the lateral resolution comparable to the longitudinal sharpness in the respective images (14μm in Fig. 2a and 7μm in Fig. 2b). The unprocessed image in Fig. 2a was obtained at a location on the sample close to that displayed in the deconvolved image. The images were captured by sampling 1024 vertical and 200 horizontal pixels, and were averaged 10 times. Each image was acquired in ~5 s.

In conclusion, we describe a systems theory model which serves as the basis for image enhancement in OCT using deconvolution. Its preliminary application illustrates iterative deconvolution of OCT images leading to increased image sharpness, with potential applications in the imaging of ophthalmic and other near transparent bio-structures containing strong, well separated echoes.

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Parallel relaxation algorithm for disparity computation

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Indexing terms: Image processing, Stereo image processing

A new energy function is introduced for the binocular disparity computation. The energy function uses the phase-magnitude of the image to detect the shift for a pair of images, and satisfies major natural constraints and requirements for implementing parallel relaxation. The multiscale concept is applied and the mean field approximation is used for obtaining a compact representation.

Introduction: The final goal of binocular disparity computation is a reconstruction of a 3D world using only two images taken from two slightly differently positioned cameras.

Our approach to this problem is based on the MRF hypothesis of the image attributes such as image intensity and phase. By this hypothesis, we mathematically define an optimal matching such as a MAP (maximum a posteriori) estimate, although other estimation measures are possible. Then we suggest a computation architecture that minimises the energy function.

Multi-layer image structure: Consider a pyramid built from an N x N image plane S. The planes Si (i ∈ [0, log N)) are derived successively from the lower plane by reducing its size by one half. For each layer i, the disparity vector d is defined by d = (d, x, y) ∈ Si and d ∈ Di where Di is the range space of the disparity vector.

A set of pixels in S must be mapped into a site in S. We call the set of pixels a block and denote it by B (i.e., 0, 1, ..., log N). In this manner, a site in S can be associated to pixels in the original image plane S by the block B. Each cell in S has a block of (2i) cells in S.

With this method, we can determine d using all the phase values in the original plane S. Assuming that all the values inside a block B correspond to the central frequency of an associated bandpass filter, we find the solution in Di. This value is now used as the initial condition for Di+1, and similar operations are performed until i = 0; the final solution resides in D0.

MAP estimate: We assume that the disparity field is a real MRF within the space S, which consists of lattices B. We further introduce a line process L = (L1, L2) to the discontinuities in the horizontal and vertical directions of the disparity map. In particular, our concern is the MAP estimate of the disparity map:

\[ P(d(l, h, v)|\phi_R, \phi^R) = \frac{\pi(d(l, h, v)|\phi_R, \phi^R)}{P(\phi_R|\phi^R)} \]

where \( \phi_R \) and \( \phi^R \) stand for phase values of the left and right images obtained on the lattice S using a filter of centre frequency \( u_0 \).

The phase is limited to [−π, π] with distance 1/\( u_0 \) since a signal filtered with a lowpass filter of centre frequency \( u_0 \) consists of sinusoidal functions of frequency \( u_0 \). Therefore, corresponding points can be found when the disparity is smaller than 1/2\( u_0 \). For this region, hierarchical pyramids of input images or filtered images were used in [3]. In this Letter, we used the method of reducing the state space of the disparity vector corresponding to the filter centre frequencies.

Phase matching: P(\( \phi_R|\phi^R \), d, p, f) in eqn. 1 denotes the matching error for the two phases at a given site of the left image. If we assume that this matching error is Gaussian with variance \( \sigma_R \), then

\[ P(\phi_R|\phi^R, d, p, f) \approx \frac{1}{Z_1} \exp \left( - \sum_{(x,y) \in S_0} \sum_{i \in B} (1-q^f) \phi_i^f \left( \phi_i^R-d_i^l \right)^2 \right) \]

where \( Z_1 \) is a normalisation constant. The subscript \( v \) denotes a lattice point S, and \( d_i^l \) and \( q^f \) represent indicators of singular
points [2] for each left and right image, respectively. This indicator is set to 1 at the singleton point and 0 elsewhere. The operator $\bigotimes$ stands for logical OR, $B$ is a block in $S$ and $d(B)$ is the disparity value in $B$.

Assuming that the disparity and line processes are MRFs and considering that $\phi^d$ and $(d, p, f)$ are mutually independent, we obtain the Gibbs distribution:

$$P(d, p, f|\phi^R) = \frac{1}{Z} \exp \left\{ -\mu[(d_{x,y} - d_{x,y-1})^2(1 - l_{x,y}^h) + (d_{x,y} - d_{x-1,y})^2(1 - l_{x,y}^v)] + \gamma(l_{x,y}^h + l_{x,y}^v) \right\} \quad (3)$$

As a result, the energy will be minimal when the line processes are turned on at the actual discontinuous sites and when the disparity functional is smooth on the object surface.

Substituting eqns. 2 and eqn. 1 becomes

$$P(d, p, f|\phi^R) = \frac{1}{Z} \exp \left\{ \sum_{(x,y)\in S} \sum_{z\in B_x} \frac{1}{2\sigma^2}[(1 - q_z^R)\phi^R_z - \phi^R_x]^2 - \mu \sum_{(x,y)\in S} ((d_{x,y} - d_{x,y-1})^2(1 - l_{x,y}^h) + (d_{x,y} - d_{x-1,y})^2(1 - l_{x,y}^v)] - \gamma \sum_{(x,y)\in S} (l_{x,y}^h + l_{x,y}^v) \right\} \quad (4)$$

**Approximation of energy function:** Since the stochastic relaxation method takes too much time for computation, we will use a deterministic method. To achieve this goal, we must further simplify the energy equation by the mean field approximation [1].

Since the line process can be replaced by the disparity difference, the energy function can be represented solely by a function of $d$. Using the mean field approximation, we can define a new pdf $P^m(d_x)$ which consists of $d$ at $(x, y)$ and mean values at other positions. Thus, we can neglect $d_{x,y}^{u,v}$ when computing the mean $(d_x)$ of $d_{x,y}$,

$$P^m(d_{x,y}) = \frac{1}{Z^m} \exp[-U^m(d_{x,y})] \quad (5)$$

$$U^m(d_{x,y}) = \frac{1}{2\sigma^2} \sum_{z\in B_x} (1 - q_z^R)\phi^R_z - \phi^R_x)^2 - \mu \sum_{(x,y)\in S} ((d_{x,y} - d_{x,y-1})^2(1 - l_{x,y}^h) + (d_{x,y} - d_{x-1,y})^2(1 - l_{x,y}^v)) - \gamma \sum_{(x,y)\in S} (l_{x,y}^h + l_{x,y}^v) \quad (6)$$

where $N_{x,y}$ is the first-order neighbourhood at $(x, y)$ and $U^m(d_x)$ is the energy function we seek.

**Realisation:** A suboptimal value of $d_x$ can be obtained by

$$\frac{\partial U^m(d_{x,y})}{\partial d_{x,y}} \bigg|_{d_{x,y}=(d_x)} = 0 \quad (7)$$

The optimal solution $(d_x)$ must satisfy eqn. 7, but a closed form is impossible. Instead, we are looking for a solution using the gradient of the energy function. To proceed further, we define the auxiliary function $l_{x,y}(x - 1, y) \triangleq 1/(1+e^{\sigma (d_{x,y} - d_{x,y-1})^2})$. Putting the auxiliary function into the left term of eqn. 7, we have

$$\frac{\partial U^m(d_{x,y})}{\partial d_{x,y}} \bigg|_{d_{x,y}=(d_x)} = -\frac{1}{\sigma^2} \sum_{z\in B_x} (1 - q_z^R)\phi^R_z - \phi^R_x)^2 \frac{\partial \phi^R_x}{\partial \phi} + \sum_{(x,m)\in N_{x,y}} 2\mu(d_{x,y} - d_{x,m})(1 - l_{x,y}(k,m)) \quad (8)$$

Since the mean values are part of the calculation, an iterative procedure will be suitable for a real implementation.

**Parallel relaxation:** For finding a $(d_x)$ which satisfies eqn. 7, a gradient descent method is used: $(d_x)^{n+1} = (d_x)^n - \tau \frac{\partial U^m(d_x)}{\partial d_x}$, where $\tau$ is a parameter governing the convergence speed. Finally we obtain

$$\begin{align*}
\langle d_{x,y} \rangle^{n+1} &= \langle d_{x,y} \rangle^n \\
+ \alpha \sum_{z \in B_x} (1 - q_z^R - \phi^R_z)^2 \frac{\partial \phi^R_x}{\partial \phi} + \sum_{(x,m)\in N_{x,y}} 2\mu(d_{x,y} - d_{x,m})(1 - l_{x,y}(k,m)) \quad (9)
\end{align*}$$

where $\alpha = \pi \sigma^2$ and $\lambda = 2\alpha \mu$. For a given level, this computation can be performed in parallel.

**Experimental result:** For testing with a real scene, the Pentagon image pair is used. The image size is $512 \times 512$ and the filter scales are $\sigma = 16$, 8 and 4. The final disparity map is obtained using eqn. 9. Fig. 1 shows the Pentagon image pair and the resulting disparity map. For this test, we used parameter values of $\lambda = 2.0$, $\mu = 1.0$ and $\gamma = 7.0$ which were heuristically chosen.

**Fig. 1 512 $\times$ 512 Pentagon image pair and resulting disparity map**

**Conclusion:** In this Letter, we introduced a new multi-layer architecture for matching a stereo image. As an optimal criteria, we derived a new energy function, based upon the MAP estimate, that contains natural constraints for the disparity computation. The result is a multilayer relaxation algorithm with $O(N\log N)$ multiplications. This algorithm is relatively fast, and robust to noise. Some open problems are the enhancement of computational complexity, the reduction of recognition errors, and the automatic updating of the internal parameters.

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**References**


**Capacity of fading channel with no side information**

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**Indexing terms:** Fading, Channel capacity

A supremum to the Shannon channel capacity of a Rayleigh fading channel without side information (SI) is computed using variational methods and is shown to be asymptotically achievable for signal-to-noise ratios (SNRs) approaching either zero or infinity. A lower bound is obtained from the mutual information corresponding to a specific discrete input distribution.