Game-Theoretic Modeling of Joint Topology Control and Power Scheduling for Wireless Heterogeneous Sensor Networks

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Abstract—Wireless Heterogeneous Sensor Network (WHSN) facilitates ubiquitous information acquisition for Ambient Intelligence (AmI) systems. It is of great importance of power management and topology control for WHSN to achieve desirable network performances, such as clustering properties, connectivity and power efficiency. This paper proposes a game theoretic model of topology control to analyze the decentralized interactions among heterogeneous sensors. We study the utility function for nodes to achieve desirable frame success rate and node degree, while minimizing the power consumption. Specifically, we propose a static complete-information game formulation for power scheduling and then prove the existence of the Nash equilibrium with simultaneous move. Because the heterogeneous sensors typically react to neighboring environment based on local information and the states of sensors are evolving over time, the power-scheduling problem in WHSN is further formulated into a more realistic incomplete-information dynamic game model with sequential move. We then analyze the separating equilibrium, one of the perfect Bayesian equilibriums resulted from the dynamic game, with the sensors revealing their operational states from their actions. The sufficient and necessary conditions for the existence of separating equilibriums are derived for the dynamic Bayesian game, which provide theoretical basis to the proposed power scheduling algorithms, NEPow and BEPow. The primary contributions of this paper include applying game theory to analyze the distributed decision-making process of individual sensor nodes and to analyze the desirable utilities of heterogeneous sensor nodes. Simulations are presented to validate the proposed algorithms and the results show their ability of maintaining reliable connectivity, reducing power consumption, while achieving desirable network performances.

Note to Practitioners—Heterogeneity poses challenges to system design of WHSN. Practical applications require a distributed topology control algorithm to ensure reliability as well as power efficiency. Existing approaches for topology control either require centralized controller to obtain network graph globally, or lack of theoretical analysis. In this paper, a new game theoretic model yields decentralized optimization for joint topology control and power management. Global game equilibriums are iteratively reached by considering individual node degree, message delivery ratio and cost of increasing power. Based on mathematical game analysis, we provide two solution concepts for implementations. The NEPow scheme is derived from Nash Equilibrium of static game model. The BEPow scheme, derived from Bayesian Nash Equilibrium of incomplete-information dynamic game model, is dedicated to the scenario involving heterogeneous types (short of power or full of power) of nodes with adjustable power levels. The simulation results demonstrate that the proposed algorithms are feasible to achieve desirable performances, in terms of connectivity, packet reception rate, throughputs and power efficiency. The belief-updating algorithm in Bayesian game is shown effective for different types of sensors to determine their power levels interactively and consistently without causing performance degradation. Both algorithms are simulated in packet-level network simulator and ready to be ported to real test-bed with slight modification.

Index Terms—Bayesian Nash equilibrium, game theory, power scheduling, topology control, wireless heterogeneous sensor networks.

I. INTRODUCTION

In VIRTUE of the advances of information technologies, the last decade has witnessed rapid progress of wireless sensor networks (WSNs) [1], [2], which form the core sensing component of Ambient Intelligence (AmI) systems [3]–[7]. WSN typically consists of a collective of various wireless networked low-power devices, each of which integrates an embedded microprocessor, radio and a limited amount of storage.

For the purpose of various applications, the interconnected sensors may be homogeneous or heterogeneous in terms of sensing units, communication ability, computational capability, transmit power level and hardware complexity. In homogeneous networks, all the member nodes are identical and use same transmit power level. The nodes are equipped with the same sensing units to track the single event [8]–[11]. In heterogeneous networks, the member nodes may be different in many aspects. They could have various kinds of sensing units, different power supply, different hardware ability, and different transmission radius. The powerful nodes serve as data sinks and the energy-limited nodes serve as sensing nodes to collect physical information. The sensor network can be formed by overlaying powerful high-end nodes with limited low-end sensor. Related conceptions of WHSN were investigated in [12]–[14]. In this paper, we define a heterogeneous sensor network as a network of sensor nodes with tunable transmit power level and various energy resources.
Aml criteria require diverse sensors [4] embedded in the environment or worn by the users in an unobtrusive and distributed manner. Therefore, the sensor networks in Aml are inherently featured with heterogeneity. The user-centric paradigm [15], [16] of Aml requires that the sensor networks recognize and respond to an individual in a personalized way. Thus, self-interest is another character of the sensor nodes in WHSN, because the heterogeneous nodes may be owned by multi-authorities or multidomains without unconditional cooperation. In addition, malicious or intrusive nodes may exist in hostile environment and result in malice behaviors in the network. The selfish behaviors are practical in vehicular sensor networks [17], civil sensor networks, battlefield sensor networks [1] and large-scale personal sensor networks [18]. For instance, future intelligent combat system involves heterogeneous sensors in the air (unmanned airplanes), on the ground and under the water (unmanned underwater vehicles), in which the subsystems are organized in different domains and total selflessness is not a valid assumption. An envisioned scenario is biosensor network [18]–[20], which is employed for large-scale monitoring such as disaster relief or personal area monitoring in hospital. The body sensors owned by one individual form a Body Sensor Network (BSN), which is not supposed selfless in communication with other BSNs owned by other individuals. The constituent sensors are diversified to monitor different physiological signals. The computation abilities are quite diverse since central process units of different products may range from powerful laptop CPU to a small SoC chip.

Based on noncooperative game theory, heterogeneous sensors can be viewed as selfish agents maximizing their own utilities and topology optimization mechanisms are developed to ensure network performances resulted from local selfish decisions. One of the challenges in developing wireless heterogeneous sensor network is to ensure the reliability of wireless transmission; meanwhile the wireless communication must avoid abuse of limited resources. Therefore, heterogeneous types of sensors may have different preferences. Those sink nodes with sustainable power supply (e.g., the one attached to laptop) may prefer using higher transmit power level to achieve higher frame success rate (FSR) and longer transmission range. Unlike homogeneous wireless sensor networks, the reliability requirements of some sink nodes have higher priority over energy consumption. However, the member nodes with limited power resources may prefer using as lower power as possible, while ensuring error-free transmission. One way to enhance the communication reliability is to build up a reliable and power efficient network topology for wireless heterogeneous sensor networks. Topology control of wireless sensor networks can be categorized to three classes in terms of implementation: using transmit power control; using node sleep and wakeup mechanism such as SMAC [21]; and using hierarchical clustering like the functionalities of Personal Area Network (PAN) leader in IEEE 802.15.4 [22] Standard. In the context of wireless heterogeneous sensor networks, we consider the first type topology control through transmit power control.

The network topology in WHSN is expected to have the following desirable features. First, the formed topology must ensure high transmission reliability, which is typically investigated by studying the network connectivity graph. Second, its wireless transmissions should have lowest adverse effects, e.g., avoid interference and increase spatial reuse. This could be partially controlled through transmit power management, which in turn results in different topology patterns. Third, it should be power efficient due to limited resources. Therefore, the two terms, power management and topology control, are directly correlated, but still are referred from different standpoint and emphasis: power management is for reducing interference, enhancing energy efficiency and improving link quality from physical-layer perspective; topology control is for enhancing network connectivity from network-layer perspective.

This paper is motivated by these considerations and proposes the joint design of topology control and power scheduling for wireless heterogeneous sensor networks based on game theory. The key contributions of this paper are: firstly investigating the relationships between topology management, transmit power control and the property of network reliability in WHSN; secondly formulating the power and topology control from game theoretic perspective, and then designing a cross-layer optimization scheme to form a desirable topology for WHSN.

The rest of this paper is organized as follows. Section II is a brief review of related work on game theoretic models, topology and power control for wireless networks. Section III presents the game theoretic model of topology control considering node preferences. We then investigate solution concepts of Nash Equilibrium (NE), Static Bayesian Equilibrium (SBE), and Dynamic Bayesian Equilibrium (DBE) in Section IV, Section V, and Section VI, respectively. Section VII describes proposed algorithms of NEPow and BEPow. Section VIII validates and evaluates the methods by simulation experiments. Finally, Section IX concludes this paper.

II. RELATED WORK

A. Power Control and Topology Optimization

One of the topology control strategies is transmit power control (TPC), which dynamically adjusts the radio frequency (RF) output power level according to the environment changes or application requirements. Transmit power control schemes for wireless sensor networks can be categorized to three themes from the implementation perspective. Global network-level power control is to dynamically tune the output power for all nodes globally and a typical scheme is in [23]. Node-level power control is to set the same RF power of a node for all outgoing packets, where different nodes may use different level [24]. Packet-level power control is to specify various output power to every single outgoing packet, which is a complicated scheme [25].

Another typical taxonomy is to classify the power control methods by the situated protocol layer, such as medium access control (MAC)-layer power control based on signal-to-interference performances, routing-layer power control based on routing performances, etc. The transmit power itself is specified to the outgoing packets at the physical-layer before transmitting to the wireless channel. However, the tuning of wireless RF power may result in cross-layer consequences. At physical-layer, the power level determines the received signal quality, i.e., signal-to-noise ratio (SNR), maximum available data rate,
and the energy consumption. At MAC-layer, the power level determines the contention range of several neighboring nodes. In turn, at network-layer, the effective transmission range determined by the power strategy will influence the route selection. Furthermore, at transport or application-layer, the end-to-end performance such as latency and retransmission rate will also be affected. More importantly, for specific applications such as large-scale sensor networks, the spatial reuse factor is greatly dominated by the power levels. It is concluded that the power control shall be guided by multiple performances.

Some transmit power control methods have been proposed for various applications, and a good survey is in [26] and [27]. A simplest version, for example, BASIC power control [28] algorithm uses highest power level to send the RTS and CTS packets. The sender tells receiver it will transmit a packet via RTS, and the receiver reply the desired-sender-power via CTS. Then, the sender uses this desired-sender-power to send the DATA. Power control MAC (PCM) [29] is another MAC-layer power-control protocol based on RTS-CTS-DATA-ACK messages. Power-aware routing optimization (PARO) [30], a routing-layer packet level control scheme, determines routes which consume lowest energy. However, most of aforementioned power control methods did not take the topology optimization into account, and lack of concrete theoretical analysis.

B. Game Theory for Wireless Sensor Networks

Game theoretic mechanism has been extensively investigated recently for distributed decision making in wireless sensor networks such as power control in cellular and data networks [31]–[33], data gathering [34], bandwidth sharing [35], and congestion control [36] in ad-hoc networks. Nevertheless, few of them have involved the topology control and power scheduling for wireless heterogeneous sensor networks. All the players or nodes play a game in the system simultaneously by picking their individual strategies. Most of the researches study the properties of the NE resulting from interaction of selfish nodes whose utility functions are known a priori. Typically, the sensor networks are supposed to play an incomplete information game because the distributed nature of sensor networks do not allow the nodes have information about the strategies of other nodes globally. The methods based on game theory for network control have natural common ground with those based on convex optimization. The existence and uniqueness of NE under mild convexity assumptions on the cost function has been proved for congestion control [36]. Auction game was employed to allocate limited resources, such as efficient routing [37], [38] and channel assignment [39]. Just like noncooperative game, collaborations between different nodes are no longer taken for granted. To save limited resources, the nodes are likely to behave selfishly by refusing to forward others’ packets. Therefore, pay-for-service model, an incentive mechanism, is introduced to enable cooperative packet forwarding. Transmission power eReCursive AuCtion Mechanism (TEAM) [37] routing protocol prevented the selfish behaviors and stimulated cooperative works by paying nodes for their service. However, TEAM only calls for auctions within one hop on the initial routes created by AODV-like protocol. Incentive comPatible Auction-Based Service Scheme (iPass) [38] enable cooperative packet forwarding by paying market price to the service nodes and also can serve as an flow control mechanism. In iPass, intermediate nodes constitute a smart market and a seal-bide second-price auction algorithm determine who should obtain how much of the bandwidth at what price. Distributed power control problem in wireless networks has been viewed as noncooperative game in [31]–[33], [40]–[43]. However, the existing algorithms can not be directly applied to WHSN, because of its different features in terms of node characteristics (low power, high density, no central controller), and player preferences (connectivity, reusability, efficiency, and reliability).

III. GAME FORMULATION OF TOPOLOGY CONTROL

A game of power scheduling is an interactive decision making process between a set of self-interested nodes, which formally consists of the following elements.

- A set of players, \( N \), which may be a group of nodes or an individual node in wireless sensor networks.
- A set of actions, \( A_i \), available for the player \( i \) to make a decision. The strategy profile \( P \) of power scheduling problem is a set of power levels chosen by \( N \) nodes, \( \{p_1, p_2, \ldots, p_N\} \).
- The payoff \( \{u_1, u_2, \ldots, u_i\} \) resulted from the strategy profile.

The problem of topology control game in WHSN is to determine an optimal power strategy profile \( P = \{p_1, p_2, \ldots, p_N\} = \{p_i, p_{\neq i}\} \) to achieve a maximal utility, where \( p_i \) is the feasible power level of node \( i \), and \( p_{\neq i} \) is strategy vector taken by the neighboring nodes of \( i \).

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The utility is represented by the pure-strategy space of the opponents of player \( i \) is represented by \( p_{\neq i} = \{p_1, p_2, \ldots, p_{\neq i-1}, p_{i+1}, \ldots, p_N\} \). For heterogeneous sensor networks, we are mainly concerned with three aspects of achievable utility: FSR represented by a function of transmit power, network topology reliability represented by network connectivity, and cost of increasing power related to the transmit power. We can view the frame-success-rate as a node-to-node level reliability preference, and network connectivity as network-level reliability preference.

A. Frame Success Rate to Evaluate Link Quality and Power Efficiency

A WHSN is a node graph \( G = (N, L) \) that consists of a set of nodes \( N = \{1, \ldots, n\} \) and a set of wireless links \( L \subseteq 2^N \). A wireless link \( l \in L \) is denoted by its end nodes \( i \) and \( j \) and the distance between node \( i \) and \( j \) is \( \text{distance}_{ij} \). When a frame is transmitted by a node \( i \) at transmit power \( P_t \), it can be detected and correctly decoded if received power \( P_r \) is greater than the signal capture threshold \( P_{\text{thresh}} \)

\[
P_r = k \cdot P_t \cdot (\text{distance}_{ij})^{-t} > P_{\text{thresh}}
\]

where \( t \) is fading factor of wireless channel and \( (\text{distance}_{ij})^{-t} \) is the attenuation coefficient.

For all linear receivers, the derivative of signal-to-interference-noise ratio (SINR, \( \gamma \)) with respect transmission power has the form

\[
\frac{\partial \gamma_i}{\partial P_t} = \frac{\gamma_i}{P_t}
\]
which is valid for wireless transmissions coordinated by most of existing MAC schemes including CSMA/CA and sleep coordination MAC schemes. In general, the SINR, $\gamma_k$, for a link is

$$\gamma_k(p_k) = \frac{\text{Bandwidth}}{\text{Data rate}} \cdot \frac{(\text{distance}_{ij})^\alpha p_i}{\sum_{k \neq i} (\text{distance}_{ik})^\alpha p_k + N} \tag{3}$$

where $\sum_{k \neq i} (\text{distance}_{ik})^\alpha p_k$ is the interference from the ongoing links of other nodes or clusters, and $N$ is background noise. Note that this SINR model is applicable to sensor networks based on direct sequence code division multiple access (DS-CDMA) or TDMA/FDMA systems with respective parameter interpretations. To the end of achieving higher SINR, the environmental interference determines the individual transmit level, which in turn causes interference to other nodes.

The first concern of power control problem is to achieve a maximum of FSR, while inducing minimal power consumptions. The formulation of FSR depends on the details of the wireless transmission, including modulation scheme, coding scheme, and channel condition. For the most widely used modulation schemes, FSR is sigmoidal (S-shaped curve) and monotonically increasing with respect to signal-to-noise ratio. For instance, the bit-error rate (BER) of binary phase-shift keying (BPSK) in additive white Gaussian noise (AWGN) can be calculated as $0.5\operatorname{erfc}(\sqrt{\gamma})$ and the corresponding packet success rate is given by $(1 - 0.5\operatorname{erfc}(\sqrt{\gamma}))^M$, where $M$ is number of information bits and $\operatorname{erfc}$ is complementary Gaussian error function. The BER computations of other modulation schemes are in a similar form, such as $\operatorname{BER}(\text{CFSK}) = 0.5\operatorname{erfc}(\sqrt{0.5\gamma})$, $\operatorname{BER}(\text{DPSK}) = 0.5\exp(-\gamma)$, and $\operatorname{BER}(\text{NCFSK}) = 0.5\exp(-0.5\gamma)$. Without loss of generality, we employ

$$f(\gamma) = (1 - e^{-0.5\gamma})^M \tag{4}$$

to represent frame success level as described in [44]. This is reasonable and valid for any success rate function in a sigmoid shape and continuously differentiable.

The FSR $f(\gamma)$ is the commonly used metric in many existing power control methods, but for WHSN, the other two important aspects must be taken into account: network connectivity and power efficiency. The FSR $f(\gamma)$ ignores the disadvantages of power consumption and interference that imposes on the other nodes. Typically, the higher the transmit power level a sensor node uses, the more RF power consumption induced, and the more interference incurred to other nodes. Therefore, in order to discourage the selfish power increase, we use the metric similar to [40] with the unit of bit-per-joule to evaluate the basic power efficiency. The rationality of this function has been extensively investigated in [40], [45]. The power efficiency metric, $\zeta_i(p_i, p_{-i})$, obtained by the $i$th node is expressed as a function of $p_i$

$$\zeta_i(p_i, p_{-i}) = R \cdot f(\gamma_i)/p_i = R \cdot (1 - e^{-0.5\gamma_i})^M/p_i \text{ bit/J} \tag{5}$$

where $f(\gamma_i)$ is the function of $\gamma_i$ to represent the FSR level of receiver $i$, and $R$ is the physical information rate.

B. Network Connectivity

1) Connectivity Features and Network Reliability: Connectivity in graph theory [46] is defined as an important metric of topology reliability for a network. In heterogeneous sensor networks, power control should not only reduce power consumption and increase spatial reuse, but also ensure the network reliability for message passing. Therefore, we utilize network connectivity as a metric to evaluate the topology reliability.

The problem of assigning transmit-power levels to sensor nodes in a distributive way to ensure certain connectivity has been proved a NP hard problem. The most frequently explored theoretical problem of connectivity graph for sensor networks is to study the probability of network being connected given critical transmission range, which has not yet revealed practical decentralized algorithm for assigning transmission range to achieve certain level of connectivity probability.

Definition 1: For a heterogeneous sensor network, $k$-node connectivity ($k$-vertex cut) of graph $G$ is defined as a set $U$ of vertices, with the minimum number of $k$ nodes, whose removal will result in graph $G - U$ a disconnected or trivial graph. Moreover, $k$-edge connectivity ($k$-edge cut) of graph $G$ is a set $X$ of edges, or the minimal number of edges, whose removal results in graph $G - X$ to be disconnected.

In graph theory, although $k$-edge connectivity and $k$-node connectivity are good metrics to evaluate network topology, unfortunately, they can only be observed globally, which is inapplicable to decentralized implementation. As an alternative, it has been proved that $k$-edge-connectivity can be achieved by considering node degree problem [46], which will help us to turn the centralized $k$-edge or $k$-node features to decentralized $k$-neighbor problem. From random geometry graph perspective, the sensor network can be modeled as random geometry graph, due to its motion and wireless link variation [47]. Further, the probability of $k$-edge-connectivity has been proved highly related to the probability of $k$-node-degree, as described in Theorem 1. In [48], a probabilistic bound of the minimum node degree for a homogeneous sensor network is given by studying the stochastic process and geometric behaviors, but the implementation issues remained unsolved. Therefore, if we expect a heterogeneous sensor network to possess certain $k$-connectivity property, we can turn this global requirement to individual node degree requirement for every single node in the protocol design.

Theorem 1. (Probabilistic Connectivity Theorem): Starting with a trivial graph, a random graph adds the corresponding links as transmission range increases, and the resulting random geometric graph would almost become $k$-connected when it achieves minimum node degree $d_{\text{min}}$ of $k$, i.e., the probability, $\mathcal{P}(G \text{ is } k-\text{connected}) = \mathcal{P}(d_{\text{min}} \geq k)$ if $n$ is sufficiently large, which is proved in [48].

2) Theoretical Connectivity of Homogeneous and Inhomogeneous Sensor Networks: If $n$ uniformly distributed nodes have identical transmission range in the network area $A$, the node density is $\lambda = n/A$ for two-dimensional plane. Assume the bidirectional link can be established once two nodes are located within the transmission range of each other. Hence, the probability that a node $i$ has symmetric node degree of $d = k$ is transformed to the probability that $k$ of the $N$ nodes are located
within the transmission range \( r_i \) of node \( i \), which is subject to Poisson distribution (see the Appendix for proof)

\[
\mathcal{P}(d = k) = \frac{(\pi r_i^2 \lambda)^k}{k!} e^{-\pi r_i^2 \lambda}. \tag{6}
\]

From the viewpoint of entire network, the probability for the network to be \( k \)-connected is derived by deducting the sum probability over the cases of \( 1 \)-connected, \( \cdots \), \( k-1 \)-connected

\[
\mathcal{P}(k \text{ - connected}) = \mathcal{P}(d \geq k) = 1 - \sum_{d=k}^{d=n} \mathcal{P}(d = k) = 1 - \sum_{d=k}^{d=n} \frac{(\pi r_i^2)^d}{d!} e^{-\pi r_i^2 \lambda}. \tag{7}
\]

In the case of inhomogeneous transmission ranges in the same network area \( A \), first we assume node \( i \) is using range \( r_i \) at time \( T \) and the other sensor nodes use two different transmission ranges \( r_i, r_j \) (see the strategy space of sensor nodes at Section IV with equal probability. If \( r_i \geq r_j \), not all the nodes within the transmission area \( \pi r_i^2 \) of node \( i \) can establish bidirectional links with \( i \). The nodes within the area \( \pi r_j^2 \) enclosed by a circle with center at node \( i \) and radius of \( r_j \) definitely get symmetric link with \( i \). The nodes within the annulus \( \pi (r_i^2 - r_j^2) \) centered at \( i \) can partly get symmetric or asymmetric links depending on the fraction of using \( r_i \) or \( r_j \). As equal probability for a node using \( r_i \) or \( r_j \) is assumed, the number of symmetric neighbors with respect to \( i \) in the annulus is proportional to the area of \( \pi (r_i^2 - r_j^2)/2 \). The same analysis can be made for the case of \( r_i \leq r_j \). Therefore, the total number of symmetric links that node \( i \) can achieve is related to the area of \( \pi (r_i^2 - r_j^2)/2 + \pi r_j^2 = \pi (r_i^2 + r_j^2)/2 \), and the corresponding probability of achievable symmetric degree of \( d = k \) is

\[
\mathcal{P}(d = k) = \frac{(\lambda \pi (r_i^2 + r_j^2)/2)^k}{k!} e^{-\lambda \pi (r_i^2 + r_j^2)/2}. \]

Thus, the expected node degree (number of neighbors) of node \( i \) is given by \( E(d) = \lambda \pi (r_i^2 + r_j^2)/2 \).

As the transmission range \( r_i \) is a power function of transmission power \( p_i \), \( p_i \propto r_i^2 \). The metric of expected node degree \( \delta(p_i) \) with respect to power \( p_i \) is expressed as

\[
\delta(p_i) = b_6 \lambda \pi \left( p_i^{2/3} + p_j^{2/3} \right)/2 \tag{8}
\]

where \( b_6 \) is the simplified constant coefficients from the product of other terms. The metric of expected node degree admit the following first-order derivative with respect to \( p_i \)

\[
\frac{\partial \delta(p_i)}{\partial p_i} = 2/3 \lambda \pi p_i^{-1/3} \tag{9}
\]

which is referred in the Theorem 2 for the analysis of equilibrium.

3) Practical Connectivity Metric: In the problem of topology control via transmit power adjustment, we should relax the assumption that WHSN is always a bidirectional node graph \( G = (N, L) \). Instead, a WHSN is reconsidered as directed graph denoting the active wireless communication links in the network, where \( L = \{(s, t) : t \text{ is within the transmission range of sender } s \text{ while } s \text{ is at current transmit power level } \} \). For node-level transmit power adjustment, it is observed that \((s, t) \in L \) does not mean \((t, s) \in L\) since \( s \) and \( t \) might use different power levels. Therefore, for a node \( s \) in a WHSN, the following terms are extended from [24] as follows:

- incoming neighbor set, \( NB_i(s) = \{ t \in N : (t, s) \in L \} \);
- outgoing neighbor set, \( NB_o(s) = \{ t \in N, (s, t) \in L \} \);
- symmetric neighbor set, \( NB(s) = NB_i(s) \cap NB_o(s) \).

When the nodes are using node-level power adjustment, asymmetric links may be generated, which make the connectivity analysis more complicated. The existing study on strong connectivity game behaviors of a network such as in [49] is from global perspective, and is not suitable for a distributive topology control from individual node perspective. In order to study the game from a distributive manner, we define the practical connectivity metric of node \( i \) as a function of node degree and power levels

\[
\delta_i(i, p_i) = f ([NB_i(i, p_i)], [NB_c(i, p_i)], [NB_o(i, p_i)]) \tag{10}
\]

Every node can only observe the incoming neighbors directly and outgoing neighbors indirectly via message exchanges. In practice, the values of \( p_i, [NB_i(i, p_i)], [NB_c(i, p_i)], [NB_o(i, p_i)] \) can be acquired by the entities at different protocol layers.

C. Joint Power and Topology Control Game

For now, the Noncooperative Power and Topology Control Game (NPTG) is defined as the linear combination of aforementioned metrics NPTG = \([N, \{P_i\}, \{u_i(s)\}]\)

\[
\max u(p_i) = \max f (c_i(p_i), \delta_i(p_i)). \tag{11}
\]

The utility function reveals the node preferences while considering reliability, connectivity and power consumption. In this way, the problem is viewed as an incomplete information noncooperative NPTG, where the sensor node only has information about its own power level, neighbor number, SINR perceived from the environment and its own channel condition. If each node is assumed as a fully rational entity, NE is achieved when each node want to maximize selfpayoff and minimize the cost. For example, the nodes may increase their transmit power to get higher FSR and node connectivity. Meanwhile, they have to consider the cost of adverse power effects and power consumption. However, when the system reaches the NE, no nodes can increase its utility any more through individual effort. The best response \( BR(p_{-i}) \) of a node \( i \) corresponding to the power strategies \( p_{-i} \) of other nodes is given by

\[
BR(p_{-i}) = \arg \max u(p_i). \tag{12}
\]

For example, when node \( s \) observe \( k \) incoming neighbors, it will response by considering maximizing its own net utility, not just using high-power level \( h \) to get higher FSR and node degree.
IV. NASH EQUILIBRIUM (NE)

A. Existence of NE in NPTG

Before the derivation of an algorithm for NPTG = \([N, \{P_i\}, \{u_i(\cdot)\}]\), we should analyze the existence of the Nash equilibria in the proposed NPTG game.

Definition 2: An power profile strategy \(P = \{p_1, p_2, \ldots, p_N\}\) is a NE of NPTG = \([N, \{P_i\}, \{u_i(\cdot)\}]\) if, for every \(i \in N\), \(u_i(p_i(p_1, p_{-i})) \geq u_i(p_i', p_{-i})\) for all \(p_i' \in P_i\). That is, at the state of NE, no node can improve its payoff by individual deviation.

Definition 3: The best response function \(BR_i(p_{-i})\) of \(i\)th node is defined as a best strategy for node \(i\) to get the maximum utility against all the other nodes’ strategy \(p_{-i}\)

\[
BR(p_{-i}) = \{p_i \in P_i : u_i(p_i, p_{-i}) \geq u_i(p_i', p_{-i}) \forall p_i' \in P_i\}.
\] (13)

The best response function \(BR(p_{-i})\) for a differentiable function \(u_i(p_i, p_{-i})\) can be derived from the partial derivative of \(u_i\) with respect to \(p_i\).

Theorem 2. (NE Existence Theorem): A NE exists in the game NPTG = \([N, \{P_i\}, \{u_i(\cdot)\}]\) if, for all \(i = 1, 2, \ldots, N\)

1) \(P_i\) is a nonempty, convex, and compact subset of same Euclidean space \(\mathbb{R}^N\);

2) every utility function \(u_i(p)\) is continuous in \(P_i\) and quasiconcave with respect to its corresponding strategies \(p_i\).

Proof: The theorem to identify the existence of the NE is obtained from [50]. The first condition is satisfied because each node has a strategy space defined continuously over the interval \([0, P_{\text{max}}]\), i.e., \(P_i\) is a nonempty, convex, and compact subset of the same Euclidean space \(\mathbb{R}^N\).

As the net utility is in the form of

\[
u_i(p_i, p_{-i}) = a \cdot \frac{f(p_i)}{p_i} + b \cdot \delta(p_i, p_{-i}) \quad (14)\]

where \(a, b\) is the constant coefficient to tune the weight of various preferences. The first term, sigmoidal power efficiency function has been proved a concave function with respect to power \(p_i\) [31]. The metric of FSR, \(f(p_i)\) is characterized by increasing sigmoid shape, and continuously differentiable, with \(f(0) = 0, f'(0) = 0, f(+\infty) = 1\).

Note the path loss exponent \(\lambda\) is greater than 2 for typical environments and the value of \(2/\lambda\) is less than 1. The connectivity metric in the second term is a concave function at the interval \([P_{\text{min}}, P_{\text{max}}]\), because of \(\partial^2\delta(p_i)/\partial p_i^2 < 0\) and the negative second-order derivative, \(\partial^2\delta(p_i)/\partial p_i^2 = \frac{2}{\lambda(2/\lambda - 1)}\lambda p_i^{2/\lambda - 2} \leq 0\). It is known that the summation of two concave functions is still a concave function. Hence, the second-order derivative of \(u_i\) with respect to \(p_i\) is less than zero, \(\partial^2 u_i/\partial p_i^2 < 0\), which implies the best responses are concave in \(p_i\). Because a concave function is by definition quasi-concave, we can derive that \(u_i(p)\) is quasi-concave in \(p_i\).

It has been shown that the equilibrium resulted from the metric of power efficiency is Pareto efficient [40]. As the optimal node degree is assumed convergent to an optimal number globally, it is trivial that the derived equilibrium from the best response is Pareto efficient, given the number of nodes is sufficiently large.

As the maximizers \(\gamma^*\) and \(p_i^*\) are one-to-one corresponding, incorporating (2), (5), (8), and (9), \((\gamma^*, p_i^*)\) is the unique maximizer to the first-order derivative of \(u_i(p)\), \(f'(\gamma^*)\gamma^* = f(\gamma^*) + p_i^{2/\lambda + 1}\). Therefore, the best response is unique. The equilibrium is unique because the best response is a standard function [31], [51], which satisfy the conditions of positivity, monotonicity, and scalability. Furthermore, let us compute the cross derivative \(\partial^2 u_i(p_i)/\partial p_i \partial p_j\). It is found that \(\partial^2 u_i(p_i)/\partial p_i \partial p_j > 0\) for the feasible strategy space \([P_{\text{min}}, P_{\text{max}}]\), where \(P_{\text{supmod}}\) corresponds to the solver of \(\partial^2 f(\gamma)/\partial \gamma^2 = 0\). Applying Taylor series expansion to the exponential function in \(\partial^2 u_i(p_i)/\partial p_i \partial p_j\) by estimation, we find that \(\partial^2 u_i(p_i)/\partial p_i \partial p_j < 0\) only for small \(p_i\) in \([0, P_{\text{supmod}}]\), and \(\partial^2 u_i(p_i)/\partial p_i \partial p_j > 0\) for most of the feasible region in \([P_{\text{supmod}}, P_{\text{max}}]\). Hence, the NPTG is a supermodular game by definition within the strategy space \([P_{\text{supmod}}, P_{\text{max}}]\), which confirms the existence and convergence of equilibrium.

B. Numerical Illustrations

We give an extremely simplified numerical example of deriving NE from game matrix. Note that it is merely for the purpose of illustrating aforementioned idea and does not stand for the practical implementation.

1) Connectivity Metric Only: Assume the two nodes in Fig. 1 only consider the connectivity metric, and further assume one symmetric degree has the payoff of unit 1, one incoming degree contributes payoff of unit 0.5, and one outgoing degree contributes utility of unit 0.5. This is reasonable because unidirectional link cannot guarantee a reliable message exchange without acknowledgement mechanism. Then, the normal form game is represented in the Table I, assuming each node need to choose from two transmit power levels \((I, h)\) for lower level and \(h\) for higher level). It is easy to get the NE of this game: \((I, h)\) and corresponding payoff \((1, 1)\), by underlining relative dominant-strategies.

2) Connectivity Metric With Power Consumption: Assume transmit power increase of one unit will induce 1 unit power consumption, that is, if a node increase one unit power, its corresponding consumption will increase one unit. Again the same power consumption function in (2), (5), (8), and (9) is applied. Thus, the NEs can be obtained by 0.8 for 3 nodes, 0.6 for 4 nodes, and 0.4 for 5 nodes. The game is represented in the Table II, assuming each node need to choose from two transmit power levels \((1, 2)\) for lower level and \((2, 4)\) for higher level). It is easy to get the NE of this game: \((1, 2)\) and corresponding payoff \((1, 1)\), by underlining relative dominant strategies.
TABLE I

<table>
<thead>
<tr>
<th>A \ B (Power Level)</th>
<th>l</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>l</td>
<td>(0, 0) (disconnected)</td>
<td>(0.5, 0.5) (unilateral)</td>
</tr>
<tr>
<td>h</td>
<td>(0.5, 0.5) (unilateral)</td>
<td>(1, 1) (bidirectional)</td>
</tr>
</tbody>
</table>

TABLE II

<table>
<thead>
<tr>
<th>A \ B (Power Level)</th>
<th>l</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>l</td>
<td>(-1, -1)</td>
<td>(0, -0.5)</td>
</tr>
<tr>
<td>h</td>
<td>(-0.5, 0)</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

scenario of Fig. 1 is examined by considering both connectivity metric and power consumption. The strategic game is depicted in Table II.

Definition 4: The resulting power profile strategy \( P = \{p_1, p_2, \ldots, p_N\} \) is an Iterated Dominant Strategy Equilibrium (IDE) of \( NPTG = (N, \{P_i\}, \{u_i(\cdot)\}) \) if, for every \( i \in N \)

\[ u_i(p_i^*, p_{-i}) > u_i(p_i, p_{-i}), \quad \forall p_{-i} \in P_{-i} \]

and the power strategy \( p_i \) of player \( i \) is said to be strictly dominated by his strategy \( p_i^* \) in this case.

Interestingly, there is one iterated dominant strategy equilibrium \((h, h)\) for this game by iteratively eliminating dominated strategies defined in Definition 4, but there are three NEs: \((l, h)\), \((h, l)\), \((h, h)\). Therefore, the NE of this game is not unique.

V. STATIC BAYESIAN EQUILIBRIUM (SBE)

Both the IDE and NE studied in previous sections bear the same assumptions that each player knows all elements of the game, including the characteristics and payoff functions of all the participants. However, this assumption is not true for practical heterogeneous sensor networks, because: 1) different types of sensors may have different preferences; 2) the types of sensors are not common knowledge; and 3) the types of a node may change over time because of power drainage. Therefore, we introduce Bayesian NE to topology control game with incomplete information.

A. Modeling Private Types and Preference

We represent the remaining energy of each node by a set of discrete energy levels and classify the nodes to different types, \( \Theta = \{\theta_i\} \). For the simplicity of analysis, we consider a heterogeneous network with two types of nodes in terms of remained energy class, Rich or Poor. This does not limit the energy to only two classes, because each node actually holds a probability function on the types of remained energy. It is an interchangeable role for a node to be either type \( R \) or type \( P \), depending on its energy resources. For example, if it is equipped with a newly full charged battery at the very beginning, its type could be rich compared with a low-battery node, but it is still “Poor” compared with the node attached to PDA or a laptop. When time elapses, the battery of a node may be used up after a period of communication. Therefore, node \( i \)’s marginal power-consumption depends on its types that only node \( i \) knows. To avoid confusion, the Rich type in this paper has sustainable power supply and the Poor type has limited battery. Both types of sensors are overlaid in the same area.

Note the type, \( R \) or \( P \), is determined by energy resource of the node, which is not optional. In contrast, the strategies of a node, i.e., transmission power levels, are determined by node itself, which could be any power level.

Type \( R \) (Rich) with full power prefers to achieve higher FSR and longer transmission range. Type \( P \) (Poor) with limited battery prefers saving as much energy as possible. Both types of players have a set of strategies available to maximize its own preferences in response to the strategies of its opponents. Based on the aforementioned analysis of reliability metric and connectivity metric, the same action decision will bring different payoff to a node if it is in a state of different type. For Poor type, the preference is to ensure the reliability for error-free communication firstly, then reduce power consumption and then maintain a certain level of connectivity, where “certain” means minimum node degree to ensure bidirectional link. For rich type, it does not care about the power consumption so much, and prefer higher reliability and connectivity.

B. Static Game

For static game model of the power scheduling problem, a generic assumption is that all nodes have knowledge of the common prior probability \( \mu \) about the types of all players, and note that this assumption is not true in the dynamic Bayesian game. We consider a two-player game with two nodes \( i \) and \( j \), and in order to make a clear illustration, we use he to stand for \( i \), and she for \( j \). Node \( i \) knows exactly his own types and which type \( j \) has, but she cannot know her own types and which type \( i \) has. Based on the aforementioned analysis of reliability metric and connectivity metric, the same action decision will bring different payoff to a node if it is in a state of different type. For Poor type, the preference is to ensure the reliability for error-free communication firstly, then reduce power consumption and then maintain a certain level of connectivity, where “certain” means minimum node degree to ensure bidirectional link. For rich type, it does not care about the power consumption so much, and prefer higher reliability and connectivity.

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in the implementation of power control, since the current hardware transceiver generally only tune the output power to limited levels.

C. Bayesian-Nash Equilibrium for Continuous Strategy Space Game

Assume the strategies of \(i\) and \(j\) can be chosen from continuous space, then we can rewrite the aforementioned payoff for both \(P\) and \(R\) types in the following form. Node \(i\) will get utility according to his own types as a function of continuous action \(p_i\)

\[
\begin{align*}
    u_P(p_i,p_j) &= \frac{\alpha f(p_i,p_j)}{\alpha p_i} + b(p_i,p_j) \\
    u_R(p_i,p_j) &= \frac{\beta f(p_i,p_j)}{\beta p_i} + b(p_i,p_j)
\end{align*}
\]

(16) (17)

where \(\alpha, \beta\) are the weights to tune different requirements on the power consumption between \(P\) type and \(R\) type. Let \(\alpha \leq \beta \leq 1/p_i\) to discourage excessive power usage for \(P\) type. For example, if let \(\beta p_i = 1\), it means the \(R\) type does not care about its power consumption.

Then, if \(i\) is \(R\), it will maximize his utility, \(\max u_R(p_i,p_j)\), given any \(p_j\). It will solve \(\max u_P(p_i,p_j)\), if it is Poor. Hence, in order to choose his desirable action \(p_i=\mathbb{R}\) in Rich status and \(p_i=\mathbb{P}\) in Poor status, his best response is given by

\[
    p_i=\mathbb{R} = \arg \max u_R(p_j) \quad \forall \theta(i) = \mathbb{R}
\]

(18)

\[
    p_i=\mathbb{P} = \arg \max u_R(p_j) \quad \forall \theta(i) = \mathbb{P}
\]

(19)

where the subscript \(\mathbb{R}\) is \(P\) means in the case of node \(i\) to be type \(P\).

Without loss of generality, for node \(j\), assume she is always regular Poor type and \(j\) does not know exactly \(i\)'s type, but she believes \(i\)'s cost function is \(u_R\) with probability of \(\mu\). Assuming FSR is only a function of transmit power level, node \(j\)'s objective function \(u_i(\mu,p_i,p_j)\) is \(\mu (\alpha f(p_j)/(\alpha p_j))) + (1-\mu) (\beta f(p_j)/(\beta p_j)) + b(p_i=\mathbb{R},p_j)\). The action can be derived from the first-order condition of this equation

\[
    p_i^* = \arg \max u(\mu,p_i,p_j),
\]

(20)

For now we have three equations (18), (19), and (20) and three unknown variables \(p_i=\mathbb{R}, p_i=\mathbb{P},\) and \(p_i\), then the solutions of this Bayesian game come up with the following steps.

- Node \(j\) uses \(p_i^*\) under the belief of the types of node \(i\).
- Node \(i\) uses \(p_i=\mathbb{R}\) if he is Rich, use \(p_i^*\) if he is Poor.
- This is a typical back induction method for the Bayesian game, and can be written as \((p_i^*, p_i=\mathbb{R}; p_i=\mathbb{P})\).

D. Bayesian-Nash Equilibrium for Discrete Strategy Space Game

The former section gives an illustration of one stage continuous game. However, it is generally impossible for the current transceivers to tune output power \(p_i\) in continuous space. For example, the TelosB node can tune the radio output power from 32 available discrete levels. Without loss of generality, it is a common practice in game theory to analyze the game from a simple two-by-two game matrix. This two-by-two game can be extended to multiplayer and multi-action game in the algorithm design, which will be implemented in the power scheduling protocol of WHSN. For simplification of illustration, each node in this section is assumed to have two pure strategies irrespective of its types: use either Low\(I\) power level or High\(h\) power level. Note that the action space, \(I\) or \(h\), is different from the types, \(R\) or \(P\), because each type can choose either action \(I\) or \(h\).

1) Specific Game Scenario: We begin with a special two-node scenario in Fig. 1 as an illustration on the static Bayesian game. The extensive form of the game tree is in Fig. 2, where initially Nature assigns a prior probability \(\mu\) \(1-\mu\) to player \(i\) being \(R\) or \(P\), respectively, and this information can be seen by all players. Nodes have to choose available actions from discretized space and we study the one-stage discrete extensive game as follows.

The payoff matrix is shown in Table III with respect to the node \(i\) and \(j\). In the first entry of the table, we use \((u_i=\mathbb{R},u_j=\mathbb{I})\) to represent the utility if \(i\) is Rich with strategy \(I\) and node \(j\) is regular Poor with strategy \(I\). Similarly, the other entries are in the form of \((u_i=\mathbb{R},u_j=\mathbb{I})\). Note that subscript of \(j\) is omitted because it is assumed regular Poor all the time. For instance, the payoff of a \(R\) node \(i\) is related to the achievable power efficiency and connectivity, \(f(p_i)/p_i + \delta_I\). For simplicity of expression, the weight \(\alpha, \beta\) in the utility function is omitted in the analysis, which has no effects on the validity of the results. When both players use \(I\) strategies, the payoff is \((0,0)\) in the scenario of Fig. 1. Because of link disconnection, while two players use \((I, h)\) strategy, the payoff become \((\delta_I(I,h), f(h)/h + \delta_I(I,h))\).

For node \(j\), there are totally four possible choices, \((h\text{ if Rich, } I\text{ if Poor})\), \((\text{if Rich, } I\text{ if Poor})\), \((\text{if Rich, } h\text{ if Poor})\), \((\text{if Rich, } h\text{ if Poor})\).
When node $i$, plays his strategy ($h$ if Rich, $l$ if Poor), regular node $j$ will have the expected payoff of using pure strategy $l$

$$E_{u_j}(l) = \mu u_j(h,l) + (1 - \mu)u_j(l,l) = \mu \delta_j(h,l)$$

and her expected payoff of using $h$

$$E_{u_j}(h) = \mu u_j(h,h) + (1 - \mu)u_j(l,h)$$

$$= \mu \left( \frac{f(h)}{h_{\alpha}} + \delta_j(h, h) \right) + (1 - \mu) \left( \frac{f(h)}{h_{\alpha}} + \delta_j(l, h) \right)$$

$$= \frac{f(h)}{h} + \mu \delta_j(h, h) + (1 - \mu) \delta_j(l, h).$$

(21)

Under the condition of believing the action of $i$ ($h$ if Rich, $l$ if Poor), $j$ will determine her strategy by comparing the expected payoff $E_{u_j}(l)$ and $E_{u_j}(h)$. That is, $a_j(h) = h$ if $E_{u_j}(h) \geq E_{u_j}(l)$, $a_j(l) = l$ if $E_{u_j}(l) < E_{u_j}(h)$. As $\mu \leq 1$ and $\delta_j(h, h) \leq \delta_j(l, h)$, the inequation $E_{u_j}(h) \geq E_{u_j}(l)$ hold for any $\mu$. Therefore, the best response of player $j$ is to play $h$, under the strategy assumption of $i$, ($h$ if Rich, $l$ if Poor).

However, from the perspective of $i$, if $j$ use $h$, $l$ will not be the best response for Poor node $i$, i.e., ($l$ if Poor) does not hold, since $u_{i \leftarrow P}(h, h) \geq u_{i \leftarrow P}(l, h)$, i.e., $f(h)/(h_{\alpha}) + \delta_j(h, h) > f(l)/(l_{\alpha}) + \delta_j(l, h)$. Thus $i$ will move on to play $h$.

Hence, $(i, a_i = (h$ if Rich, $l$ if Poor), $a_j = h)$ is not a BNE.

When node $i$ plays strategy ($l$ if Rich, $h$ if Poor), node $j$ has expected payoff of using $l$, $E_{u_j}(l) = \mu u_j(h, l) + (1 - \mu)u_j(l, l) = (1 - \mu)\delta_j(l, h)$, and get her expected payoff of using $h$, $E_{u_j}(h) = \mu u_j(h, l) + (1 - \mu)u_j(l, h) = \frac{f(h)}{h_{\alpha}} + \mu \delta_j(h, h) + (1 - \mu)\delta_j(l, h)$. Node $j$ chooses her strategy, $a_j(h) = h$ if $E_{u_j}(h) \geq E_{u_j}(l)$, $a_j(l) = l$ if $E_{u_j}(l) < E_{u_j}(h)$ and the former condition is satisfied.

Conversely, from the perspective of $i$, if $j$ use $h$, node $i$ will reconsider his strategy ($l$ if Rich, $h$ if Poor), because $l$ is not the best response for Rich type of $i$ to maximize his utility, since $u_{i \leftarrow P}(h, h) \leq u_{i \leftarrow P}(l, h)$, i.e., $\delta_j(l, h) \leq f(h)/(h_{\alpha}) + \delta_j(l, h)$. Hence, $(i, (l$ if Rich, $h$ if Poor), $h)$ is not a BNE.

If Rich node $i$ plays $l$ for both types, ($l$ if Rich, $h$ if Poor), regular node $j$ has dominant strategy of playing $h$, since $u_j(\mu, a_i = l, a_j = h) = u_j(1 - \mu, a_i = l, a_j = h)$ are always great than $u_j(\mu, a_i = h, a_j = h, a_j = h)$ or $u_j(1 - \mu, a_i = h, a_j = h, a_j = h)$. However, if regular node $j$ plays $h$, from the standpoint of $i$, the best choice of Rich node $i$ is to play $h$, since $u_{i \leftarrow P}(h, h) \leq u_{i \leftarrow P}(l, h)$. Therefore, $(i, (l$ if Rich, $h$ if Poor), $h)$ is not a BNE.

The examination of the action of $i$ ($h$ if Rich, $l$ if Poor) is quite similar to its ($l$ if Rich, $h$ if Poor). If node $i$ plays ($h$ if Rich, $l$ if Poor), node $j$ has expected payoff of using $l$, $E_{u_j}(l) = \mu u_j(h, l) + (1 - \mu)u_j(l, l) = \delta_j(l, h)$, and get her expected payoff of using $h$, $E_{u_j}(h) = \mu u_j(h, l) + (1 - \mu)u_j(l, h) = \frac{f(h)}{h_{\alpha}} + \delta_j(l, h)$. As $E_{u_j}(h) \geq E_{u_j}(l)$, the best response of node $j$ is playing $h$. Conversely, if regular node $j$ plays $h$, from the standpoint of $i$, both ($h$ if Rich) and ($h$ if Poor)

are his best strategy, because $u_{i \leftarrow P}(h, h) \leq u_{i \leftarrow P}(l, h)$ and $u_{i \leftarrow R}(l, h) \leq u_{i \leftarrow R}(h, h)$. Therefore, $(i, l$ if Rich, $h$ if Poor), $a_j = h)$ is a pure BNE for the game scenario in Fig. 1.

2) General Game Scenario: Note that the above discussion is only valid for the case in Fig. 1, where $l$ strategy will cause unsuccessful transmission for the far distance. The solution concept can also be obtained by observing the Table III, because strategy $(h, h)$ dominates all other strategies in both sub-tables.

If we put $i$ and $j$ within the mutual transmission range at lower power level, as shown in Fig. 3, the analysis is not as simple as previous scenario, but in the similar way. The payoff matrix is listed in Table IV, where neither player has a strictly dominant strategy. For example, we cannot determine ordinal relation between payoff $u_j(h, h)$ and $u_j(l, l)$ of node $j$, dependent on the comparison of $f(h)/(h_{\alpha}) + \delta_j(h, h)$ versus $f(l)/(l_{\alpha}) + \delta_j(l, h)$.

We give an analysis example on one strategy of $i$, ($h$ if Rich, $l$ if Poor). The other three cases are omitted but could be derived in the same way. If node $i$ plays ($l$ if Rich, $h$ if Poor), node $j$ has expected payoff of using $l$, $E_{u_j}(l) = \mu u_j(h, l) + (1 - \mu)u_j(l, l) = f(l)/(l_{\alpha}) + \delta_j(l, h)$, and get her expected payoff of using $h$, $E_{u_j}(h) = \mu u_j(h, l) + (1 - \mu)u_j(l, h) = f(h)/(h_{\alpha}) + \delta_j(h, h)$. The relation between $f(l)/(l_{\alpha})$ and $f(h)/(h_{\alpha})$ is not determinate, nor is $E_{u_j}(h)$ and $E_{u_j}(l)$. Then, different cases and conditions should be analyzed.

a) Suppose $E_{u_j}(h) \geq E_{u_j}(l)$, the best response of node $j$ is playing $h$ and we have $f(h)/(h_{\alpha}) + \delta_j(h, h) \geq f(l)/(l_{\alpha}) + \delta_j(l, h)$, i.e., $\alpha \geq (f(l)/l - f(h)/h)/(\delta_j(h, h) - \delta_j(l, h))$.}

![Fig. 3. Game of two players with symmetric links.](image-url)
Let us conversely check the validity of \( h_2 \) if Rich, \( h_1 \) if Poor, given \( a_j = h_1 \). (i) The validity of \( h_2 \) if Rich is dependent on whether it is true for \( u_{i-}\overline{\rho}(h_1, h) \geq u_{i-}\overline{\xi}(h_1, h) \) i.e., \( f(h)/(h\beta) + \delta_i(h_1, h) \geq f(l)/(l\beta) + \delta_i(h_1, h) \). Then, the condition for validity of \( h_2 \) if Rich is \( \beta \geq (f(l)/l - f(h)/h)/(\delta_i(h_1, h) - \delta_i(h_1, h)) \). (ii) The validity of \( h_1 \) if Poor is dependent on the truth of \( u_{i-}\overline{\rho}(h_2, h) \geq u_{i-}\overline{\xi}(h_2, h) \), i.e. \( f(h)/(h\alpha) + \delta_i(h_2, h) \geq f(l)/(l\alpha) + \delta_i(h_1, h) \). The inequity holds as it is coincident with the \( E_{u_j}(h) \geq E_{u_j}(l) \), under the assumption on symmetric node degree \( \delta_i(h_1, h) = \delta_i(h_1, l) \) and \( \delta_i(h_2, h) = \delta_i(h_2, l) \).

b) Suppose \( E_{u_j}(h) \leq E_{u_j}(l) \), the best response of node \( j \) is playing Low and we have \( f(h)/(h\alpha) + \delta_i(h_2, h) \leq f(l)/(l\alpha) + \delta_i(h_1, h) \). Given \( a_j = l \), (High if Poor) is valid only if \( u_{i-}\overline{\rho}(h_1, h) \geq u_{i-}\overline{\xi}(l, h) \), i.e. \( f(h)/(h\alpha) + \delta_i(h_1, h) \geq f(l)/(l\alpha) + \delta_i(l, l) \). However, it contradicts with \( E_{u_j}(h) \leq E_{u_j}(l) \), under the condition of \( \delta_i(h_2, h) = \delta_i(h_2, l) = \delta_i(h_1, l) \). Therefore, \( h_1 \) if Poor does not hold if \( a_j = l \).

To summarize the cases a) and b), \( \beta \geq (f(l)/l - f(h)/h)/(\delta_i(h_1, h) - \delta_i(h_1, l)) \), \( a_j = h_1 \) is a pure BNE, where the constraint is applied to \( \beta \).

3) Mixed Strategy BNE: We observe that there is no pure strategy BNE when \( \beta \leq (f(l)/l - f(h)/h)/(\delta_i(h_1, h) - \delta_i(h_1, l)) \). Instead, a mixed strategy BNE exists in this case under particular conditions as follows. Let \( \omega, \nu \) be the probability of player \( i \) and \( j \), respectively, for playing \( h \). The expected utility of \( j \) while playing \( h \) is

\[
E_{u_j}(h) = \omega u_j(h, l) + (1 - \omega) u_j(h, h) + \omega(1 - \mu) u_j(h, h) + (1 - \omega)(1 - \mu) u_j(h, h)
\]

and expected utility of \( i \) using \( l \) is

\[
E_{u_j}(l) = \omega u_j(l, l) + (1 - \omega) u_j(l, l) = f(l)/(l\alpha) + \delta_i(h_2, l) + (1 - \omega) \delta_i(l, l),
\]

Let \( E_{u_j}(h) = E_{u_j}(l) \), we have the probability \( \omega^* \) of \( i \) playing \( h \) as

\[
\omega^* = \frac{f(l)/(l\alpha) - f(h)/(h\alpha) + \delta_i(l, l) - \delta_i(h, l)}{\delta_i(h, h) + \delta_i(l, l) - \delta_i(h, l)}. 
\]

On the contrary, from the standpoint of \( i \), given \( j \)'s probability of \( \nu \) playing \( h \), the expected utility of \( i \) while playing \( h \) is \( \mu f(h)/(l\beta) + (1 - \mu)(1 - \nu) f(h)/(h\beta) + \delta_i(h_1, h) + (1 - \nu) \delta_i(h_1, l) \), and the expected utility of \( i \) while playing \( l \) is \( \mu f(l)/(l\beta) + (1 - \mu) f(l)/(l\alpha) + \nu \delta_i(h_2, l) + (1 - \nu) \delta_i(l, l) \).

By imposing \( E_{u_i}(h) = E_{u_i}(l) \), the probability \( \nu^* \) of \( j \) playing \( h \) is

\[
\nu^* = \frac{\nu f(l)/(l\alpha) - \mu f(h)/(h\beta) + (1 - \mu)(f(l)/(l\alpha) - f(h)/(h\beta)) + \delta_i(l)/(l\beta) - \delta_i(h_1, h)}{\delta_i(h_1, h) + \delta_i(l)/(l\beta) - \delta_i(h_1, l)}. 
\]

Therefore, \( (\omega^*, \nu^*, \mu, \beta) \) is the mixed-strategy BNE for the proposed game. From the above analysis, we know that there is no pure-strategy while \( \beta \leq (f(l)/l - f(h)/h)/(\delta_i(h_1, h) - \delta_i(l, h)) \), but the corresponding mixed-strategy \( (\omega^*, \nu^*, \mu, \beta) \) exists.

VI. Dynamic Bayesian Equilibrium (DBE)

By far, we have shown the existence conditions of Bayesian equilibrium for static game with simultaneous move. If we relax the belief assumption of static game, i.e., if the prior probability of types are not global common knowledge, the power scheduling problem should be studied in the context of dynamic game with sequential move. The remained energy represented by type is private information and only known by the node itself. Therefore, the node should hold belief-updating system about the energy class of neighboring nodes and the beliefs are independent over all nodes. In the dynamic Bayesian game, although the information of player types is private knowledge, the opponents are able to update their beliefs on the types in the process of game playing. We assume the aforementioned static game is repeated infinitely in the time scale, because generally no nodes can predict when nodes will leave the network or when will join the network. The utility functions of the Rich nodes and Poor nodes are the same as the ones in the continuous-function static game.

A. DBE Conceptions

A Bayesian Game consists of the following components:

- a set of players: \( N = \{1, \ldots, n\} \);
- strategy space for each player \( A_i : A = \times\in N A_i \);
- type set for player \( i \), \( \Theta_i \), which is private information;
- conditional probability function \( \mu_i(\Theta_i | \Theta) \);
- payoff function \( u_i : A \times \Theta \rightarrow \mathbb{R} \) that is determined by the Cartesian product of \( A \) and type \( \Theta \).

Given a dynamic Bayesian game represented by \( (N, \{A_i\}, \{\Theta_i\}, \{\mu_i\}, \{u_i\}) \), player \( i \)'s strategy is a probability distribution \( \sigma_i(\cdot | \theta) \) over action \( a_i \) for each type \( \theta \), to maximize his expected utility: \( \max \sum_{\theta \in \Theta_i} \mu_i(\theta | \theta) \sum_{\theta \in \Theta_i} \prod_{j \in N \setminus i} \sigma_j(\cdot | \theta_j) u_i(a_i, \theta) \). The objective of player \( i \) is to map his types to his actions (i.e., transmit power level in the context of WHSN): \( \sigma_i : \Theta_i \rightarrow \sigma(A_i) \). For example, the behavior strategy of Rich node \( i \) is \( \sigma_i(a_i(t) | \theta_i, \theta_i(t)) \), where \( \theta_i(t) \) is the action history of \( i \) in response to his opponent \( j \). Then, the objective of \( i \) is to figure out the mixed strategy at each stage using belief updates \( \mu_i(t) \) and action history \( \theta_i(t) \), to maximize his current payoff. We note that the resulted probabilistic strategy is changing with time depending on his latest belief about \( j \). Therefore, a proper belief-updating rule should be built up for these players.

B. Forming Rational Beliefs Using Bayesian Updating

Initially, a player has a prior belief assigned by Nature based on Harsanyi transformation. Concerning the types of other nodes, he will update his beliefs in the course of the game based on the observation of action history of his opponents. The belief of a node about the types of other players, \( \mu_i(\Theta - \omega | \Theta) \), is modeled with Bayesian rules. The updating can be based on many observations. For example, in WHSN, we can identify if a neighbor node is undergoing heavy traffic all the time by overhearing the traffic, and then further tell if it is Rich or Poor based on those observations.
Therefore, the objective of the Bayesian belief updating is to infer the types \( \theta_i(t) \) of the other players continuously at every time step, given a set of observed action history of its opponents \( a_{-i}(1 : t), i \in N, t = 1, 2, \ldots, \) during the course of \([1 : t]\). This can be represented by deriving the posterior probabilistic distribution \( P(\theta_i(t) | a_{1:t-1}) \), or namely the belief of \( \mu(\theta_{-i}(t)) \). Specifically, for a game between two nodes \( i \) and \( j \), we use \( P_j(\theta_i) \) to represent \( j \)'s belief about the possibility of \( i \) being type \( \theta \) and \( P_i(\theta_j) = q_i (1 : t) \) is \( i \)'s action history with respect to \( j \). According to the Bayesian rule, we are able to derive \( j \)'s posterior belief about the types of \( j \)'s opponents as follows:

\[
\mu_j(\theta_i) = \frac{P_j(\theta_i | a_i(t), \theta_i(t))}{\sum_{\theta} P_j(\theta_i | a_i(t), \theta_i(t))} = \frac{P_j(a_i(t) | \theta_i(t)) P_i(\theta_i(t))}{\sum_{\theta} P_j(a_i(t) | \theta_i(t), \theta_i(t)) P_i(\theta_i(t)).} \tag{22}
\]

As the computational complexity of calculating \( P_j(\theta_i | a_i(t)) \) would increase over time, we would like to have an algorithm only dependent on the previous time step \((t - 1)\). By Chapman–Kolmogorov equation, we can reuse the approximation of \( P_j(\theta_i | a_i(t))_{t-1} \) available at time \((t - 1)\) to generate an approximation of \( P_j(\theta_i | a_i(t))_t \) at time \( t \) (without incurring confusion, we rewrite \( a_i(t), \theta_i(t) \) as \( a_{1:t} \))

\[
P_j^*(\theta_i(t) | a_{1:t}) = \frac{P_j^*(a_i(t) | \theta_i(t)) P_j(\theta_i(t) | a_{1:t-1})}{P(\theta_i(t) | a_{1:t-1})} = \frac{\eta P_j^*(a_i(t) | \theta_i(t)) P_j(\theta_i(t) | a_{1:t-1})}{P(\theta_i(t) | a_{1:t-1})}, \tag{23}
\]

where \( \eta \) is the normalized constant ensuring that \( P_j(\theta_i(t) | a_{1:t-1}) \) sum up to one over all \( \theta_i \), and \( P(\theta_i(t) | a_{1:t-1}) \) is modeling type transition probability when the battery level or power source of a node is changing. Plugging \( P_j(\theta_i(t) | a_{1:t-1}) \) to \( P_j^*(\theta_i(t) | a_{1:t}) \), the posterior probabilities are derived recursively from \( P_j^*(\theta_i(t-1) | a_{1:t-1}) \) to \( P_j^*(\theta_i(t) | a_{1:t}) \). From (23) and (22), regular node \( j \) needs to observe \( i \)'s action \( a_{1:t}(i) \) continuously in order to determine \( i \)'s type. In the protocol design of WHSN, this is feasible by piggybacking the current transmit power level in the packet header. Any nodes capturing this packet can figure out the possible type of the sender based on the Bayesian belief update algorithm.

### C. Perfect Bayesian Equilibrium Analysis

Basic signaling game is a subclass of the Bayesian game with incomplete information where the follower or receiver specify its move depending on an observable characteristic, i.e., the signal, from the leader or sender. Since the network communication is always there in the process of game playing, the power-scheduling problem is actually a signaling game, which may admit two types of Bayesian equilibriums.

- Separating equilibrium: different types of nodes signal their ability using different actions according to their payoff functions. For instance, if node \( i \) is Rich, he will use higher power more likely, and use lower power if he is getting poor.
- Pooling equilibrium: different types of nodes send the same signal with respect to their actions. That is, both rich node \( i \) and poor node \( j \) may use same power level to make his type indistinguishable to his opponents.

The proposed Bayesian game model of power scheduling is a repeated signaling game with observable actions and incomplete information. In this section, we will show that the multi-stage game admit a separating PBE. The definition of a PBE is extended from [52] as follows. A PBE of a signaling game is a strategy profile \( \sigma^* \) and posterior beliefs \( \mu_q(a_j) \) such that

\[
\begin{align*}
(1) & \quad \forall \theta_i, \sigma^*_i(\cdot | \theta) \in \mathop{\arg \max}_{a_i} h_i(a_i, \sigma^*_j, \theta), \\
(2) & \quad \forall a_i, \sigma^*_j(\cdot | \theta) \in \mathop{\arg \max}_{a_j} \sum_{\theta} \mu(\theta | a_i) u_j(a_i, a_j, \theta), \\
(3) \quad \mu_j(\theta | a_i) = P(\theta | \sigma_i^*(a_i | \theta)) / \sum_{\theta' \in \Theta} P(\theta' | \sigma_j^*(a_j | \theta')) \quad \text{if} \quad \sum_{\theta' \in \Theta} P(\theta' | \sigma_j^*(a_j | \theta')) > 0. \tag{24}
\end{align*}
\]

C1 (Condition 2) and C2 (Condition 2) are the perfection requirements of sequentiality imposed on each player. That is, at every single information set of any stage, strategies must be optimal, given the beliefs on observed strategies. C1 states that player \( i \) response optimally considering \( j \)'s action and C2 claims that player \( j \) is getting best response with respect to \( i \)'s action and \( j \)'s belief on \( i \). Therefore, the PBE deals with not only what the players act, but also what they believe. BL (Condition of belief) says the beliefs should be consistent with the strategies.

The perfect Bayesian equilibrium is by definition much stronger than the Bayesian Nash Equilibrium. First, note that the equilibrium is a strategy profile and a belief system, which add restriction on the belief system. Second, it requires that the strategies have sequential rationality given the belief updating. Third, it requires the belief system is consistent, wherever possible, given the strategy profile.

**Proposition 3**: A separating equilibrium exists if and only if the conditions in (26) are satisfied.

**Proof**: We examine the separating equilibrium of the proposed Bayesian game, in which different types of node \( i \) will choose different actions and the follower node \( j \) also believe the observed action of \( i \) reveals \( i \)'s types

\[
\mu_j(\theta_i = \text{Rich} | a_i = h) = 1, \quad \mu_j(\theta_i = \text{Poor} | a_i = l) = 1.
\]

If the Rich type \( i \) uses higher power, it will reveal his type Rich to his opponents and get the payoff \( u_i(a_i = h, \cdot) = f(h)(/h/\beta) + \delta_i(h, l) \) (\( j \) will play \( l \) under the conditions in (26)). If the Rich node \( i \) select lower power, he would convince his opponent \( j \) that he was Poor and will get the payoff \( u_i(a_i = l, \cdot) = f(l)(/l/\beta) + \delta_i(l, h) \) (\( j \) will play \( h \) under the conditions in (26)). Hence, the necessary condition for the existence of a separating equilibrium is \( u_i(a_i = h, \cdot) \geq u_i(a_i = l, \cdot) \)

\[
f(h)/(h/\beta) + \delta_i(h, l) \geq f(l)/(l/\beta) + \delta_i(l, h). \tag{25}
\]

On the contrary, under the condition that (25) is satisfied, we consider the strategy and belief updating from the standpoint of follower \( j \). Node \( j \) continuously updates her belief on \( i \)'s
type based on the observation of $i$’s action (High). Node $j$ infers that $i$ is Rich and choose $l$ to save power, if $u_j(a_i = h, a_j = l, \theta_i = R) = f(l)/(l + h) + \delta_j(h, l) \geq u_j(a_i = h, a_j = h, \theta_i = R) = f(h)/(l + h) + \delta_j(h, h)$. If $j$ believe $i$ is Poor after observing $i$’s low power, $j$ will choose $h$ to maximize her own payoff, if $u_j(a_i = l, a_j = l, \theta_i = P) = f(l)/(l + h) + \delta_j(l, l) \leq u_j(a_i = l, a_j = h, \theta_i = P) = f(h)/(l + h) + \delta_j(l, h)$. By combining the two inequations, we have $\delta_j(l, h) - \delta_j(l, l) \geq f(l)/(l + h) - f(h)/(l + h) \geq \delta_j(h, h) - \delta_j(l, h)$. The constraints (26) is sufficient and necessary for the existence of a separating equilibrium

$$\begin{align*}
  u_j(a_i = h, \cdot) &\geq u_j(a_i = l, \cdot) \\
  u_j(a_i = l, a_j = l, \theta_i = R) &\geq u_j(a_i = l, a_j = h, \theta_i = R) \quad (26) \\
  u_j(a_i = l, a_j = l, \theta_i = P) &\leq u_j(a_i = l, a_j = h, \theta_i = P)
\end{align*}$$

Hence, we derive the constraints for a signaling game to admit a separating equilibrium, which is the necessary condition in the algorithm.

VII. GAME THEORETIC TOPOLOGY CONTROL ALGORITHM

Based on game theoretic analysis of NE and Bayesian Nash Equilibrium, we propose two algorithms for two scenarios of WHSN, respectively. The first one is Nash Equilibrium Power control (NEPow) derived from NE analysis in Section IV. NEPow is designed for a WHSN, in which the member sensors may have various transmit power levels without considering the types of the sensor devices. The second one is Bayesian Nash Equilibrium Power control (BEPow) derived from perfect Bayesian equilibrium analysis in Section VI-C. BEPow is feasible for the WHSN with different types of sensors and different sensors could use various transmit power levels. The difference between NEPow and BEPow is that the belief-updating algorithm of BEPow is able to help sensors to infer the types of its opponents, and thus determine their actions interactively. This is particular useful if the sensor networks consist of different devices such as PDA, laptop, tiny motes, etc., because the energy resources of these devices is not constant, and may change when time elapsed. Hence, the sensors should determine its transmit level not only depending on the strategies of its opponents, but also depending on its own available power resources. BEPow makes the game more realistic for WHSN by considering status transition and strategy interaction between the nodes.

A. NEPow

The NEPow topology control game runs across routing-layer, MAC-layer and physical-layer. The value of transmit power is calculated in routing-layer, based on the connectivity information acquired at routing-layer and SINR information collected at MAC-layer. Then, the value of desirable power level is passed to physical-layer and applied to the outgoing packets. The routing-layer is an extension based on [25] as illustrated in Algorithm 1, it maintains routing daemons at every power level, and then chooses the node’s transmit power based on the utility maximization

$$BR(p_{i-1}) = \arg \max u_j(p_i, p_{i-1}).$$

We assume the minimum necessary node degree $k$ is the magic number 6 for one-hop symmetric neighbors (its rationale was discussed in [53]) to ensure certain level connectivity.

Algorithm 1 Algorithm NEPow for node $i$

Initialization

- $p_i$ is power of node $i$ and has $l$ levels in $[P_{\text{min}}, P_{\text{max}}]$.
- $RT_q$ is the routing table corresponding to the $q$th power level of node $i$.
- $\text{deg}(p_i)$ is the number of neighbors of node $i$ including incoming degree and outgoing degree.
- start a routing daemon for each power level (the $q$th routing daemon builds and maintains routing table $RT_q$).

Setting the power level at routing-layer

- Repeat until termination.
- Update routing table entries which is triggered by receiving data from neighboring nodes or link broken, etc.
- Update $f(p_i)/p_i \cdot \delta_i(p_{i-1})$.
- Solve the best response $p_i$ such that $p_i = \arg \max u_i(p_i, p_{i-1})$.
- Set the transmit power level to $p_i$.
- Set $RT_q$ as the master routing table.
- Wait until timer is expired.

B. BEPow

BEPow is expected to infer different sensor types of the neighboring nodes, and the node determines its decision according to its own state and observed actions. Therefore, the BEPow differs from NEPow that it uses belief-updating sub-algorithm to update beliefs and then plays a repeated game. Assume the sufficient condition in (26) is satisfied, there exists separating equilibrium for the power scheduling game as illustrated in Algorithm 2.

Algorithm 2 Algorithm BEPow for node $i$

Initialization

- Initialize $p_i$, $RT_q$, $\text{deg}(p_i)$, routing daemon, as described in NEPow.
- Set prior belief about the different sensor nodes.

Set the power level at routing-layer

- Repeat until termination.
- Update neighbor information upon receiving data.
- Update $f(p_i)/p_i \cdot \delta_i(p_{i-1})$.
- Update $\text{bed}(i)$ upon observing its opponent’s action, according to (23) and check if conditions are satisfied by inequation (26).
- If satisfied, determine the best response $p_i$ by separating equilibrium in (24), such that $p_i = \arg \max \{u_i(p_i, p_{i-1}), \text{bed}(i)\}$.
- If not satisfied, play NEPow.
- Set the transmit power level to $p_i$.
- Set $RT_q$ as the master routing table.
- Wait until timer is expired.
connectivity requirements. To guarantee connectivity extensively in [53], where the optimal value of an appropriate metric of average node degree has been examined to reduce the reliability of wireless transmission. Note that the nodes. Small node degree will reduce the connectivity and thus force the sensors to increase transmit power to reach distant nodes. This simulation demonstrates that the joint power management and topology control game is able to balance the preference between connectivity payoff and power consumption in a decentralized way. In addition, it increases throughput of wireless communications, as shown in Fig. 5, because the game playing process increases the spatial reuses, while ensuring error-free transmission by iterative power scheduling. For the transient performance of the proposed topology control algorithm, Fig. 6 shows the continuously updated power levels of examined nodes in (a) and the node degree in (b). The proposed power-scheduling algorithm is able to converge very fast and assign proper transmit power levels for the nodes to operate within the predefined minimum-node-degree. The network scale has been given, and $k = 9$ guarantees connectivity for a network with the number of nodes, $N$, ranging from 50 to 500. For a network of uniformly distributed sensor nodes, the network could remain connected with an even smaller $k$ using NEPow.

2) Runtime Performance Analysis: The network performance using proposed NEPow topology control algorithm is evaluated from various perspectives. To examine the runtime performance, we study the transient behaviors of the nodes during the process of game playing to adjust to right transmit power levels. We used the metrics of instant throughput and instant jitter to examine the goodness of data delivery, as shown in Fig. 5. We examined the process of updating power levels in Fig. 6 to see the efficiency and stability of proposed NEPow algorithm. The average transmit-power over all nodes is reduced by 45% in this case, compared with the case without power control. From the resulting patterns, we can see after applying the proposed topology control algorithm, the average node degree is reduced close to the predefined minimum-node-degree and the power consumption is reduced as well.

3) Remarks: This simulation demonstrates that the joint power management and topology control game is able to balance the preference between connectivity payoff and power consumption in a decentralized way. In addition, it increases throughput of wireless communications, as shown in Fig. 5, because the game playing process increases the spatial reuses, while ensuring error-free transmission by iterative power scheduling. For the transient performance of the proposed topology control algorithm, Fig. 6 shows the continuously updated power levels of examined nodes in (a) and the node degree in (b). The proposed power-scheduling algorithm is able to converge very fast and assign proper transmit power levels for the nodes to operate within the predefined minimum-node-degree. The network scale has been given, and $k = 9$ guarantees connectivity for a network with the number of nodes, $N$, ranging from 50 to 500. For a network of uniformly distributed sensor nodes, the network could remain connected with an even smaller $k$ using NEPow.

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achieve desirable connectivity. The system is stable in terms of the commonly used performance metrics including power control signals, connectivity and throughput. In the NPTG, the method is typically termed pricing in game theory and it is an effective scheme to deal with the selfish behaviors. By pricing, each node will consider the cost and social effects during the course of increasing its own utility, which in turn induces implicit cooperation among all the nodes. From the comparisons, we can see NEPow is able to control the topology, maintain reliable connectivity, keep stable transmissions, and significantly reduce the cost of increasing power.

C. Belief Updates of BEPow

The belief update behaviors of BEPow were also simulated using the same setup as the former scenario. We simulated a network with both static and slowly moving sensors.

1) Static Sensor Network: The process of instant belief-updating of sensor nodes using BEPow algorithm is plotted in Fig. 7(a). While observing belief-updating of the follower nodes, such as node 9 in Fig. 7(a), we can see there is some time delay (around 2 or 3 time steps on average) for the follower to update its belief on its opponents. This delay occurs because the follower node must first GET the action of its opponents, and then UPDATE its belief using the joint information of current observed action and last belief about its opponents. If the sensors are static or quasi-static compared with the network scale, the delay of updating belief does not affect the network performances.

2) Mobile Sensor Network at Lower Motion Speed: A simulation scenario with mobile sensors was carried out to study the transient behavior of BEPow algorithm. The mobile node N4 was 30 meters away from static node N3 at first, and N3 believed N4 was Rich since N3 observed N4 had been using higher power levels for long time. At the fifth time step, N4 begun moving close to N3 at the speed of 1 m/s, and at the tenth step, N4 moved away again. N3 updated his belief on N4 being Rich according to its observation, as shown in Fig. 7(b). This simulation shows the belief-updating algorithm is valid for identifying Rich or Poor type of mobile sensors at lower motion speed.

IX. CONCLUSION

This paper examines the topology control problem in heterogeneous sensor networks from game theoretical perspective and proposes a joint power and topology control algorithm to improve system performances. Three desirable characteristics, reliability, connectivity and power efficiency, are considered in designing the power and topology control game. The strategies played by the nodes reflect the trade-off between node preferences including FSR, node degree and power consumption. We proved the existence of both Nash equilibriums and perfect-Bayesian-Nash-equilibriums, and gave the sufficient and necessary conditions to derive the Equilibriums. The decentralized decision-making nature of sensor nodes admits a separating equilibrium by a further stringent game perfection. The proposed game theoretic framework could be extended to other tractable problems in WHSN with elaborate design. Note that the types defined in this paper are not limited to Rich or Poor, and could be various meaningful types depend on different applications. For instance, for joint congestion and power control problems, the types could be congested or not congested. The congestion can be featured with delay or round-trip time. If a node found all existing neighboring nodes are congested, it can increase power to search for more neighboring nodes.

For the future work, we should incorporate node sleep/wakeup schemes for topology control, and further take into account clustering behaviors of wireless heterogeneous sensor networks. We will finally plug the algorithms to real test-bed and study the performances, which is hopeful to achieve valuable experience for practitioners.

APPENDIX

A. Proof of Equation (6)

Proof: Let us begin with one-dimensional case. The $N$ nodes are uniformly distributed with network range $[0, x_m]$. Node $i$ is placed at $x_i$ and thus the transmission range of node $i$ is $[x_i - r_i, x_i + r_i]$. The probability that a node is randomly deployed within $[x_i - r_i, x_i + r_i]$ is $2r_i/x_m$. Hence, the probability that $k$ nodes is randomly deployed within $[x_i - r_i, x_i + r_i]$ is subject to binomial distribution, binomial $(d, n, 2r_i/x_m)$, $\mathbb{P}(d = k) = (n! / k!(n - k)!) (2r_i/x_m)^k (1 - 2r_i/x_m)^{n-k}$.

As $n$ approaches $+\infty$ and $r_i \ll x_m$, the limit of $\mathbb{P}(d = k)$ is expressed as a Poisson distribution, $\lim_{n \to +\infty} \mathbb{P}(d = k) = ((n \cdot 2r_i/x_m)^k / k!) e^{-2r_i/x_m}$.

As the node density is $\lambda = n/x_m$, $n \cdot 2r_i/x_m = 2r_i \lambda$, we have, $\mathbb{P}(d = k) = (2r_i \lambda)^k / k! e^{-2r_i \lambda}$.

The term $2r_i$ represents the range covered by node $i$. Thus, in a two-dimensional case, we replace the line interval with area covered by node $i$, $\pi r_i^2$, and the node density is $\lambda = n/A$. Then, the probability that there are $k$ nodes in the area $\pi r_i^2$, covered by $i$ is $\mathbb{P}(d = k) = (\pi r_i^2 \lambda)^k / k! e^{-\pi r_i^2 \lambda}$.

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