33, $\Pi_{I_n}$ and $\Pi_{T_n}$ are calculated. Let $P_{T_{n,m}(n)}^{[k]}$ and $P_{I_{n,m}(n)}^{[k]}$ respectively denote the probability of the transmissions from the nodes $T_n$ and $I_n$ on the channel $m$, i.e., the state changes from the node $m(n)$ to the nodes $T_n$ and $I_n$:

$$P_{T_{n,m}(n)}^{[k]} = \Pi_{T_{n,m}(n)}^{[k]} P_{m,0}^{[n]} \left(1 - P_{I_{n,m}(n)}^{[n]}\right)$$

$$P_{I_{n,m}(n)}^{[k]} = \Pi_{I_{n,m}(n)}^{[k]} P_{m,1}^{[n]} \left(1 - P_{d_{n,m}}^{[n]}\right).$$

The $k$-th SU will successfully transmit data on each channel $m$ at the stage $n$ (with probability $Q_{T_{n,m}}^{[k]}$) provided that its state transits from nodes $m(n)$ to $T_n$ for $1 \leq n \leq \delta$ (with probability $P_{T_{n,m}(n)}^{[k]}$), and all other SUs do not collide its communications. Assume that $T_{n,m}^{[k]}$ represents the probability of the SU $\ell$ does not transmit on the channel $m$ in a time slot. Hence, the $k$-th SU successfully transmits data on the channel $m$ at the stage $n$ with probability

$$Q_{T_{n,m}}^{[k]} = \prod_{\ell \neq k} P_{T_{n,m}(n)}^{[\ell]} A_{m}^{[\ell]}. \quad (37)$$

If we omit the superscript $[k]$, $Q_{T_{n,m}}^{[k]}$ is simplified to $Q_{T_{n,m}} = P_{T_{n,m}(n)} A_{m}^{n-1}$. At the stage, the SU transmits for $R_{T_n}$ time units with the constant rate of $C_R$. The average throughput of each SU follows as

$$r = \frac{1}{T} \sum_{m} \sum_{n=1}^{\delta} Q_{T_{n,m}} A_{n}^{n-1} R_{T_n} C_R. \quad (38)$$

Finally, we need to formulate $A_{m}$. To this end, the proposed Markov model is modified so that the SU would not be able to send on the channel $m$ in any stages. Fig. 9 depicts the $n$-th stage of the pruned Markov model. As can be observed, if the SU enters the stage $m(n)$, it will be immediately routed to the next handoff state. Using this figure and following the same steps taken in the Appendices A and B, the steady states probabilities of being at $T_n$, $I_n$, or eventually TE is obtained. Then, we have

$$A_{m} = \Pi_{\text{TE}} + \sum_{n=1}^{\delta} \Pi_{T_n} + \Pi_{I_n}. \quad (39)$$

To find the average interference time, $t_I$, note that each SU encounters

$$t_I^{[k]} = \frac{1}{T N_p} \sum_{n=1}^{\delta} \Pi_{I_{n,m}} R_{T_n}$$

level of interference in each time slot. But, these random variables are not independent; because SU $k_2$ can transmit on the occupied channel $m$, where it has been mistakenly interfered by SU $k_1$ in the previous sensing stages. Therefore,

$$\frac{1}{T N_p} \sum_{k=1}^{N_s} \sum_{n=1}^{\delta} \Pi_{I_{n,m}} R_{T_n}$$

is an upper bound of the interference time of the network. Let $B_{I_{n,m}}$ be the probability no SUs cause interference on the channel $m$ at the stage $n$. From (39), we have

$$B_{I_{n,m}} = \prod_{k=1}^{N_s} \left(1 - P_{I_{n,m}(n)}^{[k]}\right) = \left(1 - P_{I_{n,m}(n)}\right)^{N_s}. \quad (40)$$

Hence, the average interference time of the network is:

$$t_I = \frac{1}{T N_p} \sum_{m=1}^{N_s} \sum_{n=1}^{\delta} \left(1 - B_{I_{n,m}}\right) R_{T_n}.$$

**References**


