A Robust Model based Algorithm for Detection of Singularities in Fingerprint Images

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Abstract. The performance of fingerprint recognition is heavily depending on the reliable extraction of singularities. Common algorithms are based on a Poincaré-Index estimation which is a numerical method. These algorithms are only robust when certain heuristics and rules are applied. In this paper we present a model based approach for the detection of singular points. The presented method exploits the geometric nature of linear differential equation systems. Our method is robust against noise in the input image and is able to detect singularities even if they are partly occluded. The algorithm proceeds by fitting parameters for a given patch and then analyses these parameters. The parameters give a measure for the significance of the underlying structure and the type of a possible singularity.

1 Introduction

Fingerprint matching is a very suitable method for identifying people. The application of fingerprint based personal authentication and identification is steeply increasing. In general, personal verification or identification based on fingerprints mainly consists of acquisition, feature extraction, matching and a final decision [5].

1.1 Methods for Extraction of Singularities

Karu and Jain [4] referred to a Poincaré-Index method. However, there are principal weaknesses adhered to the method. Many rules and heuristics have been proposed by different authors (e.g. [17]) in order to make the method robust against noise and minor occlusions. Another method, described in [9] exploits the fact that partitioning the orientation image in regions characterized by homogeneous orientations implicitly reveals the position of singularities. Nilson et al. [11] identify singular points by their symmetry properties by applying complex filters.

1.2 Application of Fingerprint Singularities

In general, matching of fingerprint images is a difficult problem [3], mainly due to the large variability in different impressions of the same finger (i.e. displacement,
rotation, distortion, noise, ...). A way to relax the problem is to pre-align the
fingerprints using singularities [6]. Another Application where Singularities are
needed is the classification of Fingerprint images [5].

1.3 Model based detection of Singularities

In [13] Rao et al. proposed a novel algorithm for singular point detection in
flow fields. Their main idea is to locally approximate a flow pattern by a two-
dimensional linear differential equation. This allows a parametric representation
of different types of phase portraits, and their classification is possible based on
the extracted parameters.

In this paper we present a novel method for the detection of singularities based
on the work of Rao et al. In comparison, our method is robust against noise
in the input image and is able to detect singularities even if they are partly
occluded. Additionally, we present methods for detection and recognition of all
types of singularities in fingerprint images, whereas Rao et al. presented a model
for vortices only. This model based attempt is new to the field of fingerprint
singularity detection.

1.4 Outline

In section 2 an explanation of phase portraits is given. We analyse the weaknesses
of the original algorithm and propose a robust method.

In section 3 we explain how this algorithm can be applied to fingerprint images.

Section 4 shows the conducted experiments. In the first part, we demonstrate the
abilities of our robust method using synthetic data. Furthermore we show the
application of the algorithm on several fingerprints with occlusions and noise.

Finally, in the last section a summary of the proposed method (5) is given.

2 Two-Dimensional Linear Phase Portraits

Phase portraits are a powerful mathematical model for describing oriented tex-
tures, and therefore have been applied by many authors [2,13,16]. Linear phase
portraits can be expressed by the following differential equation system:

\[
\frac{dx}{dt} = \dot{x} = p(x, y) = cx + dy + f \\
\frac{dy}{dt} = \dot{y} = q(x, y) = ax + cy + e
\]  

(1)

By varying the parameters of these equations we can describe a set of oriented
textures comprising saddles, star nodes, nodes, improper nodes, centers and
spirals [7] (examples are given in figure 1). The orientation of these fields is
given by:

\[
\phi(x, y) = \tan^{-1} \left( \frac{dy}{dx} \right) = \tan^{-1} \left( \frac{\dot{y}}{\dot{x}} \right) = \tan^{-1} \left( \frac{ax + by + e}{cx + dy + f} \right)
\]  

(2)
Fig. 1. Classification of different phase portraits based on the eigenvalues of $A$ [12].
(a) spiral, (b) center, (c) improper node, (d) star node, (e) node, (f) saddle.

Equations 1 can further be represented in a more convenient matrix notation as:

$$\dot{X} = A \ast X + B$$

where $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $A = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$, $B = \begin{bmatrix} f \\ e \end{bmatrix}$ (3)

$A$ is called the characteristic matrix of the system. A point at which $\dot{x}$ and $\dot{y}$ are zero is called a critical point $(x_0, y_0)$ [7]. The elements of the characteristic matrix are used to determine six flow patterns. [12]. The type of the flow pattern is determined by the eigenvalues of the characteristic matrix.

2.1 Parameter extraction

Rao and Jain [12] presented an algorithm for the parameter extraction of oriented textures. However, the non-linear least squares computation required in their algorithm is computationally expensive and prone to local minima. In [15] Shu et al. presented a linear formulation of an algorithm which computes the critical points and parameters for a two dimensional phase portrait. Because their approach is linear there exists a closed form solution. The reader is referred to [12, 15, 16] for more details. Recent applications of the algorithm can be seen in [8, 18]. Throughout this paper we refer to this algorithm as the linear least squares algorithm.

2.2 Algorithm Analysis

In [16] Shu et al. refined their algorithm and presented a detailed analysis of their algorithm. From this analysis and our own experiments (see section 4) two conclusions can be drawn:

1. The presented algorithm works well in the case of Gaussian distributed noise. In the presence of occlusions, the algorithm may fail to extract the correct parameters.
2. The method has non-uniform sensitivity to noise - depending on the position of the point. The sensitivity in regions close to the singular point is low, whereas the sensitivity in regions away from the singular points is increased.
2.3 RANSAC based approach

Although the roots of the linear phase portrait estimation algorithm can be tracked back to the year 1990 [15], only recently several authors applied this algorithm in their work. For example in [8], the authors applied this method in order to extract a high level description of fingerprint singularities and direction fields thereof. As mentioned above, there are conceptional weaknesses adhered to this algorithm. In order to improve the performance of the original algorithm we propose the following RANSAC [1] based approach:

1. Randomly select 6 triplet data points \((x, y, \zeta)\) from the oriented texture and compute the model parameters using the linear least squares algorithm.
2. Verify the computed model by using a voting procedure. Every pixel lying within a user given threshold \(t\), increases the vote.
3. If the vote is big enough, accept fit and exit with success.
4. Repeat 1-3 for \(n\) times

The number of iterations \(n\) can be computed using the following formula [1]:

\[
    n = \frac{\log(1 - z)}{\log[1 - (1 - \epsilon)^m]}
\]

Where \(z\) is the confidence level, \(m\) is the number of parameters to be estimated and \(\epsilon\) is the outlier proportion.

3 Application to Fingerprint Singularities

As already mentioned, deltas and loops cannot be modeled by using linear phase portraits. Our approach for modeling deltas is to double the orientation angle and then to fit the parameters in the doubled angle orientation field. In a first step, loops can be detected as whorls in the image. In a further step, we analyse if one part of the detection contains a homogenous region. (See figure 2(c) for an example)

![Figure 2](image)

**Fig. 2.** The orientation fields of a delta (a). The doubled angle orientation fields can be seen in (b). The delta is modeled in the doubled angle orientation field as a saddle type singularity. Loops (c) are modeled in two parts, first the half whorl type and second a homogenous region.
3.1 Parameter analysis

Once parameters for a given sub window are fitted, these parameters must be inspected for their meaning. This inspection is done using the eigenvalues of the characteristic matrix $A$. In general the ratio of the two eigenvalues $\frac{\lambda_1}{\lambda_2}$ is expressing the aspect ratio of a oriented pattern around a given singularity. In order to prevent physically impossible parameters to be fitted, we introduce a threshold for this ratio. In Table 1 an overview of the classification and the thresholds are given.

<table>
<thead>
<tr>
<th>Appearance</th>
<th>Eigenvalues</th>
<th>Thresholds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whorl</td>
<td>complex eigenvalues</td>
<td>$\frac{1}{3} &lt; \frac{\lambda_1}{\lambda_2} &lt; 3$</td>
</tr>
<tr>
<td></td>
<td>$\lambda_1 = \Re + j\Im$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\lambda_2 = \Re - j\Im$</td>
<td></td>
</tr>
<tr>
<td>Delta</td>
<td>real distinct eigenvalues $\lambda_1$ and $\lambda_2$</td>
<td>$\frac{1}{3} &lt; \frac{\lambda_1}{\lambda_2} &lt; 4$</td>
</tr>
<tr>
<td></td>
<td>with opposite sign</td>
<td></td>
</tr>
<tr>
<td>Loop</td>
<td>upper part only: complex eigenvalues</td>
<td>$\frac{1}{3} &lt; \frac{\lambda_1}{\lambda_2} &lt; 3$</td>
</tr>
<tr>
<td></td>
<td>$\lambda_1 = \Re + j\Im$</td>
<td>$\Re &lt; 0.2$</td>
</tr>
<tr>
<td></td>
<td>$\lambda_2 = \Re - j\Im$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Classification-Schema for Fingerprint Singularities: The classification is done based on the extracted parameters of the model. Whorls are detected in the original orientation field which is extracted gradient based. Detection of deltas is done in the doubled angle orientation field. In case of the loop, first a half center is detected, followed by the detection of a homogeneous region. Thresholds are introduced in order to prevent the fitting of physically impossible parameters.

4 Experimental Results

4.1 Parameter Fitting Examples

In the following, a small comparison between the original linear least squares algorithm and the one proposed by us is given. In Figure 3 we test the robustness against occlusion. In Figure 4 we conducted a test in order to explain the importance of uniform sensitivity of the parameter estimation algorithm.

The estimation of the orientation field is accomplished by the gradient based algorithm of [14]. Figure 5 shows the fitting capability of our algorithm on fingerprint singularities. Note how the parameters have been extracted correctly although the image contains many defects.

4.2 Singular Point Detection in Fingerprint Images

In this subsection we present results of the detection and recognition algorithm. The used images have been taken from the Fingerprint Verification Competition.
Fig. 3. **Occlusion test:** in this example we compare the robustness of the two algorithms against occlusions. First an orientation field of center type has been created. On both sides of the orientation field an occlusion has been simulated by replacing the original values by random values \([a]\). While the original algorithm fails \([b]\) our new approach \([c]\) extracts the parameters precisely.

Fig. 4. **Uniform sensitivity test:** in \([a]\) the orientation has been extracted using a gradient based method. In figure \([b]\) one can see the fitting result of the linear least squares algorithm. Figure \([c]\) shows the fitting results of our proposed method. Note that our algorithm is uniform sensitive and thus is robust to perturbations in the input image.

Fig. 5. **Fitting test:** parameter fitting for a delta type singularity. The orientation field in figure \([a]\) and \([c]\) was generated by using a gradient based method described in [14]. Occlusions are regions where the gradients can not be correctly extracted. In figure \([b]\) and \([c]\) it is visualized how the proposed algorithm was capable of fitting the correct model parameters. Note that our method not only detects the singular point robustly, but also smooths the orientation field around the singularity.
Robust Singularity Detection

[10]. Figure 6 visualizes the detection results on a whorl type fingerprint. In table 2 result of the detection capability on 109 images can be seen.

Fig. 6. This example shows the detection of Fig. 7. The proposed model fitted to a a whorl and two deltas. The position of the loop, the upper part as circle and the lower sliding window is marked with a rectangle. part as homogeneous area.

<table>
<thead>
<tr>
<th>type</th>
<th>true positive</th>
<th>false positive</th>
<th>true negative</th>
<th>false negative</th>
<th>accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta</td>
<td>34</td>
<td>13</td>
<td>58</td>
<td>0</td>
<td>87.6%</td>
</tr>
<tr>
<td>whorl</td>
<td>29</td>
<td>3</td>
<td>76</td>
<td>0</td>
<td>97.2%</td>
</tr>
<tr>
<td>loop</td>
<td>72</td>
<td>8</td>
<td>19</td>
<td>8</td>
<td>85.0%</td>
</tr>
</tbody>
</table>

Table 2. The tests have been done on the FVC2000 database using 109 images from the db1_a and db2_a folders (69 loops, 31 whorls, 3 tended archs and 6 archs).

5 Conclusion

We presented a model based method for singular point detection in fingerprint images. Our proposed method is robust to noise and occlusions in the input image. The algorithm proceeds by fitting model parameters at each location of a sliding window and then analyses this model parameters. On positions of singular points, a detection can be performed by the analysis of these parameters. We performed several tests on synthetic and natural images in order to point out the mentioned capabilities of our algorithm. We also showed how the algorithm is able to reconstruct the orientations near singular points. In the last part of this paper we showed detection results of the proposed algorithm.

Future work includes the testing of our method with larger datasets. Furthermore we aim at merging our model based approach with existing numerical methods.

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References