BER-Based Adaptation in Composite Fading Channel

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Abstract—Bit error rate (BER)-based constrained optimization of average spectral efficiency for adaptive Multi-level Quadrature Amplitude Modulation (MQAM) with service rate and power constraint is studied under composite Gamma-Lognormal fading channel condition. We obtain the corresponding optimal schemes for power and rate adaptation for continuous and discrete rate MQAM. The optimal power for continuous rate QAM is shown to be a combination of two functions; one of which with water-filling nature and the other proportional to $\log(1/\text{BER})$, whereas for discrete rate QAM the solution is proportional to $\log(1/\text{BER})$ over each rate region. We have also shown that transmission rate is a monotonically increasing function of BER.

Keywords—Based, MQAM; power adaptation; rate adaptation; composite fading channel

I. INTRODUCTION

Dynamic power and rate allocation through adaptive modulation is known as a powerful means to gain high spectral efficiencies over time-varying channels. A common objective in optimizing the transmit power and rate in fading environments is to maximize the average number of bits per symbol that can be reliably sent through the channel. It is shown [1] that the ergodic capacity of fading channels can be achieved by variable rate and power transmission. According to [2], the Shannon capacity of a fading channel is also achieved by just varying the transmit power. The optimum power allocation strategy corresponding to the ergodic channel capacity is shown to be water-filling [3]. Likewise, in [4]-[5] the optimal power adaptation corresponding to maximum spectral efficiency for adaptive continuous QAM subject to a power constraint is obtained to be water-filling in power.

In [6] the problem of maximizing ergodic capacity subject to a given outage probability and an average power constraint is addressed for applications that require a minimum guaranteed quality of service. For a given basic rate accommodated over $(1-\varepsilon)$ percent of channel states the optimum power allocation is shown to be a combination of water-filling and channel inversion.

The problem of finding the optimum power allocation for the class of fading channels with slow and fast time scales is considered in [7], where the standard block flat fading model [2] with additive white Gaussian noise (AWGN) is used for the slow varying part of the channel gain, whereas the fast fading is considered within each block. In [7] the optimum power allocation is obtained for maximizing the expected capacity over the slow fading with two constraints of minimum average service rate over a block and maximum long-term average power. It is shown there that a combination of channel inversion and the so-called soft water-filling is the solution to this optimization problem.

To acquire the channel state at the transmitter, all the above methods rely on the channel estimation at the receiver and the feedback to provide the channel information to the transmitter. The effect of imperfect channel estimation on the performance of MQAM BER performance and on variable rate and power schemes has been considered in [5], [8] and [9]. In [10] the problem of maximizing the spectral efficiency of MQAM over flat fading channels subject to average power constraint and instantaneous BER is addressed under imperfect channel estimation. In a different approach to adaptation under imperfect channel estimation, [11] considers BER-based MQAM in which the rate and SNR-target vary with BER.

In this paper we consider a class of optimization problems for rate and power allocation in a wireless communication system where adaptive QAM is used for rate adaptation. Invoking the fact that BER is a function of both the channel gain and the estimate thereof, we will present the optimization of service outage spectral efficiency of MQAM over a composite channel with adaptation based on BER. In order to model the slow and fast fading channel, the composite channel considered for BER-based adaptation is Gamma-lognormal distributed. The optimal power and rate allocation problem will also be solved for discrete rate MQAM. In summary, the contributions of this work are:

- Power and rate adaptations are based on BER (unlike SNR-based adaptation schemes in [1]-[2], [4]-[7] and other similar works).
- Minimum service rate constraint in addition to average power constraint is incorporated in the average spectral efficiency maximization problem.
- The optimum BER boundaries are found for the case of discrete rate adaptation.
- A closed form expression for spectral efficiency in composite fading channel with Gamma-lognormal-distributed-SNR is provided, considering the service rate constraint.

The paper is organised as follows: in section II we solve the BER-based continuous and discrete rate MQAM optimization problems. The numerical results for each of the above optimization problems are given in section III, and the conclusion will be drawn in section IV.

II. ADAPTATION TO BER

In this section, we find, the optimum variable power and rate transmission using adaptive Multi-level Quadrature...
Amplitude Modulation (MQAM). Employing MQAM the transmitter sends its data at the rate of \( R \approx \log_2(M)/T \), where \( T \) is the symbol time duration and \( M \) is the constellation size [1]. Let \( B \) denote the received signal bandwidth and assume the Nyquist pulses \( (B = 1/T) \) are employed to send the data. In this paper we allow \( M \) to take on any real value larger than 2. Practical restrictions on \( M \) will be considered in the following section.

Here we relax the assumption that the channel gain is known perfectly to the receiver or the transmitter. One practical approach in such cases is to adapt the transmission power and rate based on an imperfect channel estimate \( \hat{\gamma} \). An alternative method is to perform adaptation based upon the BER, bearing in mind that instantaneous BER is in general a function of both \( \gamma \) and \( \hat{\gamma} \) [2]. We assume that the transmitter has the deterministic knowledge of BER through an ideal feedback from the receiver as well as the BER probability distribution function \( \Phi(\text{BER}) \). Furthermore the transmitter is assumed to have knowledge of the probability density function (pdf) of the received SNR. We will solve this problem for continuous and discrete rate adaptation considering the transmit power as a function of BER.

Assuming that the transmitter and the receiver have knowledge of BER but transmitter has no deterministic knowledge of \( \gamma \), invoking [4, eq. 21], we can define the BER-based Conditional Spectral Efficiency (CSE) as

\[
\mathcal{K}_b(S_{\text{BER}}, \text{BER}) = \mathbb{E}_r \left\{ \log_2 \left( 1 + \frac{\kappa_{\text{BER}} \gamma S_{\text{BER}}}{\bar{S}} \right) \right\}
\]

where \( \kappa_{\text{BER}} = c_2/\ln(1/c_1_{\text{BER}}) \) and \( \mathbb{E}_r \{ \cdot \} \) denotes expectation with respect to \( \gamma \). \( S_{\text{BER}} \) is the transmit power with average \( \bar{S} = E_{\text{BER}}(S_{\text{BER}}) \), and \( c_1 \) and \( c_2 \) are positive real constants. We will use \( \mathcal{K}_b \) instead of \( \mathcal{K}_b(S_{\text{BER}}, \text{BER}) \) when the context is clear.

Defining Average Spectral Efficiency (ASE) as \( \mathbb{K}_e = \mathbb{E}_{\text{BER}} \{ \mathcal{K}_b \} \) we introduce the optimization problem here as

Maximize \( \mathbb{K}_e = \mathbb{E}_{\text{BER}} \{ \mathcal{K}_b \} \), subject to

\[ E_{\text{BER}} \{ S_{\text{BER}} \} \leq \bar{S} \quad (a) \]
\[ S_{\text{BER}} > 0 \quad (b) \]
\[ \Pr \{ \mathcal{K}_b < \mathcal{K}_0 \} \leq \epsilon \quad (c) \]

\( \mathcal{K}_0 \) denotes the basic service rate; constraint (c) implies that the CSE is guaranteed to be greater than a given basic service rate of \( \mathcal{K}_0 \), with the probability of \( 1 - \epsilon \).

Gamma-Lognormal distribution allows for modelling of composition of slow and fast fadings [14], [16]. However it is shown in [14] that the Gamma-lognormal distribution can be approximated by the Lognormal distribution. Assuming that the SNR has lognormal distribution with parameters \( \mu \) and \( \sigma \) as

\[
f(\gamma) = \frac{e^{-(\ln\gamma - \mu)^2/2\sigma^2}}{\gamma \sigma \sqrt{2\pi}}, \quad (2)\]

\( \mathcal{K}_b \) can be approximated as

\[
\mathcal{K}_b = \frac{\Gamma(\alpha x/2) \sum_{i=1}^{[\alpha]} \Gamma(1 - \alpha x/2)}{A \Gamma(\alpha x/2)}, \quad (3)\]

where \( A = e^{\mu^2/2\sigma^2} \ln 2 \sqrt{2\pi}, \quad \sigma = \sigma^{-2}, \quad x = (S_{\text{BER}} \kappa_{\text{BER}})^{-1} \), and \( \Gamma(\cdot) \) is the gamma function. Operator \([\cdot]\) rounds to the nearest larger integer value.

### A. BER-B Based Continuous Rate and Power Adaptation

Prior to solving the optimization problem, we solve the following sub-problem, omitting the constraint (c),

Maximize \( \mathbb{K}_e = \mathbb{E} \{ \mathcal{K}_b \} \), subject to

\[ E_{\text{BER}} \{ S_{\text{BER}} \} \leq \bar{S} \]
\[ S_{\text{BER}} \geq 0. \]

To solve this sub-problem based on the usual Lagrangian-based constraint optimization, we introduce the following lemma, whose proof is given in the appendix.

**Lemma:** The above sub-problem reduces to the solution of the following fixed-point equation

\[
\frac{S_{\text{BER}} \lambda}{\bar{S}} = \frac{1}{A \lambda} \left( x^\alpha e^{\alpha x/2} \Gamma(\alpha + 1) \Gamma(-\alpha x/2) \right) \quad (4)
\]

which has a unique positive answer for every \( \kappa_{\text{BER}} > A(\alpha/2)^{\alpha} \lambda \Gamma(\alpha + 1) \), and \( \lambda \), the Lagrangian multiplier, can be found from \( E_{\text{BER}} \{ S_{\text{BER}} \lambda \} = \bar{S} \).

We will denote the solution of the above sub-problem as \( S_{\text{BER}} \lambda \) in the rest of this paper.

In order to add the probabilistic constraint (c) to the sub-problem, we introduce the sets of good and bad BERs as \( \mathcal{B}_e = \{ \text{BER} : \text{BER} \geq \text{BER}_e \} \) and \( \bar{B}_e = \{ \text{BER} : \text{BER} < \text{BER}_e \} \) respectively, where \( \text{BER}_e \) is the solution of \( \Phi(\text{BER}_e) = \epsilon \), and
\( \Phi(.) \) denotes the cumulative distribution function of BER. Also by setting \( x_0 \) as the solution of \( \mathbb{K}_b = \mathcal{A}_0 \) in (3), it can be seen that while \( \mathbb{K}_b \) is a monotonically decreasing function of \( x \) (see appendix), the solution of the optimization problem, must satisfy the condition \( S(\text{BER}) \geq \frac{1}{\mathcal{K}(\text{BER})x_0} \) in order to guarantee that \( \mathbb{K}_b \geq \mathcal{A}_0 \). Equivalently we should have

\[
\bar{S} \geq \int_{\mathbb{B}_e} \frac{1}{\mathcal{K}(\text{BER})x_0} \Phi(\text{BER})d\text{BER} \triangleq S_{\min}(\mathcal{A}_0, \mathcal{B}_e). \tag{5}
\]

Extending the results of [7], it is straightforward to show that the optimum power allocation \( S(\text{BER}) \) corresponding to the optimization problem, is given by

\[
\frac{S(\text{BER})}{\bar{S}} = \begin{cases} 
\frac{1}{\mathcal{K}(\text{BER})x_0} & \text{BER} \in \mathcal{B}_e \\
0 & \text{otherwise}.
\end{cases} \tag{6}
\]

For \( \bar{S} = S_{\min}(\mathcal{A}_0, \mathcal{B}_e) \); and when \( \bar{S} > S_{\min}(\mathcal{A}_0, \mathcal{B}_e) \), the optimum power allocation policy is given by

\[
\frac{S(\text{BER})}{\bar{S}} = \begin{cases} 
\frac{1}{\mathcal{K}(\text{BER})x_0} & \text{BER}_e \leq \text{BER} \leq \max\left\{\text{BER}_e, \text{BER}^*\right\} \\
S_{\langle \text{BER}, \mathcal{A}_0 \rangle} & \text{otherwise}
\end{cases} \tag{7}
\]

where \( \text{BER}^* \) is the solution of

\[
\frac{1}{\mathcal{K}(\text{BER}^*)x_0} = S_{\langle \text{BER}^*, \mathcal{A}_0 \rangle}, \tag{8}
\]

and \( \mathcal{A}_0 \) is found from average power constraint (a).

\section*{B. Adaptation Based on BER in Discrete Rate Condition}

Here we obtain the optimal power and rate adaptation based on BER, assuming that data rate can be chosen from a fixed set of values \( k_i, i = 0, 1, \cdots, N-1 \). That is, the fixed rate \( k_i \) is transmitted over the error probability range of \( [b_i, b_{i+1}) \), where \( b_i, i = 0, 1, \cdots, N-1 \) denote the BER boundaries. This simplifies the \( \mathbb{K}_b \) of the previous section to \( k_i, i = 0, 1, \cdots, N-1 \), which we refer it as block-rate in this paper.

The objective in this section is to maximize the ASE given an average power and service block-rate constraint, \( \Pr(k_i < \mathcal{A}_0) \leq \varepsilon \). We find the optimal BER boundaries, \( b_i \), that divide the range of bit error rate \( \text{BER} \) into \( N \) regions (or blocks).

We assume that the transmitter has perfect knowledge of BER through an ideal feedback from the receiver. Furthermore the transmitter is assumed to have statistical knowledge of the received SNR. Invoking [3, eq. 44], we derive the transmission power as a function of BER using

\[
\frac{S(\text{BER}, \gamma)}{\bar{S}} = \left(\frac{2^{k_i} - 1}{\mathcal{K}(\text{BER})}\right), \tag{9}
\]

and based upon this assumption we define the power adaptation function based on BER as the expectation of power with respect to SNR

\[
\frac{S(\text{BER})}{\bar{S}} \triangleq \mathbb{E}_\rho\left\{S(\text{BER}, \gamma) \mid \text{BER}\right\} = \left(\frac{2^{k_i} - 1}{\mathcal{K}(\text{BER})}\right)\mathbb{E}(1/\gamma). \tag{10}
\]

In order to find the optimum BER boundaries, the Lagrangian, \( J(b_0, \cdots, b_{N-1}) \), can be written as

\[
J(b_0, \cdots, b_{N-1}) = \sum_{i=0}^{N-1} \left[ b_i \int_{b_i}^{b_{i+1}} \Phi(\text{BER})d\text{BER} \right] + \lambda \left[ \sum_{i=0}^{N-1} \int_{b_i}^{b_{i+1}} \frac{S(\text{BER})}{\bar{S}} \Phi(\text{BER})d\text{BER} - 1 \right] \tag{11}
\]

where the optimal regions \( b_i \) are found by solving the following set of equations

\[
\frac{\partial J}{\partial b_i} = 0, \quad i = 0, \cdots, N-1. \tag{12}
\]

Solving the set of equations (12) we obtain

\[
b_i = c_2 \exp \left\{ \chi \left( z(k_i) - z(k_{i-1}) \right) \right\}, \quad i = 1, \cdots, N-1, \tag{13}
\]

where \( \chi \) is found from the average power constraint, and \( z(k_i) = (2^{k_i} - 1)\mathbb{E}(1/\gamma) \). Similar to the previous subsection, defining \( S_{\min,d} \) as

\[
S_{\min,d}(\mathcal{A}_0, \mathcal{B}_e) \triangleq \int_{b_i}^{b_{i+1}} \frac{z(\mathcal{A}_0)}{\mathcal{K}(\text{BER})} \Phi(\text{BER})d\text{BER}, \tag{14}
\]

the necessary condition to satisfy the service block-rate constraint is \( \bar{S} \geq S_{\min,d}(\mathcal{A}_0, \mathcal{B}_e) \).

Assuming that the \( i^{th} \) BER region is the first region that \( \mathcal{A}_0 < k_i \), it is straightforward to show that, for \( \bar{S} > S_{\min,d}(\mathcal{A}_0, \mathcal{B}_e) \), the optimal power allocation with discrete rate adaptation, and with the basic service block-rate of \( \mathcal{A}_0 \), when \( b_0 \leq b_i < \text{BER}_e < b_{i+1}, \quad i \in \{0, \cdots, N-1\} \) is

\[
\frac{S(\text{BER})}{\bar{S}} \triangleq \begin{cases} 
\frac{z(k_i)}{\mathcal{K}(\text{BER})} & k_i \geq \mathcal{A}_0 \\
S_{0}(\text{BER}) & k_i < \mathcal{A}_0
\end{cases} \tag{15}
\]

where \( S_{0}(\text{BER}) \) is
respectively. Setting the service block-rate to $b_i$ and $b_j$ with $b_j < B_i$, the power is
\[
\frac{S(\text{BER})}{S} = \begin{cases} 
 z(k_i)/\kappa(\text{BER}) & \text{BER} < b_i, \forall i < l \\
 z(k_i)/\kappa(\text{BER}) & \text{BER} > b_j, \forall j > i \\
 z(\mathcal{K}_0)/\kappa(\text{BER}) & \text{BER} < b_j \\
 0 & \text{otherwise}.
\end{cases}
\]

and if $\text{BER}_e < b_0$ the power is
\[
\frac{S(\text{BER})}{S} = \begin{cases} 
 z(\mathcal{K}_0)/\kappa(\text{BER}) & \text{BER}_e < b_j \\
 z(k_i)/\kappa(\text{BER}) & b_j < B_i, \forall i > j
\end{cases}
\]

for any $i = 0, \ldots, l, \ldots, N - 1$. And when $\bar{S} = S_{\min,d}(\mathcal{K}_0, \mathcal{B}_e)$:
\[
\frac{S(\text{BER})}{S} = \begin{cases} 
 z(\mathcal{K}_0)/\kappa(\text{BER}) & \text{BER} \in \mathcal{B}_e \\
 0 & \text{otherwise}.
\end{cases}
\]

Assuming $\text{BER}_0$ as the BER which corresponds to $\mathcal{X}_0$, Fig. 1 depicts the optimum power allocation for continuous and discrete rate adaptation under different setting as follows:

1) $S(\text{BER}) = \bar{S}$, and $\text{BER}_e = \text{BER}_0$. 2) $S(\text{BER}) > \bar{S}$, and $\text{BER}_e < \text{BER}_0$. 3) $S(\text{BER}) > \bar{S}$, and $\text{BER}_0 < \text{BER}_e$. 4) $S(\text{BER}) > \bar{S}$, and $\bar{S}$ is high enough for $\text{BER}^* < \text{BER}_0$ the power allocation has only water-filling nature.

III. NUMERICAL RESULTS

Here we assume that BER is Gaussian distributed with mean and variance of $\mu_b$ and $\sigma_b^2$ respectively. Setting the service rate and the service block-rate to $\mathcal{K}_0 = 4^{\text{Bits/symbol}}$, and $\varepsilon = 0.07$, the optimum BER-based power allocation policy for continuous and discrete rates are depicted in Figs. 2, where dashed and solid curves correspond to continuous and discrete rate, respectively.

Assuming $\mu_b = 10^{-2}$ and $\sigma_b^2 = 0.1$, $\sigma^2 = 6$ and $\mu_b = 0$, Fig. 3 depicts the average spectral efficiency of the discrete and continuous rate adaptation for the above settings.

IV. CONCLUSION

We derived the BER-based optimum power and rate allocation in adaptive QAM schemes over Gamma-lognormal distributed SNR. With service rate and service block-rate constraint, we maximized the ASE of BER-based continuous and discrete rate adaptation, respectively. We showed that the optimal power and continuous rate adaptation based on BER is a combination of two functions; one of which is of water-filling nature, and the other is proportional to $\log(1/\text{BER})$. For discrete rate MQAM the solution was only proportional to $\log(1/\text{BER})$ over each rate interval.

We have also shown that transmission rate is a monotonically increasing function of BER and spectral efficiency will be slightly decreased be restricting the transmission rate to take on only 8 different discrete values.

APPENDIX

Assuming lognormal distribution of $\gamma$, equation (1) can be written as
\[
\mathbb{K}_b = \frac{1}{\sigma \ln 2 \sqrt{2\pi}} \int_0^{\infty} \frac{1}{\gamma} \ln[1 + \gamma \kappa(\text{BER}) S(\text{BER})] e^{-(\ln(\gamma) - \mu)^2/2\sigma^2} d\gamma
\]
\[
= A \int_0^{\infty} \left\{ \frac{x^{\alpha - 1}}{\alpha} \ln[1 + \gamma \kappa(\text{BER}) S(\text{BER})] e^{-ax \ln(\gamma)/\alpha} \right\} d\gamma \quad (A1)
\]
where $A, \alpha$ are as defined before. As $e^{-ax \ln(\gamma)/\alpha} = e^{-a\gamma/\alpha}$ for any $\gamma > 3$, using [17, 3.353.5] and [17, 3.383.10]
\[
\mathbb{K}_b = \frac{\Gamma(\alpha) x^{\alpha} e^{\alpha x/2}}{A} \sum_{i=1}^{\alpha} \frac{1}{\alpha x/2} \Gamma(i - \alpha, \alpha x/2) \quad (A2)
\]

where $x = (S(\text{BER}) \kappa(\text{BER}))^{-1}$. Differentiating $\mathbb{K}_b$ in (A1) with respect to $x$, it is straightforward to verify that $\frac{d\mathbb{K}_b}{dx} < 0$, i.e. CSE is a decreasing function of $x$. Writing the Lagrangian equation as
L(γS(BER), λ) = [K_{p,b} \gamma S(BER, BER) - λ S(BER)] ϕ(BER) and setting \partial L(γS(BER), λ)/∂S(BER) = 0, the optimal power control policy of the sub-problem is obtained from [17, 3.383.10]

\[ S(BER, λ) = \frac{1}{Aλ^-1} \left( x^α e^{αx/2} - γ(α+1)Γ(α, αx/2) \right). \]  

(A3)

In order to show that (A3) has a unique positive answer, defining η(x) as

\[ η(x) = x^α e^{αx/2} - γ(α+1)Γ(α, αx/2). \] 

(A4)

(A3) can be rewritten as

\[ η(x) = \frac{Aλ^-1}{κ(BER)} = \int_0^∞ \frac{x^α e^{αx/2} - γ(α+1)Γ(α, αx/2)}{1+γ/λ} dγ. \]  

(A5)

As \[ \frac{dη(x)}{dx} = \int_0^∞ \frac{x^α+1 e^{-x/λ}}{(x+γ)^2} - dγ > 0, \forall x, \] η(x) is a strictly monotonically increasing and positive function of x. As η(0) = 0, and \[ \lim_{x→∞} η(x) = Γ(α+1)(α/2)^α, \] [17, 3.326. 2], equation (A3) has a unique positive solution for every \[ κ(BER) > A(α2^α/2) λ/Γ(α+1). \]

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