AN EXTENDED SPIKING NEURAL P SYSTEM FOR FUZZY KNOWLEDGE REPRESENTATION

JUN WANG\textsuperscript{1}, LI ZOU\textsuperscript{2}, HONG PENG\textsuperscript{3} AND GEXIANG ZHANG\textsuperscript{4}

\textsuperscript{1}School of Electrical and Information Engineering
Xihua University
Chengdu 610039, P. R. China
wangjun@mail.xhu.edu.cn; ph66@tom.com

\textsuperscript{2}School of Computer and Information Technology
Liaoning Normal University
No. 850, Huanghe Road, Dalian 116029, P. R. China

\textsuperscript{3}School of Mathematics and Computer Engineering
Xihua University
Chengdu 610039, P. R. China

\textsuperscript{4}School of Electrical Engineering
Southwest Jiaotong University
No. 111, Erhaun Road, North Section 1, Chengdu 610031, P. R. China
zhgxdylan@126.com

Received February 2010; revised June 2010

Abstract. In order to extend capability of spiking neural P systems (SN P systems) to represent fuzzy knowledge and further to process fuzzy information, we propose an extended spiking neural P system in this paper, called fuzzy spiking neural P system (FSN P system). In the FSN P system, two types of neurons (fuzzy proposition neuron and fuzzy rule neuron), certain factor and new spiking rule are considered, and content of neuron is fuzzy number instead of natural number (the number of spikes) in SN P systems. Due to graphical nature and advantages of SN P systems, the FSN P system is especially suitable to model fuzzy production rules in a rule-based system. An example is used to illustrate fuzzy reasoning process based on the FSN P system. Due to distributed and parallel computing and dynamical firing characteristics of FSN P system, it can exhibit potential advantages on fuzzy reasoning. In addition, we compare the FSN P system with other methods on fuzzy knowledge representation and fuzzy reasoning.

Keywords: Fuzzy spiking neural P system, Fuzzy number, Fuzzy production rules, Fuzzy reasoning

1. Introduction. Membrane computing (or called P systems) belongs to natural computing. Formally, P systems are constructed by the structure and functioning of living cells, as well as from the way cells are organized in tissues, organs and organisms [1, 2, 3]. Hence, there are three main classes of P systems, i.e., cell-like P systems, tissue-like P systems and neural-like P systems. Recently, spiking neural P systems (SN P systems) belonged to neural-like P systems, which were proposed by Ionescu et al. [4], were widely investigated [5, 6, 7, 8]. SN P systems are incorporated into membrane computing from the way that biological neurons communicate through electrical impulses of identical form (spikes). Intuitively, we have a directed graph where the nodes represent the neurons and the edges represent the synaptic connections among the neurons. The flow of information is carried by the action potentials, which are encoded by spikes that are contained in the neurons and can be sent from presynaptic to postsynaptic neurons according to specific rules. SN P systems possess several attractive advantages for practical applications, such
as distributed and parallel computing model, high understandability, dynamical firing behavior, synchronization, nonlinear, non-deterministic, etc.

Because the knowledge acquired by human is often imprecise, many real-world problems can be easily handled by humans, however, they are too difficult to be handled by machines [9, 10, 11]. In many cases, imprecise knowledge can be expressed by fuzzy production rules, and based on extension principle, we can finish fuzzy reasoning of fuzzy production rules [12, 13, 14]. However, fuzzy production rules for expressing fuzzy knowledge is not straightforward and easily understandable, fuzzy reasoning process of fuzzy production rules usually is very complicated, and fuzzy number computing based on extension principle also is complex. The graphical nature of SN P systems, their distributed and parallel computing ability and neuron’s dynamic firing mechanism are attractive for fuzzy knowledge representation and dynamic fuzzy reasoning. Whether or not SN P systems can be used to express fuzzy knowledge and model dynamic reasoning process? This is an interesting problem, and our work mainly focuses on the problem. However, existing SN P systems and their variants lack ability to process fuzzy information and express fuzzy knowledge. Our motivation is to extend SN P systems so that they possess the ability. Our idea is to introduce some mechanisms into original definition of SN P systems, such as two types of neurons, fuzzy number (as content of neuron), certain factor and new spiking rule. Therefore, we propose an extended SN P system, called fuzzy SN P system (denoted by FSN P system) to express fuzzy production rules in a rule-based system and model its dynamic reasoning process. The advantages of the proposed FSN P system are as follows.

1. Due to directed graph structure of FSN P system, graphical representation of fuzzy production rules in a rule-based system modeled by FSN P system is very easily understandable.
2. FSN P system can model dynamic fuzzy reasoning process of fuzzy production rules in a rule-based system in more intelligent manner due to dynamic firing mechanism of neurons in FSN P system.
3. Since SN P system holds distributed and parallel computing ability, fuzzy reasoning process based on FSN P system has potential advantage of parallel fuzzy reasoning.

The paper is organized as follows. In Section 2, some concepts of fuzzy production rules are briefly reviewed and we recall SN P systems. In Section 3, we present a FSN P system for expressing fuzzy knowledge. In Section 4, FSN P system is used to model fuzzy production rules in a rule-based system and its fuzzy reasoning process. In Section 5, we compare FSN P system with other knowledge representation and fuzzy reasoning methods. Section 6 provides an illustrating example for fuzzy reasoning based on FSN P system. Conclusions are drawn in Section 7.

2. Preliminaries. In this section, we briefly review fuzzy production rules, which will be involved in this paper. We can refer to [9, 12] for more details. And then, we discuss SN P systems in standard form and in computing version. A more detailed description of SN P systems can be found in [4, 5, 6].

2.1. Fuzzy production rules. In many cases, information cannot be assessed precisely in a quantitative form due to their uncertainty. For processing uncertain information, fuzzy set theory proposed by Zadeh [15] is an important tool. In real-world practice, we can construct fuzzy production rules of a knowledge-based system based on fuzzy set, e.g., fuzzy control rules of a complex control system. Formally, denote all fuzzy production rules as \( R = \{ R_1, R_2, \ldots, R_n \} \), and their a fuzzy production rule \( R_i \) has form

\[
R_i : \text{IF } p_j \text{ THEN } p_k
\]
in which, \( p_j \) and \( p_k \) are two fuzzy propositions. From the logical point of view, \( R_i \) can be represented by \( p_j \rightarrow p_k \), and its truth values may be one of \( TV = \{ \text{“absolutely true”, “almost certain”, “very true”, “pretty true”, “likely”, “rather true”, “sort of true”, “more or less true”, “a little true”, “hardly true”, “almost impossible”, “absolutely false”} \}, \) such as “rather true”. The fuzzy propositions mentioned above as well as truth values can be explained by fuzzy sets of the universe of discourse \( U \) (sometime called fuzzy numbers), i.e., if \( U = \{x_1, x_2, \ldots, x_n\} \) is discrete, then a fuzzy set \( A \) on \( U \) is

\[
\{\mu_A(x_1)/x_1, \mu_A(x_2)/x_2, \ldots, \mu_A(x_n)/x_n\},
\]

where, for any \( x_i \in U \), \( \mu_A(x_i) \) is membership degree of \( x_i \) included by \( A \). If \( U \) is continues, then the membership function of a fuzzy set \( A \) on \( U \) is \( \mu_A : U \rightarrow [0,1] \). Figures 1 and 2 show some membership functions of fuzzy numbers in \( TV \), respectively, in which, \( U = [0,1] \).

In fuzzy set theory, the notion of \( \alpha \)-cut \( A_\alpha \) of a fuzzy set \( A \) can help us to understand relationship between fuzzy set and classical set. Formally, \( \alpha \)-cut \( A_\alpha \) of a fuzzy set \( A \) is defined by: for \( 0 \leq \alpha \leq 1 \),

\[
A_\alpha = \{x \mid \mu_A(x) \geq \alpha, x \in U\}.
\]

Figure 3 shows a \( \alpha \)-cut \( A_\alpha \) of a fuzzy set \( A \). Based on \( \alpha \)-cut \( A_\alpha \) of a fuzzy set \( A \), \( A \) can be rewritten by

\[
A = \int_0^1 \alpha A_\alpha.
\]

A fuzzy set \( A \) is called a convex fuzzy set if there exits unique \( x \in U \) such that \( \mu_A(x) = 1 \). For any convex fuzzy set \( A \), \( \alpha \)-cut \( A_\alpha \) has the form \( A_\alpha = [a_1^\alpha, a_2^\alpha] \). Based on
(1), the “and” and “or” operations of two fuzzy numbers $A$ and $B$ are as following:

$$A \otimes B = \int_0^1 \alpha[a_1^\alpha \land b_1^\alpha, a_2^\alpha \land b_2^\alpha] , \quad A \oplus B = \int_0^1 \alpha[a_1^\alpha \lor b_1^\alpha, a_2^\alpha \lor b_2^\alpha]$$

where symbols “$\otimes$” and “$\oplus$” are the “and” and “or” operators, respectively; “$\land$” and “$\lor$” are the general “min” and “max” operators, respectively.

2.2. SN P systems. Formally, a SN P system of degree $m \geq 1$, is a construct of the form \cite{4}:

$$\Pi = (O, Q, \text{syn}, \text{in}, \text{out})$$

where,

1) $O = \{a\}$ is the singleton alphabet (the object $a$ is called spike);

2) For any $\sigma_i \in Q = \{\sigma_i| i = 1, 2, \ldots, m\}$, neuron $\sigma_i$ has the form $\sigma_i = (n_i, r_i)$, $i \in \{1, 2, \ldots, m\}$, where

   (i) $n_i \geq 0$ is the initial number of spikes contained by the neuron $\sigma_i$;

   (ii) $r_i$ is a finite set of rules of the form $E/a^c \rightarrow a^p; d$, where $E$ is a regular expression over $a$, $c \geq 1$ and $p \geq 0$, with $c \geq p$; if $p = 0$, then $d = 0$, too.

3) $\text{syn} \subseteq \{1, 2, \ldots, m\} \times \{1, 2, \ldots, m\}$ with $(i, i) \notin \text{syn}$ for $i \in \{1, \ldots, m\}$ (synapses).

4) $\text{in}$ and $\text{out}$ are input and output neurons, respectively.

In above SN P systems, the rule of the form $E/a^c \rightarrow a^p; d$ is called firing rule or spiking rule. Neuron’s firing mechanism can be explained as follows. If a neuron $\sigma_i$ contains $k$ spikes, $a^k \in L(E)$ (regular language) and $k \geq c$, then its a rule $E/a^c \rightarrow a^p; d \in r_i$ can be applied; applying the rule means that $c$ spikes are consumed, thus only $k - c$ spikes remain in the neuron $\sigma_i$; the neuron $\sigma_i$ is fired and it produces $p$ spikes after $d$ time units. The parameter $d$ in firing rule reflects time delay mechanism of SN P systems, which is from biological neuron, i.e., the length of axon may cause a time delay before a spike reaches its target. When a neuron fires, if $d = 0$, then the spikes produced by it are emitted immediately; otherwise, if $d = 1$, then the spikes are emitted in the next step. In the case $d \geq 1$, if its firing rule is used in step $t$, then in steps $t, t + 1, \ldots, t + d - 1$ the neuron is closed, and it cannot receive new spikes (if a neuron has a synapse to a closed neuron and sends spikes along it, then the spikes are lost). In the step $t + d$, the neuron spikes and becomes again open, hence it can receive spikes. The $p$ spikes emitted by a neuron $\sigma_i$ are replicated and they go to all neurons $\sigma_j$ such that $(i, j) \in \text{syn}$ (each $\sigma_j$ receives $p$ spikes). If an enabled rule has $d = 0$ (the rule is called a forgetting rule), then no spike is emitted and the neuron cannot be closed.

SN P systems operate in a nondeterministic and maximally parallel manner using a global clock. However, each neuron is only able to fire at most one rule per step since the rule must cover all the spikes currently in the neuron. It is possible that two (or more) rules are applicable in a given step. In this case, the applied rule is selected.
non-deterministically. The system operates in a maximally parallel manner at each step in that all neurons that are fireable must fire (applying some rules).

The initial configuration of the system is described by the numbers \( n_1, n_2, \ldots, n_m \) of spikes existing in the counterpart neurons, with all neurons being open. During the computation, a configuration is described by both the number of spikes in each neuron and the state of the neuron; more precisely, by the number of steps to count down until it becomes open (this number is zero if the neuron is already open). Using the rules above, we can define transitions from one configuration to another configuration. A transition between two configurations \( C_1 \) and \( C_2 \) is denoted by \( C_1 \Rightarrow C_2 \). A computation halts if it reaches a configuration where all neurons are open and no rule can be used. The sequence of transitions from the initial to final configurations is called a computation.

3. **FSN P System.** In this section, we present an extended spiking neural P system, called fuzzy spiking neural P system (FSN P system), to represent fuzzy production rules of a rule-based system and model its fuzzy reasoning process. In FSN P system, we consider two types of neurons (fuzzy proposition neuron and fuzzy rule neuron) and certain factor, and content of neuron is fuzzy numbers instead of natural number (the number of spikes).

Formally, a FSN P system of degree \( m \geq 1 \), is a construct of the form

\[
\Pi = (O, P, R, Q, \text{syn, in, out})
\]

where

1) \( O = \{a\} \) is the singleton alphabet (the object \( a \) is called spike);
2) \( P = \{p_1, p_2, \ldots, p_k\} \) is a finite set of fuzzy propositions, where \( p_i \) represents \( i \)th fuzzy proposition, \( 1 \leq i \leq k \);
3) \( R = \{R_1, R_2, \ldots, R_n\} \) is a finite set of fuzzy production rules of a rule-based system, where \( R_j \) represents \( j \)th fuzzy rule, \( 1 \leq j \leq n \);
4) \( Q \) is a finite set of neurons, \( Q = \{\sigma_1, \sigma_2, \ldots, \sigma_m\} \), in which, \( Q = Q_1 \cup Q_2, Q_1 \cap Q_2 = \emptyset \)

\( Q_1 = \{\sigma_1, \sigma_2, \ldots, \sigma_k\} \) is a set of fuzzy proposition neurons, i.e., every neuron \( \sigma_i \) in \( Q_1 \) corresponds to a fuzzy proposition \( p_i \) in \( P \), \( 1 \leq i \leq k \). \( Q_2 = \{\sigma_{k+1}, \sigma_{k+2}, \ldots, \sigma_{k+n}\} \) is a set of fuzzy rule neurons, i.e., every neuron \( \sigma_{k+j} \) in \( Q_2 \) corresponds to a fuzzy production rule \( R_j \) in \( R \), \( 1 \leq j \leq n \), \( m = k + n \). In fuzzy rule neurons \( Q_2 \), the operations “and” or “or” will be performed. Each neuron has the form of \( \sigma_i = (A_i, C_i, r_i), 1 \leq i \leq m \), where
   (i) \( A_i \), a fuzzy number, represents potential value contained in neuron \( \sigma_i \).
   (ii) In rule neuron \( \sigma_{k+j}, C_i \), a fuzzy number, represents certain factor of the corresponding fuzzy production rules \( R_i \), \( 1 \leq i \leq n \). In proposition neuron \( \sigma_i, C_i \) is ignored, \( 1 \leq i \leq k \).
   (iii) \( r_i \) represents a firing rule of the neuron \( \sigma_i \) (also known as a spiking rule) with the form \( a^\mu \rightarrow a^\nu \), where \( \mu \) and \( \nu \) are fuzzy numbers. For any proposition neuron, \( \mu \) and \( \nu \) are true value \( A_i \) of the proposition that the neuron corresponds to. For any rule neuron, \( \mu \) is logical “and” (denoted by \( \otimes \) ) or “or” (denoted by \( \oplus \)) of the inputs (fuzzy numbers) received by the neuron, and \( \nu = \mu \otimes C_i \).
5) \( \text{syn} \subseteq \{1, 2, \ldots, m\} \times \{1, 2, \ldots, m\} \) with \( i \neq j \) for all \( (i, j) \in \text{syn} \) for \( 1 \leq i, j \leq m \) (synapses between neurons);
6) \( \text{in}, \text{out} \subseteq Q_1 \) are the input and output neuron sets, respectively.

Since FSN P system inherits majority of mechanisms of SN P systems, it also possesses several excellent features as follows: (i) a directed graph structure, (ii) a distributed and parallel computing model, (iii) synchronized, (iv) non-deterministic, (v) neuron’s dynamic firing behavior, etc. However, compared with SN P systems, FSN P system has
some differences as follows. (i) FSN P system consists of two type neurons, proposition neuron and rule neuron. A proposition neuron corresponds to a proposition in knowledge base of a rule-based system, while a rule neuron corresponds to a fuzzy production rule of the rule-based system. (ii) The content of each neuron is a fuzzy number (that represents its potential value). (iii) Each rule neuron is assigned a certain factor, which is a fuzzy number and actually is certain factor of a fuzzy production rule associated with the rule neuron. (iv) New type of firing rule $a^\mu \rightarrow a^\nu$ and new firing mechanisms are employed. (v) Time delay mechanism is omitted, i.e., each firing rule has $d = 0$.

In FSN P system, its new firing mechanism can be described as follow. When a neuron contains a spike with $A_i > 0$, its rule $a^\mu \rightarrow a^\nu$ is fired. For a proposition neuron, $\mu = \nu = A_i$, while for a rule neuron, $\mu = A_i$ and $\nu = \mu \otimes C_i$. Thus, after firing, the potential value $\mu$ of the spikes contained in the neuron is consumed and a spike with the value $\nu$ is emitted and transmitted along its synapses. It is noticed that time delay is not considered for the firing rule (because the time delay is not necessary for knowledge representation). So, the spike is immediately emitted after the rule is excited.

From above analysis, we can see that the presented FSN P system not only inherits numerous advantages of SN P systems, but also holds the ability to process fuzzy information and express fuzzy knowledge. Specially, due to dynamic firing and spiking mechanisms of its neurons, we will see that FSN P system can more intelligently model dynamic fuzzy reasoning process of fuzzy production rules in a rule-based system.

4. Representation of Fuzzy Production Rules Using FSN P System. In this section, FSN P system will be used to model fuzzy production rules in a rule-based system. A simple type of fuzzy production rules can be modeled by a FSN P system, shown in Figure 4. Moreover, Figures 4(a) and 4(b) represent its fuzzy reasoning process. Generally speaking, a simple type of fuzzy production rules usually has the following form:

$$R_i: \text{IF } p_j, \text{ THEN } p_k (\text{CF } = B).$$

In the model of Figure 4, proposition neuron $\sigma_j$ corresponds to proposition $p_j$ of antecedent part in the fuzzy production rule $R_i$, which contains (initial potential) a fuzzy number $A$ (i.e., truth value of proposition $p_j$). Neuron $\sigma_k$ also is a proposition neuron, which corresponds to proposition $p_k$ of consequent part in the fuzzy production rule $R_i$. The neuron $\sigma_i$ is a rule neuron, which corresponds to the rule $R_i$, where $C$ is certain factor of the rule $R_i$ and denoted by a fuzzy number. When neuron $\sigma_j$ is excited, its potential value $A$ is consumed and immediately sent to neuron $\sigma_i$ (where, $\mu = \nu = A$). Then, after receiving the potential value $A$, neuron $\sigma_i$ fires its spiking rules. After the rule is performed, its potential value $\mu(= A)$ is consumed. At the same time, a potential value $\nu(= \mu \otimes C)$ is sent to neuron $\sigma_k$. Finally, neuron $\sigma_k$ receives a potential value $B = A \otimes C$, shown in Figure 4(b). If neuron $\sigma_k$ is an output neuron, it will fire its rule to export its potential value $B$. Suppose $A_\alpha$ and $C_\alpha$ are $\alpha$-cuts of fuzzy numbers $A$ and $C$. 

![Figure 4](image.png)

Figure 4. The simple fuzzy production rule modeled by FSN P system
First type of composite fuzzy production rules is
Type 1 \( R_i \): IF \( p_1 \) and \( p_2 \) and \( \ldots \) and \( p_k \) \((\text{CF} = C)\).

In Type 2, composite rule modeled by using FSN P system is shown in Figure 7(a). Figure 7(b) is its graphical form. Assume truth value of proposition \( p_1 \) is fuzzy number \( C \), respectively, \( A_\alpha = [a_1^\alpha, a_2^\alpha], C_\alpha = [c_1^\alpha, c_2^\alpha], \alpha \in [0, 1] \), then truth value of proposition neuron \( \sigma_k \) is \( B = A \otimes C \), namely,

\[
A \otimes C = \int_0^1 \alpha [a_1^\alpha \wedge c_1^\alpha, a_2^\alpha \wedge c_2^\alpha] \tag{3}
\]

In knowledge base of a rule-based system, in addition to simple fuzzy production rules mentioned above, composite fuzzy production rules are frequently used to express fuzzy knowledge. The antecedent or consequent part of a composite fuzzy production rule contains “and” and “or” connectors. Typically, there are four types of composite fuzzy production rules. Next, we will use FSN P system to model the four types of composite fuzzy production rules. First type of composite fuzzy production rules is

Type 1 \( R_i \): IF \( p_1 \) and \( p_2 \) and \( \ldots \) and \( p_k \) \((\text{CF} = C)\).

The composite rule modeled by a FSN P system is shown in Figure 5(a). Figure 5(b) is its graphical form, where the neuron with \( \otimes \) will perform logical “and” operation. Assume truth values of propositions \( p_1, p_2, \ldots, p_k \) are fuzzy numbers \( A_1, A_2, \ldots, A_k \) respectively, where \( [a_{i1}^\alpha, a_{i2}^\alpha] \) is \( \alpha \)-cut of fuzzy number \( A_i \), \( 1 \leq i \leq k-1, \alpha \in [0, 1] \). The certain factor of the rule \( R_i \) is fuzzy number \( C \), where \( [c_1^\alpha, c_2^\alpha] \) is \( \alpha \)-cut of fuzzy number \( C, \alpha \in [0, 1] \). The reasoning process of the fuzzy production rule modeled by using FSN P system is shown in Figure 6. The computation process is as follows. The proposition neurons \( \sigma_1, \sigma_2, \ldots, \sigma_{k-1} \) are fired by their spiking rules, and then they send their potential values \( A_1, A_2, \ldots, A_{k-1} \) to the rule neuron “\( \otimes \)”. After the rule neuron receives all the inputs, it performs logical “\( \otimes \)” operation \( (\mu = A_1 \otimes A_2 \otimes \ldots \otimes A_{k-1}) \), and then it fires its spiking rule and sends potential value \( \nu (= \mu \otimes C) \) to neuron \( \sigma_k \). Therefore, through dynamic firing of proposition neurons and rule neurons in the FSN P system, truth value of proposition \( p_k \) is \( A_k = A_1 \otimes A_2 \otimes \ldots \otimes A_{k-1} \otimes C \), i.e.,

\[
A_k = \int_0^1 \alpha [a_{11}^\alpha \wedge a_{21}^\alpha \wedge \ldots \wedge a_{(k-1)1}^\alpha \wedge c_1^\alpha, a_{12}^\alpha \wedge a_{22}^\alpha \wedge \ldots \wedge a_{(k-1)2}^\alpha \wedge c_2^\alpha] \tag{4}
\]

Second type of composite fuzzy production rules is
Type 2 \( R_i \): IF \( p_1 \) THEN \( p_2 \) and \( p_3 \) and \( \ldots \) and \( p_k \) \((\text{CF} = C)\).

In Type 2, composite rule modeled by using FSN P system is shown in Figure 7(a). Figure 7(b) is its graphical form. Assume truth value of proposition \( p_1 \) is fuzzy number...
Figure 6. Fuzzy reasoning process of Type 1 of composite fuzzy production rules using FSN P system.

Figure 7. Type 2 of composite production rules.

Third type of composite fuzzy production rules is

\[ R_i: \text{IF } p_1 \text{ or } p_2 \text{ or } \ldots \text{ or } p_{k-1} \text{ THEN } p_k \ (\text{CF} = C). \]

The composite rule can be modeled by using FSN P system, shown in Figure 9(a). Figure 9(b) is its graphical form, where the neuron with “\(\oplus\)” performs logical “or” operation. Assume truth values of propositions \(p_1, p_2, \ldots, p_{k-1}\) are fuzzy numbers \(A_1, A_2, \ldots, A_{k-1}\), respectively, where \([a_{1i}^\alpha, a_{2i}^\alpha]\) is \(\alpha\)-cut of fuzzy number \(A_i\), \(1 \leq i \leq k-1\), \(\alpha \in [0,1]\). The fuzzy reasoning process of the fuzzy rule can be modeled by FSN P system.
system, shown in Figure 10. The proposition neurons $\sigma_1$, $\sigma_2$, $\ldots$, $\sigma_{k-1}$ are fired by their spiking rules respectively, and then they send potential values $A_1$, $A_2$, $\ldots$, $A_{k-1}$ to a rule neuron. Subsequently, the rule neuron carries out logical “$\oplus$” operation on all receiving inputs ($\mu = A_1 \oplus A_2 \oplus \ldots \oplus A_{k-1}$), and then it fires its spiking rule and sends a potential value $\nu (= \mu \otimes C)$ to neuron $\sigma_i$. As a result, truth value of proposition $p_k$ is $A_k = (A_1 \oplus A_2 \oplus \ldots \oplus A_{k-1}) \otimes C$, that is

$$A_k = \int_0^1 \alpha \left[ (a_{11}^\alpha \lor a_{21}^\alpha \lor \ldots \lor a_{(k-1)1}^\alpha) \land c_1^\alpha, (a_{12}^\alpha \lor a_{22}^\alpha \lor \ldots \lor a_{(k-1)2}^\alpha) \land c_2^\alpha \right]$$

Fourth type of composite fuzzy production rules is

Type 4 $R_i$: IF $p_1$ THEN $p_2$ or $p_3$ or $\ldots$ or $p_k$ ($CF = C$).

For decision-making control, this composite fuzzy production rule is useless because it has no practical significance. Therefore, such a fuzzy composite fuzzy production rule is unnecessary in fuzzy knowledge representation.

5. **An Illustrating Example for Fuzzy Reasoning.** In this section, we give an example to illustrate that FSN P system is suitable for expressing fuzzy production rules in
A rule-based system and can accomplish their fuzzy reasoning process in more intelligent manner.

Suppose $p_1, p_2, p_3, p_4, p_5, p_6$ and $p_7$ are seven propositions in knowledge base of a rule-based system. The knowledge base consists of seven fuzzy production rules as follows.

1. If $p_1$ THEN $p_2$ (CF = almost certain)
2. If $p_1$ THEN $p_3$ (CF = almost certain)
3. If $p_1$ THEN $p_4$ (CF = pretty true)
4. If $p_2$ THEN $p_5$ (CF = pretty true)
5. If $p_2$ THEN $p_6$ (CF = rather true)
6. If $p_3$ and $p_4$ THEN $p_6$ (CF = sort of true)
7. If $p_4$ THEN $p_6$ and $p_7$ (CF = very true).

We employ FSN P system to model above fuzzy production rules, shown in Figure 11. In Figure 11, there are seven proposition neurons ($\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6$ and $\sigma_7$) and seven rule neurons ($\sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}$ and $\sigma_{14}$). On each proposition neuron $\sigma_i$, we label the “$p_i$”, which indicates proposition $p_i$ associated with neuron $\sigma_i$. The words labeled on each rule neuron $\sigma_j$ indicate certain factor of the corresponding fuzzy production rule (denoted by a fuzzy number), for example, “almost certain” on neuron $\sigma_8$ indicates that certain factor of fuzzy production rule $R_1$ associated by neuron $\sigma_8$ is “almost certain”.

The FSN P system has only an input neuron $\sigma_1$ and three output neurons ($\sigma_1, \sigma_2$ and $\sigma_3$).

In the following, we describe dynamic fuzzy reasoning process based on FSN P system. We denote system configuration by two vectors $v_p = (v_{p1}, v_{p2}, v_{p3}, v_{p4}, v_{p5}, v_{p6}, v_{p7})$ and $v_r = (v_{r1}, v_{r2}, v_{r3}, v_{r4}, v_{r5}, v_{r6}, v_{r7})$, where $v_{pi}$ ($i = 1, 2, \ldots, 7$) expresses the value of spike (neuron’s state) contained in proposition neuron $\sigma_i$, and $v_{rj}$ ($i = 1, 2, \ldots, 7$) expresses the value of spike (neuron’s state) contained in rule neuron $\sigma_{j+7}$. We introduce a state value “unknown”: if a neuron contains no spike, its content (or state) is unknown. Suppose initial input of the system is “very true” that is imported into neuron $\sigma_1$. The dynamic reasoning process can be described as follows.

1. When $t = 1$, neuron $\sigma_1$ receives input value “very true”. System configuration is as follows: (see Figure 11):
   
   $v_p(1) = (\text{very true, unknown, unknown, unknown, unknown, unknown, unknown, unknown})$
   $v_r(1) = (\text{unknown, unknown, unknown, unknown, unknown, unknown, unknown, unknown})$
(2) When \( t = 2 \), neuron \( \sigma_1 \) fires and produces a spike with value “very true”, and then transports the spike into rule neurons \( \sigma_8, \sigma_9 \) and \( \sigma_{10} \). Now, system configuration is updated as follows: (see Figure 12):
\[
\nu_p(2) = (\text{unknown, unknown, unknown, unknown, unknown, unknown, unknown})
\]
\[
\nu_r(2) = (\text{very true, very true, very true, unknown, unknown, unknown, unknown})
\]

(3) When \( t = 3 \), rule neurons \( \sigma_8, \sigma_9 \) and \( \sigma_{10} \) synchronously fire and produce a spike with values “very true” \( \otimes \) “almost certain” = “very true”, “very true” \( \otimes \) “almost certain” = “very true”, “very true” \( \otimes \) “pretty true” = “pretty true”, respectively. And then, they transport a spike into proposition neurons \( \sigma_2, \sigma_3 \) and \( \sigma_4 \), respectively. Now, system configuration is updated as follows: (see Figure 13):
\[
\nu_p(3) = (\text{unknown, very true, very true, pretty true, unknown, unknown, unknown})
\]
\[
\nu_r(3) = (\text{unknown, unknown, unknown, unknown, unknown, unknown, unknown})
\]

(4) When \( t = 4 \), proposition neurons \( \sigma_2, \sigma_3 \) and \( \sigma_4 \) synchronously fire. Neuron \( \sigma_2 \) produces a spike with value “very true”, and then transports the spike into rule neuron \( \sigma_{13} \). Neuron \( \sigma_3 \) produces a spike with value “very true”, and then transports the spike into rule neuron \( \sigma_{14} \). Since rule neuron \( \sigma_{13} \) receives a spike from neurons \( \sigma_2 \) and \( \sigma_3 \) respectively, the content of neuron \( \sigma_{13} \) is “very true” \( \otimes \) “pretty true” = “pretty true”. Now, system configuration is updated as follows: (see Figure 14):
\[
\nu_p(4) = (\text{unknown, unknown, unknown, unknown, unknown, unknown, unknown})
\]
\[
\nu_r(4) = (\text{unknown, unknown, unknown, very true, very true, pretty true, pretty true})
\]

(5) When \( t = 5 \), rule neurons \( \sigma_{11}, \sigma_{12} \) and \( \sigma_{13} \) synchronously fire. Neuron \( \sigma_{11} \) produces a spike with value “very true” \( \otimes \) “pretty true” = “pretty true”, and then transports the spike into proposition neuron \( \sigma_8 \). Neuron \( \sigma_{12} \) produces a spike with value “very true” \( \otimes \) “rather true” = “rather true”, and then transports the spike into proposition neuron \( \sigma_6 \). Neuron \( \sigma_{13} \) produces a spike with value “pretty true” \( \otimes \) “sort of true” = “sort of true”, and then transports the spike into proposition neurons \( \sigma_6 \). Likewise, neuron \( \sigma_{14} \) produces a spike with value “pretty true” \( \otimes \) “very true” = “pretty true”, and then transports the spike into proposition neurons \( \sigma_7 \). Since proposition neuron \( \sigma_8 \) receives a spike from rule neurons \( \sigma_{12}, \sigma_{13} \) and \( \sigma_{14} \) respectively, the content of neuron \( \sigma_8 \) is “rather true” \( \oplus \) “sort of true” \( \oplus \) “pretty true” = “pretty true”. Now, system configuration is updated as follows: (see Figure 15):
\[
\nu_p(5) = (\text{unknown, unknown, unknown, unknown, pretty true, pretty true, pretty true})
\]
\[
\nu_r(5) = (\text{unknown, unknown, unknown, unknown, unknown, unknown, unknown})
\]

**Figure 11.** The FSN P system model for representing fuzzy production rules and first step of its fuzzy reasoning process \((t = 1)\)
When \( t = 6 \), proposition neurons \( \sigma_5, \sigma_6 \) and \( \sigma_7 \) synchronously fire. Neuron \( \sigma_5 \) produces a spike with value "pretty true" and exports into environment (i.e., exports truth value "pretty true"). Neuron \( \sigma_6 \) produces a spike with value "pretty true" and exports into environment (i.e., exports truth value "pretty true"). Likewise, Neuron \( \sigma_7 \) produces a spike with value "pretty true" and exports into environment (i.e., exports truth value "pretty true"). Now, system configuration reaches following halt configuration: (see Figure 16):
Figure 15. Fifth step of fuzzy reasoning process \((t = 5)\)

\[
\begin{align*}
\nu_p(5) &= (\text{unknown, unknown, unknown, unknown, pretty true, pretty true, pretty true}) \\
\nu_r(5) &= (\text{unknown, unknown, unknown, unknown, unknown, unknown, unknown})
\end{align*}
\]

Figure 16. Sixth step of fuzzy reasoning process \((t = 6)\)

\[
\begin{align*}
\nu_p(6) &= (\text{unknown, unknown, unknown, unknown, unknown, unknown, unknown}) \\
\nu_r(6) &= (\text{unknown, unknown, unknown, unknown, unknown, unknown, unknown})
\end{align*}
\]

Since system configuration already reaches halt configuration, the system halts. Consequently, reasoning results are “pretty true” (for proposition \(p_5\)), “pretty true” (for proposition \(p_6\)) and “pretty true” (for proposition \(p_7\)).

From above fuzzy reasoning process, we can see that due to dynamic firing and spiking mechanisms of neurons in FSN P system, dynamic fuzzy reasoning can work in more intelligent manner. In addition, since each neuron is an individual component in FSN P system and FSN P system has parallel computing ability, fuzzy reasoning based on FSN P system possesses parallel reasoning ability.

6. Compared with Other Methods. Fuzzy production rule is widely used to express fuzzy and uncertain knowledge in some practical areas, such as process control, expert system, fault diagnosing, etc. The fuzzy production rule often is denoted as a fuzzy IF-THEN rule form, where antecedent part and consequent part are the concepts expressed by fuzzy sets. When we use fuzzy production rules to represent fuzzy knowledge base of a rule-based system, there exits such a problem that fuzzy knowledge represented by them has not simple and understandable structure so that we can easily understand the expressed contents. However, it is well known that for fuzzy knowledge representation problem, many practical areas require the knowledge representation methods that have high understandability, such as expert system, fault diagnosing, etc. Therefore, due to its directed graph structure and other mechanisms, FSN P system is especially suitable
for expressing fuzzy knowledge and the expressed model has simple and high understand-
ability.

Fuzzy reasoning of fuzzy production rules is a mechanism to deduce a consequent based
on observations that do not exactly match with antecedent part of the rule. This kind
of reasoning is also known as approximate reasoning and it play a major role in human
reasoning because in many situations human beings have to make decisions based on
uncomplete and fuzzy information. There are mainly two types of fuzzy reasoning meth-
ods. One is reasoning methods based on Zadeh’s compositional rule of inference method
[16, 17], while other is similarity-based reasoning methods [18]. So far, many fuzzy rea-
soning methods have been presented [17, 19, 20, 21, 22]. Generally, fuzzy reasoning of
fuzzy production rules usually is a very complicated process. However, FSN P system is
a distributed and parallel computing model, and the firing and spiking mechanisms of its
neurons can make dynamic fuzzy reasoning process modeled by it work in more intelligent
manner. Moreover, the fuzzy reasoning process holds potential parallel computing abil-
ity. It is noticed that although SN P systems have parallel computing ability, we cannot
realize the ability on current computer, so we think the fuzzy reasoning based on FSN P
system is potentially parallel.

7. Conclusions. As a novel distributed and parallel computing model, SN P systems
possess many attractive advantages. Currently, how to extend the application areas of
SN P systems has attracted attention of researchers. Since existing SN P systems and
the variants mainly focus on some computing theory problems such as computing effec-
tiveness and computing ability, their definition form and mechanism do not directly be
used in many practical problems. Due to the graphical nature, parallel computing ability
and dynamic firing mechanism of SN P systems, our work focuses on fuzzy information
processing and fuzzy knowledge representation by using SN P systems. In this paper,
we present an extended SN P system by introducing some new mechanisms into original
definition of SN P systems, called fuzzy spiking neural P system (FSN P system). The
researches demonstrate that FSN P system is especially suitable to express fuzzy produc-
tion rules and finish their dynamic fuzzy reasoning process in more intelligent manner.
In addition, we emphasize the advantages of FSN P system by an illustrating example
and comparison analysis between FSN P system and other methods (fuzzy knowledge
representation methods and fuzzy reasoning methods).

In this paper, we discuss FSN P system only limited to model fuzzy production rules
of a rule-based system by using FSN P system. In order to deal with fuzzy and uncertain
knowledge and solve the problem that propositions in antecedent of a rule-based system
have different importance, weighted fuzzy production rule has been developed. Based on
FSN P system, how to propose a parameterized FSN P system to model weighted fuzzy
production rule is our further work. Since SN P systems are a distributed and parallel
computing model, how to incorporate the features of SN P system (specially, its dynamic
firing mechanism) to develop a parallel reasoning algorithm is our another further work.
In addition, we know that since one of main mechanisms of SN P systems is from impulse
neurons, this provides a way to develop learning mechanism of FSN P system by use of
references in the research fruits of artificial neural network. Our further work will focuses
on this problem.

Acknowledgment. This work was partially supported by the Open Research Fund of
Key Laboratory of Sichuan Signal and Information Processing, Xihua University (SZJJ200
9-016), Research Fund of Sichuan Provincial Key Discipline of Power Electronics and
Electric Drive, Xihua University (SZD0503-09-0), Foundation of Sichuan Provincial Key
Discipline of Computer Software and Theory, and Research Fund of Key Laboratory of
Xihua University (No. XDZ0818-09). Talent Cultivation Project of Xihua University
(R0920906). The authors also gratefully acknowledge the helpful comments and sugges-
tions of the reviewers, which have improved the presentation.

REFERENCES